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## A Simple and Efficient Method for Computing Data Cubes

Viet Phan-Luong  
 Université Aix-Marseille  
 LIF - UMR CNRS 6166  
 Marseille, France  
 Email: viet.phanluong@lif.univ-mrs.fr

**Abstract**—Based on a construction of the power set of the fact table schemes, this paper presents an approach to represent and to compute data cubes using a prefix tree structure for the storage of cuboids. Though the approach is simple, the experimental results show that it is efficient in run time and storage space.

*Keywords*-Data warehouse; Data cube; Data mining.

### I. INTRODUCTION

In data warehouse, a data cube of a fact table with  $n$  dimensions and  $m$  measures can be seen as the result of the set of the Structured Query Language (SQL) group-by queries over the power set of dimensions, with aggregate functions over the measures. The result of each group-by query is an aggregate view, called a cuboid, of the fact table. The concept of data cube represents important interests to business decision as it provides aggregate views of measures over multiple combinations of dimensions. As the number of cuboids in a data cube is exponential to the number of dimensions of the fact table, when the fact table is big, computing a cuboid is critical and computing all cuboids of a data cube is exponentially cost in time [1][5]. To improve the response time, the data cube is usually precomputed and stored on disks [6]. However, storing all the data cube is exponential in space. Research in Online Analytical Processing (OLAP) focuses important effort for efficient methods of computation and representation of data cubes.

#### A. Related work

There are approaches that represent data cubes approximately or partially [9][10][16][18]. The other approaches search to represent the entire data cube with efficient methods to compute and to store the cube [2][7][21]. The computing time and storage space can be minimized by reducing redundancies between tuples in cuboids [20] or based on equivalence relations defined on aggregate functions [11][15] or on the concept of closed itemsets in frequent itemset mining [14] or by coalescing the storage of tuples in cuboids [19].

In the approaches to efficiently compute and store the cube, the computation is usually organized over the complete lattice of subschemes of the fact table dimension scheme, in

such a way the run time and the storage space can be optimized by reducing redundancies [2][8][11][12][13][15][17]. The computation can traverse the complete lattice in a top-down or bottom-up manner. [7][16]. For grouping tuples to create cuboids, the sort operation can be used to reorganize tuples: tuples are grouped over the prefix of their scheme and the aggregate functions are applied over the measures. By grouping tuples, the fact table can be horizontally partitioned, each partition can be fixed in memory, and the cube computation can be modularized.

To optimize the cuboid construction, in top-down methods [7][20], the cuboids over the subschemes on a path from the top to the bottom in the complete lattice can be built in only one lecture of the fact table sorted over the largest scheme of the path. An aggregate filter is used for each subscheme. The filter contains, at each time, only one tuple over the subscheme with the current value of aggregated measures (or a non aggregated mark). When reading a new tuple of the fact table, if over the subscheme, the new tuple has the same value as the filter, then only the value of the aggregated measures is updated. Otherwise, the current content of the filter is flushed out to disk, and before the new tuple passes into the filter, the subtuple over the next subscheme (next on the path from the top to the bottom) goes into the next subscheme filter.

To minimize the storage space of a cuboid, only aggregated subtuples with aggregated measures are directly stored on disk. Non-aggregated subtuples are not stored but represented by references to the (sub)tuples where the non aggregated tuples are originated.

The bottom-up methods [11][13][15][20] walk the paths from the bottom to the top in the complete lattice, beginning with the empty node (corresponding to the cuboid with no dimension). For each path, let  $T_0$  be the scheme at the bottom node and  $T_n$  the scheme at of the top node of the path (not necessary the bottom and the top of the lattice, as each node is visited only once). These methods begin by sorting the fact table over  $T_0$  and by this, the fact table is partitioned into groups over  $T_0$ . To minimize storage space, for each one of these groups, the following depth-first recursive process is applied [20].

If the group is single, then the only element of the group is represented by a reference to the corresponding tuple in

the fact table, and there is no further process: the recursive cuboid construction is pruned.

Otherwise, an aggregated tuple is created in the cuboid over  $T_0$  and the group is sorted over the next scheme  $T_1$  in the path (with larger scheme) to be partitioned into subgroups. The creation of a real tuple or a reference in the cuboid corresponding to each subgroup over  $T_1$  is similar to what we have done when building the cuboid over  $T_0$ .

When the recursive process is pruned at a node  $T_i, 0 \leq i \leq n$ , or reaches to  $T_n$ , it resumes with the next group of the partition over  $T_0$ , until all groups of the partition are processed. The construction resumes with the next path, until all paths of the complete lattice are processed, and all cuboids are built.

Note that in the above optimized bottom-up method, in all cuboids, if references exist, they refer directly to tuples in the fact table, not to tuples in other cuboids. This method, named Totally-Redundant-Segment BottomUpCube (TRS-BUC), is reported in [20] as a method that dominates or nearly dominates its competitors in all aspects of the data cube problem: fast computation of a fully materialized cube in compressed form, incrementally updateable, and quick query response time.

### B. Contribution

This paper presents a simple and efficient approach to compute and to represent data cube without sorting the fact table or any part of it, neither partitioning the fact table, nor computing the complete lattice of subschemes, nor sophisticated techniques to implement direct or indirect references of tuples in cuboids. The efficient representation of data cube is not only a compact representation of all cuboids of the data cube, but also an efficient method to get the original cuboids from the compact representation. The main ideas of the proposed approach are:

1) Among the cuboids of a data cube, there are ones that can be easily and rapidly get from the others, with no important computing time. We call the latter the prime and next-prime cuboids. These cuboids will be computed and stored on disks.

2) The prime and next-prime cuboids are computed and stored on disk using a prefix tree structure for compact representation. To improve the efficiency of research through the prefix tree, this work integrates the binary search tree into the prefix tree.

3) To compute the prime and next-prime cuboids, this work proposes a running scheme in which the computation of the current cuboids can be speeded up by using the previously computed cuboids.

The paper is organized as follows. Section 2 introduces the concept of the prime and next-prime schemes and cuboids. Section 3 presents the structure of the integrated binary search prefix tree used to store cuboids. Section 4 presents the running scheme to compute the prime and next-prime

cuboids and shows how to efficiently get any other cuboids from the prime and next-prime cuboids. Section 5 reports the experimentation results. Finally, conclusion and further work are in Section 6.

## II. PRIME AND NEXT-PRIME CUBOIDS

This section defines the main concepts of the present approach to compute and to represent data cubes.

### A. A structure of the power set

A data cube over a scheme  $S$  is the set of cuboids built over all subsets of  $S$ , that is the power set of  $S$ . As in most of existing work, attributes are encoded in integer, let us consider  $S = \{1, 2, \dots, n\}$ ,  $n \geq 1$ . The power set of  $S$  can be recursively defined as follows.

1) The power set of  $S_0 = \emptyset$  (the empty set) is  $P_0 = \{\emptyset\}$ .

2) For  $n \geq 1$ , the power set of  $S_n = \{1, 2, \dots, n\}$  can be defined recursively as follows:

$$P_n = P_{n-1} \cup \{X \cup \{n\} \mid X \in P_{n-1}\} \quad (1)$$

$P_n$  is the union of  $P_{n-1}$  (the power set of  $S_{n-1}$ ) and the set of which each element is built by adding  $n$  to each element of  $P_{n-1}$ . Let us call  $P_{n-1}$  the *first-half power set* of  $S_n$  and the second operand the *last-half power set* of  $S_n$ .

**Example:** For  $n = 3$ ,  $S_3 = \{1, 2, 3\}$ , we have:

$$P_0 = \{\emptyset\}, \quad P_1 = \{\emptyset, \{1\}\}, \quad P_2 = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\},$$

$$P_3 = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{3\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}.$$

The last-half power set of  $S_3$  is:

$$\{\{3\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}.$$

### B. Last-half data cube and first-half data cube

Consider a fact table  $R$  (a relational data table) over a dimension scheme  $S_n = \{1, 2, \dots, n\}$ . In view of the first-half and the last-half power set, suppose that  $X = x_1, \dots, x_i \subseteq S_n$  is an element of the first-half power set of  $S_n$ . Let  $Y$  be the smallest element of the last-half power set of  $S_n$  that contains  $X$ . Then,  $Y = X \cup \{n\}$ . If the cuboid over  $Y$  is already computed in the attribute order  $x_1, \dots, x_i, n$ , then the cuboid over  $X = x_1, \dots, x_i$  can be done by a simple sequential reading of the cuboid over  $Y$  to get data for the cuboid over  $X$ . So, we call:

– A scheme in the last-half power set a *prime scheme* and a cuboid over a prime scheme a *prime cuboid*. Note that all prime schemes contain the last attribute  $n$  and any scheme that contains attribute  $n$  is a prime scheme.

– For efficient computing, the prime cuboids are computed by pairs with one dataset access for each pair. Such a pair is composed of two prime cuboids. The scheme of the first one has attribute 1 and the scheme of the second one is obtained from the scheme of the first one by deleting attribute 1. We call the second prime cuboid the next-prime cuboid.

– The set of all cuboids over the prime (or next-prime) schemes is called the last-half data cube. The set of all remaining cuboids is called the first-half data cube. In this

TABLE I. FACT TABLE R1

RowId	A	B	C	D	M
1	a2	b1	c2	d2	m1
2	a3	b2	c2	d2	m2
3	a1	b1	c1	d1	m1
4	a1	b1	c2	d1	m3
5	a3	b3	c2	d3	m2

approach, the last-half data cube is computed and stored on disks. Cuboids in the first-half data cube are computed as queries based on the last-half data cube.

### III. INTEGRATED BINARY SEARCH PREFIX TREE

The prefix tree structure offers a compact storage for tuples: the common prefix of tuples is stored once. So, there is no redundancy in storage. Despite the compact structure of the prefix tree, if the same prefix has a large set of different suffixes, then the search time in the set of suffixes can be important. To tackle it, this work proposes to integrate the binary search tree into the prefix structure. The integrated structure, called the *binary search prefix tree (BSPT)*, is used to store tuples of cuboids. With this structure, tuples with the same prefix are stored as follows:

- The prefix is stored once.
- The suffixes of those tuples are organized in siblings and stored in a binary search tree.

Precisely, in C language, the structure is defined by :

```
typedef struct bsptree Bsptree; // Binary search prefix tree
struct bsptree{
    Elt data; // data at a node
    LtId *ltid; // list of RowIds
    Bsptree *son, *lsib, *rsib; };
```

where *son*, *lsib*, and *rsib* represent respectively the son, the left and the right siblings of nodes. The field *ltid* is reserved for the list of tuple identifiers (*RowId*) associated with nodes. For efficient memory use, *ltid* is stored only at the last node of each path in the BSPT. With this representation, each binary search tree contains all siblings of a node in the normal prefix tree. For example, we have:

– Table I represents the fact table R1 over the dimension scheme *ABCD* and a measure *M*.

– Figure 1 represents the BSPT of the tuples over the scheme *ABCD* of the fact table R1, where we suppose that with the same letter *x*, if  $i < j$  then  $x_i < x_j$ , e.g.,  $a_1 < a_2 < a_3$ . In Figure1, the continue lines represent the son links and the dash lines represent the *lsib* or *rsib* links.

The BSPT is saved to disk with the following format:

*level > suffix : ltids*

where

- level is the length of the prefix part that the path has in common with its left neighbor,
- suffix is a list of elements, and
- ltids is a list of tuple identifiers (*RowId*).

Cuboids are built using the BSPT structure. The list of *RowIds* associated with the last node of each path allows

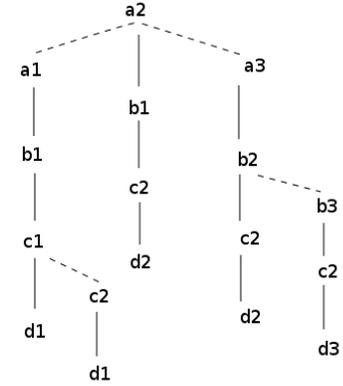


Figure 1. A binary search prefix tree

the aggregate of measures. For example, with the fact table in Table I, the cuboid over *ABCD* is saved on disk as the following.

```
0 > a1 b1 c1 d1 : 3
2 > c2 d1 : 4
0 > a2 b1 c2 d2 : 1
0 > a3 b2 c2 d2 : 2
1 > b3 c2 d3 : 5
```

#### A. Insertion of tuples in a BSPT

**Algorithm Tuple2Ptree:** Insert a tuple into a BSPT.

**Input:** A BSPT represented by node *P*, a tuple *ldata* and its list of tids *lti*.

**Output:** The tree *P* updated with *ldata* and *lti*.

**Method:**

If (*P* is null) then

create *P* with  $P \rightarrow data = head(ldata)$ ,

$P \rightarrow son = P \rightarrow lsib = P \rightarrow rsib = NULL$ ;

if  $queue(ldata)$  is null then  $P \rightarrow ltid = lti$

else  $P \rightarrow son = Tuple2Ptree(P \rightarrow son, queue(ldata), lti)$ ;

Else if ( $P \rightarrow data > head(ldata)$ ) then

$P \rightarrow lsib = Tuple2Ptree(P \rightarrow lsib, ldata, lti)$ ;

else if ( $P \rightarrow data < head(ldata)$ ) then

$P \rightarrow rsib = Tuple2Ptree(P \rightarrow rsib, ldata, lti)$ ;

else if  $queue(ldata)$  is null then

$P \rightarrow ltid = insert(P \rightarrow ltid, lti)$ ;

else  $P \rightarrow son = Tuple2Ptree(P \rightarrow son, queue(ldata), lti)$ ;

return *P*;

In algorithm *Tuple2Ptree*,  $head(ldata)$  returns the first element of *ldata* and  $queue(ldata)$  returns the queue of *ldata* after removing  $head(ldata)$ .

#### B. Grouping tuples using binary prefix tree

**Algorithm Table2Ptree:** Build a BSPT for a relational table.

**Input:** A table *R* in which each tuple has a list of tids *lti*.

**Output:** The BSPT *P* for *R*

**Method:**

Create an empty BSPT  $P$ ;  
 For each tuple  $ldata$  in  $R$  with its list of tids  $lti$  do  
 $P = \text{Tuple2Ptree}(P, ldata, lti)$  done;  
 Return  $P$ ;

#### IV. COMPUTING THE LAST-HALF DATA CUBE

Let  $S = \{1, 2, \dots, n\}$  be the set of all dimensions of the fact table. To compute the last-half data cube:

– We begin by computing the first prime and next-prime cuboids based on the fact table, one over  $S$  and the other over  $S - \{1\}$ .

– Apart the first prime and next-prime cuboids (over  $S$  and  $S - \{1\}$ , respectively), for the current prime scheme  $X$  of size  $k$  (the number of all dimensions in  $X$ ), the computation of the prime and next-prime cuboids over  $X$  and  $X - \{1\}$ , respectively, is based on a previously computed prime cuboid with the smallest scheme that contains  $X$ .

– To keep track of the computation, we keep the schemes of all computed prime cuboids in a list called the running scheme and denoted by RS. So,  $X$  is appended to RS ( $S$  is the first element added to RS). To build the  $RS$ , for the currently pointed scheme  $X$  in RS, for each dimension  $j \in X, j \neq 1$  and  $j \neq n$  ( $n$  is the last dimension of the fact table), we append  $X - \{j\}$  to RS, if  $X - \{j\}$  is not yet there.

More precisely, for computing the last-half data cube, we use algorithm *LastHalfCube*.

#### Algorithm LastHalfCube

**Input:** A fact table  $R$  over scheme  $S$  of  $n$  dimensions.

**Output:** The last-half data cube of  $R$  and the running scheme RS.

**Method:**

- 0)  $RS = \text{emptyset}$ ; // RS: Running Scheme
- 1) Append  $S$  to the RS;
- 2) Using *Table2Tree* and  $R$  to generate two cuboids over  $S$  and  $S - \{1\}$ , respectively;
- 3) Set  $cS$  to the first scheme in RS; //  $cS$ : current scheme
- 4) While  $cS$  has more than 2 attributes do
- 5) For each dimension  $d$  in  $cS, d \neq 1$  and  $d \neq n$ , do
- 6) Build a subscheme  $scS$  by deleting  $d$  from  $cS$ ;
- 7) If  $scS$  is not yet in RS then append  $scS$  to RS, let  $cubo$  be the cuboid over  $cS$  (already computed);  
 If  $cubo$  is not yet in memory then load it in memory;
- 8) Using *Tuple2Ptree* and  $cubo$  to generate two cuboids over  $scS$  and  $scS - \{1\}$ , respectively;
- 9) done;
- 10) Set  $cS$  to the next scheme in RS;
- 11) done;
- 12) Return RS;

#### A. Example of running scheme

Table II shows the simplified execution of *LastHalfCube* on a table  $R$  over  $S = \{1, 2, 3, 4, 5\}$ : only the prime and the

TABLE II. GENERATION OF THE RUNNING SCHEME OVER  $S = \{1, 2, 3, 4, 5\}$ .

Prime RS	NPrime	Prime RS	NPrime	Prime RS	NPrime
12345	2345				
1 345	345				
		1 45	45		
		1 35	35	15	5
12 45	2 45			15x	
		1 45x			
		1 25	25		
123 5	23 5			15x	
		1 35x			
		1 25x			

next-prime (NPrime) schemes of the cuboids computed by the algorithm are reported. The prime schemes appended to RS (Running Scheme) during the execution of *LastHalfCube* are in the columns named Prime/RS of Table II. The first prime schemes are in the first column Prime/RS, the next ones are in the second column Prime/RS, and the final ones are in the third column Prime/RS. The final state of RS is  $\{12345, 1345, 1245, 1235, 145, 135, 125, 15\}$ . In Table II the schemes marked with x (e.g., 145x) are those already added to RS and are not re-appended to RS.

For a fact table  $R$  over a scheme  $S$  of  $n$  dimensions,  $S = \{1, 2, \dots, n\}$ , algorithm *LastHalfCube* generates RS with  $2^{n-2}$  subschemes. Indeed, we can see that all subschemes appended to  $RS$  have 1 as the first attribute and  $n$  as the last attribute. So, we can forget 1 and  $n$  from all those subschemes. By this, we can consider that the first subscheme added to RS is  $2, \dots, n-1$ . Over  $2, \dots, n-1$ , we have only one subscheme of size  $n-2$  ( $C_{n-2}^{n-2}$ ). In the loop For at point 5 of *LastHalfCube*, alternatively each attribute from 2 to  $n-1$  is deleted to generate a subscheme of size  $n-3$ . By doing this, we can consider as, in each iteration, we build a subscheme over  $n-3$  different attributes selected among  $n-2$  attributes. So, we build  $C_{n-2}^{n-3}$  subschemes. So on, until the subscheme  $\{1, n\}$  (corresponding to the empty scheme after forgetting 1 and  $n$ ) is added to RS. As

$$C_{n-2}^{n-2} + C_{n-2}^{n-3} + \dots + C_{n-2}^0 = 2^{n-2}$$

By adding the corresponding next-prime schemes, *LastHalfCube* generates  $2^{n-1}$  different subschemes. Thus, algorithm *LastHalfCube* computes  $2^{n-1}$  prime and next-prime cuboids.

#### B. Data Cube representation

For a fact table  $R$  over a dimension scheme  $S = \{1, 2, \dots, n\}$  with measures  $M_1, \dots, M_k$ , the data cube of  $R$  is represented by the three following elements:

1) The running scheme (RS): The list of the prime schemes over  $S$ . Each prime scheme has an identifier

number that allows to locate the files corresponding to the prime and next-prime cuboids in the last-half data cube.

2) The last-half data cube of which the cuboids are precomputed and stored on disks using the format to store the BSPT.

3) A relational table over  $RowId, M_1, \dots, M_k$  that represents the measures associated with each tuple of  $R$ .

Clearly, such a representation reduces about 50% space of the entire data cube, as it represents the last-half data cube in the BSPT format.

### C. Computing the first-half data cube

Let  $X$  be a scheme in the first-half power set of  $S = \{1, 2, \dots, n\}$ . For computing the cuboid over  $X$ , we base on the precomputed last-half data cube over  $S$ . The computation is processed as follows, where  $lti(t)$  denotes the list of tids of a tuple  $t$  and  $p(t)$  the prefix of  $t$  over  $X$ .

```

Let  $C$  be the stored cuboid over  $X \cup \{n\}$ ;
Let  $t1$  be the 1st tuple of  $C$  and  $ltids = lti(t1)$  ;
For each next tuple  $t2$  of  $C$  do
    If the  $p(t2) = p(t1)$  then append  $lti(t2)$  to  $ltids$ ,
    Else {
        Write  $p(t1)$  :  $ltids$  to the cuboid over  $X$ ;
         $t1 = t2$ ;  $ltids = lti(t1)$ ;
    }
Done;
```

## V. EXPERIMENTAL RESULTS

The present approach to represent and to compute data cubes is implemented in C and experimented on a laptop with 8 GB memory, Intel Core i5-3320 CPU @ 2.60 GHz x 4, 188 Go Disk, running Ubuntu 12.04 LTS. To get some ideas about the efficiency of the present approach, we recall here the experimental results reported in [20] as references, because the work [20] has experimented many existing and well known methods for computing and representing data cube as Partitioned-Cube (PC), Partially-Redundant-Segment-PC (PRS-PC), Partially-Redundant-Tuple-PC (PRT-PC), BottomUpCube (BUC), Bottom-Up-Base-Single-Tuple (BU-BST), and Totally-Redundant-Segment BottomUpCube (TRS-BUC). The experiments in [20] were run on a Pentium 4 (2.8 GHz) PC with 512 MB memory under Windows XP. The results were reported on real and synthetic datasets. In the present work, we limit our attention to only the real datasets: CovType [3] and SEP85L5 [4]. However, by reporting the results of [20], we do not want to really compare the present approach to TRS-BUC or others, as we do not have sufficient conditions to implement and to run these methods on the same system and machine.

CovType is a dataset of forest cover-types. It has ten dimensions and 581,012 tuples. The dimensions and their cardinality are: Horizontal-Distance-To-Fire-Points (5,827), Horizontal-Distance-To-Roadways (5,785),

TABLE III. EXPERIMENTAL RESULTS REPORTED IN [20]

CovType			
Algorithms	Storage space	Construction time	avg QRT
PC	#12.5 Gb	1900 sec	
PRT-PC	#7.2 Gb	1400 sec	
PRS-PC	#2.2 Gb	1200 sec	3.5 sec
BUC	#12.5 Gb	2900 sec	2 sec
BU-BST	#2.3 Gb	350 sec	
BU-BST+	#1.2 Gb	400 sec	1.3 sec
TRS-BUC	#0.4 Gb	300 sec	0.7 sec
SEP85L			
Algorithms	Storage space	Construction time	avg QRT
PC	#5.1 Gb	1300 sec	
PRT-PC	#3.3 Gb	1150 sec	
PRS-PC	#1.4 Gb	1100 sec	1.9 sec
BUC	#5.1 Gb	1600 sec	1.1 sec
BU-BST	#3.6 Gb	1200 sec	
BU-BST+	#2.1 Gb	1300 sec	0.98 sec
TRS-BUC	#1.2 Gb	1150 sec	0.5 sec

Elevation (1,978), Vertical-Distance-To-Hydrology (700), Horizontal-Distance-To-Hydrology (551), Aspect (361), Hillshade-3pm (255), Hillshade-9am (207), Hillshade-Noon (185), and Slope (67).

SEP85L is a weather dataset. It has nine dimensions and 1,015,367 tuples. The dimensions and their cardinality are: Station-Id (7,037), Longitude (352), Solar-Altitude (179), Latitude (152), Present-Weather (101), Day (30), Weather-Change-Code (10), Hour (8), and Brightness (2).

For greater efficiency, in the experiments of [20], the dimensions of the datasets are arranged in the decreasing order of the attribute domain cardinality. The same arrangement is done in the our experiments. Moreover, as most algorithms studied in [20] compute condensed cuboids, computing query in data cube needs additional cost. So, the results are reported in two parts: computing the condensed data cube and querying data cube. The former is reported with the construction time and storage space and the latter the average query response time.

Table III presents the experimental results approximately got from the graphs in [20], where “avg QRT” denotes the average query response time and “Construction time” denotes the time to construct the (condensed) data cube. However, [20] did not specify whether the construction time includes the time to read/write data to files.

Table IV reports the results of the present work, where the term “run time” means the time from the start of the program to the time the last-half (or respectively, the first-half) data cube is completely constructed, including the time to read/write input/output files.

As we do not compute the condensed cuboids, but only compute the last-half data cube and use it to represent the data cube, we can consider that the last-half data cube corresponds somehow to the (condensed) representations of data cube in the other approaches, and computing the first-half data cube corresponds to querying data cube. In this view,

TABLE IV. EXPERIMENTAL RESULTS OF THIS WORK

CovType			
	Storage space	Run time	avg QRT
Last-Half Cube	7 Gb	992 sec	
First-Half Cube	6,2 Gb	439 sec	
Data Cube	13,2 Gb	1431 sec	0.43 sec
SEP85L			
	Storage space	Run time	avg QRT
Last-Half Cube	3.6 Gb	691 sec	
First-Half Cube	3.3 Gb	243 sec	
Data Cube	6.9 Gb	934 sec	0.47 sec

the average query response time corresponds to the average run time for computing a cuboid based on the precomputed and stored cuboids. That is, the average query response time for SEP85L is  $243s/512 = 0.47$  second and for CovType  $439s/1024 = 0.43$  second, because the cuboids in the last-half data cube are precomputed and stored, only querying on the first-half data cube needs computing. Though the compactness of the data cube representation by the present approach is not comparable to the compactness offered by TRS-BUC, it is in the range of other existing methods. It is similar for the run time to build the last-half data cube of CovType. However, the run time to build the entire (not only the last-half) data cube of SEP85L seems to be better than all other existing methods. On the average query response time, it seems that the present approach offers a competitive solution, because querying data cube is a repetitive operation and improving the average query response time is one of the important goals of research on data cube.

VI. CONCLUSION, REMARKS AND FURTHER WORK

Essentially, this work represents a data cube by the last-half data cube: the set of cuboids over schemes that contain the last dimension of the fact table, called prime (or next-prime) cuboids. All other cuboids, those over schemes that do not contain the last dimension, are obtained by a simple projection of the corresponding cuboids in the last-half data cube. The binary search prefix tree (BSPT) structure is used to store cuboids in memory and on disk. Such a structure offers not only a compact representation of cuboids but also an efficient search of tuples. Building a cuboid in the last-half data cube is reduced to building a BSPT. Building a cuboid in the first-half data cube is reduced to copying the prefixes of the BSPT of the corresponding cuboid in the last-half data cube. The BSPT allows efficient group-by operation without previous sort operation on tuples in the fact table or in cuboids. With this advantage, we can think of the possibility of incremental construction of the last-half data cube and the possibility of updating the data cube when inserting new tuples in the fact table.

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