Controlled Triangular Batches Petri Nets for hybrid mesoscopic modeling of traffic road networks under VSL control

Radhia Gaddouri, Leonardo Brenner, Isabel Demongodin

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***Abstract*— In the discrete event and hybrid systems theory, Batches Petri Nets (BPN) have been defined as an extension of Hybrid Petri nets for representing in a single node a linear relation between flow and density. This formalism and its extensions allow us to delay flows according to accumulation phenomena. Applied to mesoscopic modeling of traffic road networks, Triangular Batches Petri Nets (TBPN) have been defined as a triangular flow-density relation, allowing the representation of traffic congestion/decongestion phenomena. When variable speed limit (VSL) control is applied on the traffic road networks, i.e., a method that improves the traffic conditions by the reduction of congestion, Controlled Triangular Batches Petri Nets (CTBPN) can be used for analyzing such phenomena. In this paper, we present the hybrid formalism defined in CTBPN, i.e., the hybrid behavior of a batch in free, congestion or decongestion behavior. As an illustrative example, the congestion/decongestion phenomena on a road section and the impact of a VSL control are shown.**

I. INTRODUCTION

Nowadays the traffic demand is often greater than traffic capacity of the roads and, when it occurs, a phenomenon of traffic congestion appears. To reduce the traffic congestion without expand or build new roads, the optimization of the traffic flow becomes an important study issues in transportation systems. The most common optimization strategies are Ramp Metering Regulation (RMR) and Variable Speed Limit (VSL) [11] [12]. RMR strategy controls the upstream flow reallocating the vehicles into other parts of network where the demand is lower. VSL strategy consists in the variation of the speed limits to change the maximum capacity of the road in order to remove the congestion (reducing speed) or to reduce the decongestion time (increasing speed). VSL is computed based on different factors like traffic flow, density, and speed and it must follow the traffic flow-density (also called fundamental diagram) model. The relation traffic flow-density is studied in a macroscopic level (hydrodynamic theory) [2]. We propose in this paper to model the congestion/decongestion phenomena caused by flow-density model in a mesoscopic level (group of vehicles) using a hybrid model based on discrete events and to detail the behavior of a group of vehicles when a VSL is implied.

Among the formalisms that consists of discrete events and hybrid models, continuous and hybrid Petri nets [4] are well adapted to the modeling and analysis of performance and control of flow systems. Batches Petri Nets (BPN) [6] extend the hybrid Petri nets class by defining a new type of node, the batch node, and the concept of controllable batch, i.e., a group of entities (vehicles) moving through a transfer zone at its transfer speed. These Petri nets, by their hybrid dynamics formalization, allow transfer elements to be represented at a mesoscopic level with possibility of accumulation (or congestion) of entities (vehicles). In the BPN formalism, the dynamics of batches inside a batch place is governed by a flow-density relation representing a switching between free and accumulation behaviors. Gaddouri et al. [9] have extended and enriched these formalisms by a more general flow-density relation, i.e., a triangular form that represents in a very detailed manner the fundamental diagram of the traffic road domain [3], [1]. The batch place is extended to a *Triangular Batch place* (TB-place) defined by four continuous characteristics: a maximum speed, a maximum density, a length and a *maximum flow*.

More specifically, this paper extends the hybrid dynamics of batches inside a triangular batch place previously defined in [8], by defining controlled events, such as the variation of maximum speeds of TB-places or the modification of maximum flows associated to continuous/batch transitions. This new semantic can then be located within the context of the VSL strategy for controlling congestion in freeways or highways. The hybrid dynamics is also presented by three continuous behaviors: free, congestion and decongestion.

This paper is organized as follows. In Section 2, concepts associated with Controlled Triangular Batch Petri Nets are introduced. We propose in Section 3, new continuous-time and discrete event dynamics of controllable batches under variation of maximum speed of TB-places and according to assumptions imposed by the triangular relation of flow-density. Section 4 presents an example illustrating the congestion/decongestion phenomena of traffic flows with VSL control. Some concluding remarks compose Section 5.

II. CONTROLLED TRIANGULAR BATCHES PETRI NETS FORMALISM

We first recall in this section some concepts and definitions of Triangular Batches Petri Nets (TBPN)) [9] used in this paper. Next, Controlled Triangular Batches Petri Nets (CTBPN) is presented as an extension of a TBPN where variations of maximum speeds and maximum flows are controlled.

A. Triangular Batches Petri Nets

A Triangular Batches Petri Net (TBPN) is an extension of a Generalized Batches Petri Net (see [5], [6] and [7] for more details on this formalism), where some new characteristics related to the batch place have been added.

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***Definition** 2.1:* A *Triangular Batches Petri Net* (TBPN) is a 6-tuple $N = (P, T, Pre, Post, \gamma, Time)$ where:

- $P = P^D \cup P^C \cup P^{TB}$ is a finite set of places partitioned into the three classes of discrete, continuous and triangular batch places.
- $T = T^D \cup T^C \cup T^B$ is a finite set of transitions partitioned into the three classes of discrete, continuous and batch transitions.
- $Pre, Post : (P^D \times T \to \mathbb{N}) \cup ((P^C \cup P^B) \times T \to \mathbb{R}_{\geq 0})$ are, respectively, the pre-incidence and post-incidence matrices, denoting the weight of the arcs from places to transitions and from transitions to places.
- $\gamma : P^{TB} \to \mathbb{R}^4_{\geq 0}$ is the *triangular batch place function*. It associates with each triangular batch place $p_i \in P^{TB}$ the quadruple $\gamma(p_i) = (V_i, d_i^{max}, S_i, \Phi_i^{max})$ that represents, respectively, the maximum speed, the maximum density, the length and the maximum flow.
- $Time : T \to \mathbb{R}_{\geq 0}$ associates a non negative number with every transition:
  - if $t_j \in T^D$, then $Time(t_j) = d_j$ denotes the *firing delay* associated with the discrete transition;
  - if $t_j \in T^C \cup T^{TB}$, then $Time(t_j) = \Phi_j$ denotes the *maximal firing flow* associated with the continuous or batch transition. ■

The nodes of a TBPN [8] are represented in Fig. 1.

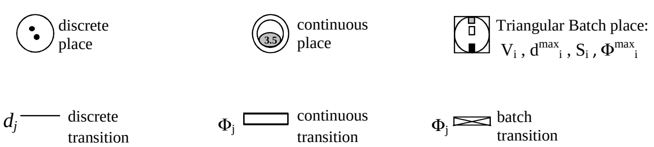

Fig. 1. Nodes of Triangular Batches Petri Nets

***Definition** 2.2:* Let a triangular batch place p_i, with $\gamma(p_i) = (V_i, d_i^{max}, S_i, \Phi_i^{max})$. A *propagation speed of congestion*, denoted W_i, and a *critical density* d_i^{cri}, are associated with p_i, defined respectively by:

$$W_i = \frac{\Phi_i^{max} \cdot V_i}{d_i^{max} \cdot V_i - \Phi_i^{max}} \tag{1}$$

$$d_i^{cri} = \frac{W_i . d_i^{max}}{V_i + W_i} \tag{2}$$

The *flow-density relation* that governs the dynamics of TB-place p_i is defined as follows:

$$\phi = \begin{cases} d.V_i & \text{if} \quad 0 \leq d \leq d_i^{cri} \\ W_i.(d_i^{max} - d) & \text{if} \quad d_i^{cri} < d \leq d_i^{max} \end{cases} \tag{3}$$

where d denotes density and ϕ denotes flow. ■

Figure 2 represents these definitions. Let us now introduced some definitions needed for the rest of this paper. Let a Triangular Batch place p_i, with $\gamma(p_i) = (V_i, d_i^{max}, S_i, \Phi_i^{max})$. An *input flow* $\phi_i^{in}(\tau)$ and an *output flow* $\phi_i^{out}(\tau)$ of place p_i are respectively: $\phi_i^{\text{in}}(\tau) = Post(p_i, \cdot) \cdot \boldsymbol{\varphi}(\tau)$ and $\phi_i^{\text{out}}(\tau) = Pre(p_i, \cdot) \cdot \boldsymbol{\varphi}(\tau)$ where $\varphi(\tau)$ is the instantaneous firing vector of continuous and batch transitions (see [7] for more details).

B. Controlled Triangular Batches Petri Nets

A Controlled Triangular Batches Petri Net (CTBPN) has the same syntax than TBPN. However we associate with CTBPN a different semantics, assuming that the maximal firing flow of continuous and batch transitions and, the maximal transfer speed of triangular batch places are control inputs.

***Definition** 2.3:* A Controlled Triangular Batches Petri Net (CTBPN) is a TBPN where the maximal transfer speed of TB-place $p_i \in P^{TB}$ and, the maximal firing flow associated with a continuous or batch transition $t_j \in T^C \cup T^B$, can varied. We denote respectively these variables: $v_i(\tau)$, with $0 \leq v_i(\tau) \leq V_i$, and $\phi_j(\tau)$, with $0 \leq \phi_j(\tau) \leq \Phi_j$. ■

It should be noted that the variation of the speed of TB-places imposes a variation of the critical density and of the maximum flow of TB-place while the propagation speed of congestion, W_i stays constant (see Fig. 2).

***Definition** 2.4:* Let TB-place p_i with $\gamma(p_i) = (V_i, d_i^{max}, S_i, \Phi_i^{max})$ with a maximal transfer speed $v_i(\tau)$ such that $0 \leq v_i(\tau) \leq V_i$. At time τ, the *controlled critical density* $d_i^{cri}(\tau)$ and the *controlled maximum flow* $\phi_i^{max}(\tau)$ are respectively defined by:

$$d_i^{cri}(\tau) = \frac{W_i . d_i^{max}}{v_i(\tau) + W_i}, \tag{4}$$

$$\phi_i^{max}(\tau) = v_i(\tau) . d_i^{cri}(\tau) \tag{5}$$

with $0 \leq \phi_i^{max}(\tau) \leq \Phi_i^{max}$ and $\frac{\Phi_i^{max}}{V_i} \leq d_i^{cri}(\tau) \leq d_i^{max}$.

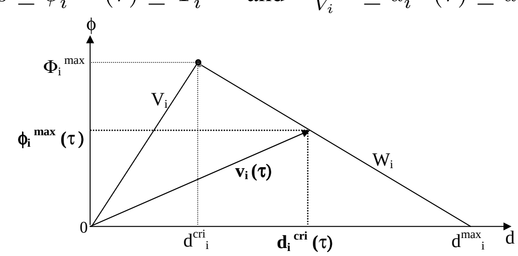

Fig. 2. Flow-density relation of a TB-place

C. Batches and markings of TB-places

A batch, i.e., a group of discrete entities characterized by continuous variables, has been defined for Batches Petri Nets. When, three continuous variables are associated with it, it is called a batch. When, four continuous variables are considered [6], it is called a controllable batch, which is a batch with a speed characteristic lower or equal to its maximum speed.

***Definition** 2.5:* A *controllable batch* $C\beta_r(\tau)$ at time τ, is defined by a quadruple, $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ where $l_r(\tau) \in \mathbb{R}_{\geq 0}$ is its *length*, $d_r(\tau) \in \mathbb{R}_{\geq 0}$ is its *density*, $x_r(\tau) \in \mathbb{R}_{\geq 0}$ is its *head position* and $v_r(\tau) \in \mathbb{R}_{\geq 0}$ is its *speed*. The *instantaneous batch flow* of $C\beta_r(\tau)$ is such that: $\varphi_r(\tau) = v_r(\tau) \cdot d_r(\tau)$. ■

Each batch place contains a series of controllable batches ordered by their head positions.

Definition 2.6: The *marking of a TB-place* at time τ is a series of controllable batches. If $p_i \in P^{TB}$ then $m_i = \{C\beta_h, \cdots, C\beta_r\}$.

Definition 2.7: A controllable batch $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ of TB-place p_i with $\gamma(p_i) = (V_i, d_i^{max}, S_i, \Phi_i^{max})$, where its head position equals to the length of p_i, i.e., $x_r(\tau) = S_i$, is called an *output controllable batch*, denoted $OC\beta_r(\tau)$. The *output density*, $d_i^{out}(\tau)$, of a TB-place is defined as follows. If at time τ, TB-place p_i has an output controllable batch $OC\beta_r(\tau)$, then $d_i^{out}(\tau) = d_r(\tau)$, else $d_i^{out}(\tau) = 0$. ■

All controllable batches composing the marking of a TB-place must respect the triangular flow-density relation (see eq.3). This condition allows us to define states of controllable batches.

Definition 2.8: (*States of batches*) Let $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ be a controllable batch of TB-place p_i, with $v_i(\tau)$ variable speed and V_i maximum speed of p_i ($v_i(\tau) \leq V_i$).

- $C\beta_r$ is in a *free state* if its density is lower or equal to the critical density of p_i: $d_r(\tau) \leq d_i^{cri}(v_i)$;
- $C\beta_r$ is in a *congested state* if its density is greater to the critical density of p_i: $d_r(\tau) > d_i^{cri}(v_i)$. ■

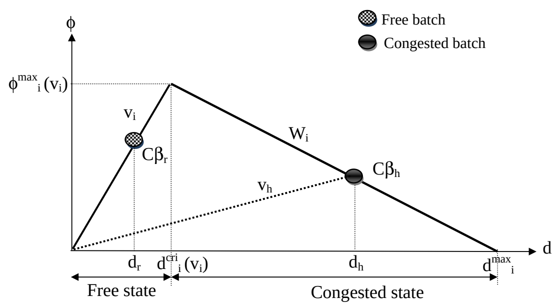

Fig. 3. Free and congested states of controllable batches

Let us now define, at time τ, two static functions which can be applied on batches of a TB-place.

- *Merge.* If two batches with the same density and the same speed are in contact, they can be merged. Let batches $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ and $C\beta_h(\tau) = (l_h(\tau), d_h(\tau), x_h(\tau), v_h(\tau))$, such that $x_r(\tau) = x_h(\tau) + l_r(\tau)$, $d_r(\tau) = d_h(\tau)$ and $v_r(\tau) = v_h(\tau)$. In this case, batch $C\beta_r(\tau)$ becomes $C\beta_r(\tau) = (l_r(\tau) + l_h(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ and batch $C\beta_h(\tau)$ is destroyed.
- *Split.* It is always possible to split a batch into two batches in contact with the same density and speed. Let batch $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ is split in two batches with the same density and the same speed as follows: $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ and $C\beta_{r'}(\tau) = (0, d_r(\tau), x_r(\tau), v_r(\tau))$.

D. Variation of maximum speeds of TB-places

In a CTBPN, characteristics and states of batches change when the maximal speed of TB-place p_i varies from $v_i(\tau)$ to $v_i'(\tau)$. It has to be noted that, at time τ, the maximal speed jumps from value v_i to value v_i'. As previously presented and according to eq. 4, when the maximal speed of place p_i increases (resp. decreases), the critical density $d_i^{cri}(v_i)$ decreases (resp. increases). Thus at time τ, two cases must be considered: the maximal speed decreases, i.e., $v_i'(\tau) < v_i(\tau)$, or the maximal speed increases, i.e., $v_i'(\tau) > v_i(\tau)$.

1) Decreasing speed $v_i'(\tau) < v_i(\tau)$

Three cases are possible when the maximal speed decreases (see Fig. 4):

- case 1: $C\beta_1 = (l_1, d_1, x_1, v_1)$ is a free controllable batch. When the speed of TB-place decreases, batch $C\beta_1$ reduces its speed but keeps its density. It stays a free batch, $C\beta_1 = (l_1, d_1, x_1, v_i')$.
- case 2: $C\beta_2 = (l_2, d_2, x_2, v_2)$ is a congested controllable batch with a higher speed than v_i' ($v_2 > v_i'$). When the speed of TB-place decreases, batch $C\beta_2$ reduces its speed to v_i' but keeps its density. It becomes a free batch, $C\beta_2 = (l_2, d_2, x_2, v_i')$.
- case 3: $C\beta_3 = (l_3, d_3, x_3, v_3)$ is a congested controllable batch with a lower speed than v_i' ($v_3 < v_i'$). When the speed of TB-place decreases, $C\beta_3$ keeps all its characteristics and stays a congested batch.

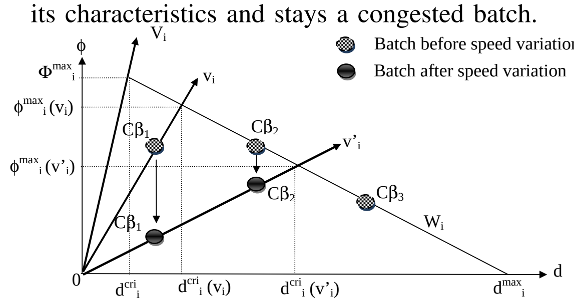

Fig. 4. Batches after a decreasing of the maximum speed of a TB-place

2) Increasing speed $v_i'(\tau) > v_i(\tau)$:

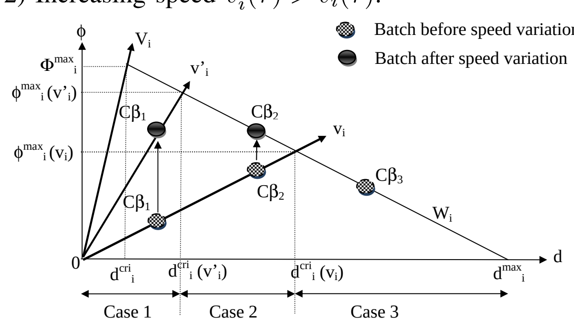

Fig. 5. Batches after an increasing of the maximum speed of a TB-place

Three cases are possible when the maximal speed increases (see Fig. 5) :

- case 1: $C\beta_1 = (l_1, d_1, x_1, v_1)$ is a free controllable batch and its density is lower than $d_i^{cri}(v_i')$. When the TB-place speed increases to v_i', batch $C\beta_1$ increases its speed to v_i' and keeps its density. It stays a free batch, $C\beta_1 = (l_1, d_1, x_1, v_i')$.
- case 2: $C\beta_2 = (l_2, d_2, x_2, v_2)$ is a free controllable batch and its density is greater than $d_i^{cri}(v_i')$. When

the TB-place speed increases to v_i', batch $C\beta_2$ keeps its density while its speed increases to speed $v_2' = (W_i \cdot (d_i^{max} - d_2))/d_2$, that respects $v_i < v_2' < v_i'$. It becomes a congested batch, $C\beta_2 = (l_2, d_2, x_2, v_2')$.
- case 3: $C\beta_3 = (l_3, d_3, x_3, v_3)$ is a congested controllable batch. When the speed of TB-place changes, this batch does not changed and stays a congested batch.

E. Events of a TB-place

The behavior of a TB-place is based on a discrete event approach with linear and constant evolutions between events. The invariant behavior state (IB-state) of a batches Petri net characterizes the global state between two timed events. Restricted to a TB-place, it corresponds to a period of time such that the input flow, the output flow, the output density and the transfer speed are constants. In a CTBPN, two controlled events have been added:

- a controlled speed event which is a triplet (p_i, v_i, τ), where p_i is a TB-place ($p_i \in P^{TB}$), $v_i \in [0, V_i]$ is the variable maximum speed of p_i and τ is the date of occurrence of this event.
- a controlled flow event which is a triplet (t_j, ϕ_j, τ), where t_j is a continuous or batch transition ($t_j \in T^C \cup T^B$), $\phi_j \in [0, \Phi_j]$ is the variable maximum firing flow of t_j and τ is the date of occurrence of this event.

More generally, in a CTBPN, the events that have to be considered during the evolution are:

- Internal events
  - i.1 - a batch becomes an output batch $C\beta_r = OC\beta_r$;
  - i.2 - two batches meet;
  - i.3 - a batch is destroyed $C\beta_r = \emptyset$.
- External events
  - e.1 a discrete transition is fired;
  - e.2 a continuous place becomes empty;
  - e.3 a discrete transition becomes enabled;
  - e.4 a batch becomes an output batch;
  - e.5 an output batch is destroyed.
- Controlled events
  - c.1 - the maximum flow of a batch or a continuous transition is modified: (t_j, ϕ_j', τ);
  - c.2 - the maximum speed of a TB-place is modified: (p_i, v_i', τ). ■

Let us now focus on the continuous dynamics of batch places.

III. Continuous-time Dynamics of controllable batches inside a TB-place

As the dynamics of batches can be controlled through the modification of maximum flows associated with continuous and batch transitions and through the variation of the maximum transfer speeds of TB-places, a new dynamics of batches circulate inside TB-places has to be defined. This dynamics is based on the theory of shockwave and takes into account the triangular flow-density relation.

Moreover, with every TB-place p_i, are associated continuous functions that represent transformation of batches: inputing, moving and exiting. These continuous functions change, by linear variations, variables of length and position, while density and speed of batches stay constant in time (i.e., these last variables only change when an event occurs, see previous section). Consequently, for any batch $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$, it holds: $\dot{d_r} = \dot{v_r} = 0$.

Let us present now the three dynamics of controllable batches.

A) Free dynamics

First we recall the definition of the free behavior, previously introduced in [6].

***Definition** 3.1:* (*Free behavior*) Controllable batch $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ of batch place p_i is in a *free behavior*, if it moves freely at its transfer speed $v_r(\tau)$. ■

Three different dynamics can occur.

***Definition** 3.2:* (*Input in free behavior*) A created controllable batch, $C\beta_r(\tau) = (0, d_r(\tau), 0, v_r(\tau))$, without contact with another batch or in contact with a downstream batch $C\beta_h(\tau)$ that has a greater speed (i.e., $v_h(\tau) \geq v_r(\tau)$), freely enters in place p_i according to:

$$\dot{x_r} = \dot{l_r} = v_r(\tau) \tag{6}$$

***Definition** 3.3:* (*Move in free behavior*) A controllable batch, $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$, which is a free batch, freely moves inside place p_i according to:

$$\dot{x_r} = v_r(\tau); \quad \dot{l_r} = 0 \tag{7}$$

***Definition** 3.4:* (*Exit in free behavior*) An output controllable batch $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), S_i, v_r(\tau))$, which has its flow equals to the output flow of p_i, or which is free with a lower batch flow than the output flow, freely exits from place p_i according to:

$$\dot{x_r} = 0; \quad \dot{l_r} = -v_r(\tau) \tag{8}$$

B) Congestion dynamics

To adapt some definitions on GBPN to CTBPN, the accumulation behavior previously introduced in [6] and [7] is now called *congestion behavior*.

***Definition** 3.5:* (*Congestion behavior*) Controllable batch $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ of batch place p_i is in a *congestion behavior*, if it cannot move at its speed but must reduces it, i.e., it starts an accumulation. Two cases can cause this behavior:

- $C\beta_r(\tau)$ is an output batch of p_i and the output flow of p_i is lower than the output batch flow ($\phi_i^{out} < \varphi_r(\tau)$).
- $C\beta_r(\tau)$ is a batch in contact with a downstream batch $C\beta_h(\tau)$ that has a lower speed ($v_h(\tau) < v_r(\tau)$). ■

Let us now focus on the dynamics of controllable batches in congestion behavior, in compliance with the previous definition.

***Definition** 3.6: (Input in congestion behavior)* A created controllable batch $C\beta_r(\tau) = (0, d_r(\tau), 0, v_r(\tau))$ in contact with a downstream batch $C\beta_h(\tau) = (l_h(\tau), d_h(\tau), x_h(\tau), v_h(\tau))$, and which is in a congestion behavior ($v_h(\tau) < v_r(\tau)$) at time τ, enters in place p_i according to equation (6), after changing its speed and its density as follows: $v_r(\tau) = v_h(\tau)$ and $d_r(\tau) = d_h(\tau)$.

Definition 3.7: (Move in congestion behavior) A controllable batch $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$, which is not a created or an output batch ($x_r(\tau) < S_i$ and $l_r(\tau) \neq 0$), in contact with a downstream batch $C\beta_h(\tau) = (l_h(\tau), d_h(\tau), x_h(\tau), v_h(\tau))$, and which is in a congestion behavior ($v_h(\tau) < v_r(\tau)$) at time τ, is splitted as follows:

- $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ and
- $C\beta_{r'}(\tau) = (0, d_{r'}(\tau), x_{r'}(\tau), v_{r'}(\tau))$ with $v_{r'}(\tau) = v_h(\tau)$, $d_{r'}(\tau) = d_h(\tau)$ and $x_{r'}(\tau) = x_r(\tau)$.

From time τ on, the evolution of both batches $C\beta_r$ and $C\beta_{r'}$ is governed by:

$$\begin{cases} \dot{x_{r'}} = v_{r'}(\tau) \\ \dot{l_{r'}} = \dfrac{d_r(\tau)}{d_r(\tau) - d_{r'}(\tau)}.(v_{r'}(\tau_1) - v_r(\tau)) \end{cases} \tag{9}$$

$$\begin{cases} \dot{x_r} = -\dfrac{d_r(\tau)}{d_r(\tau) - d_{r'}(\tau)}.(v_{r'}(\tau_1) - v_r(\tau)) \\ \dot{l_r} = -\dfrac{d_{r'}(\tau)}{d_r(\tau) - d_{r'}(\tau)}.(v_{r'}(\tau_1) - v_r(\tau)) \end{cases} \tag{10}$$

■

To leave the batch place, output batch $C\beta_r(\tau)$ must reduce its speed and should increase its density according to the output flow of p_i. To represent this situation, it is necessary to apply the split function.

Definition 3.8: (Exit in congestion behavior) An output controllable batch $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), S_i, v_r(\tau))$ of batch place p_i, which is in a congestion behavior at time τ (i.e., $\phi_i^{out} < \varphi_r(\tau)$), is split into two batches as follow:

- $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), S_i, v_r(\tau))$ and
- $C\beta_{r'}(\tau) = (0, d_{r'}(\tau), x_{r'}(\tau), v_{r'}(\tau))$ with $d_{r'}(\tau) = d_i^{max} - \frac{\phi_i^{out}}{W_i}$, $v_{r'}(\tau) = \frac{\phi_i^{out}}{d_{r'}(\tau)}$ and $x_{r'}(\tau) = S_i$.

From time τ on, both batches evolves according to:

$$\begin{cases} \dot{x_{r'}} = 0 \\ \dot{l_{r'}} = \dfrac{v_r(\tau).d_r(\tau) - \phi_i^{out}}{d_{r'}(\tau) - d_r(\tau)} \end{cases} \tag{11}$$

$$\begin{cases} \dot{x_r} = -\dfrac{v_r(\tau).d_r(\tau) - \phi_i^{out}}{d_{r'}(\tau) - d_r(\tau)} \\ \dot{l_r} = \dfrac{\phi_i^{out} - v_r(\tau).d_{r'}(\tau)}{d_{r'}(\tau) - d_r(\tau)} \end{cases} \tag{12}$$

■

C) Decongestion dynamics

The decongestion dynamics is only applied to congested batch. Of course, a created batch cannot be in a decongestion behavior.

Definition 3.9: (*Decongestion behavior*) Congested controllable batch $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ of batch place p_i is in a *decongestion behavior*, if it can move with a higher speed. Three situations can cause this behavior:

- $C\beta_r(\tau)$ is a congested output batch and the output flow of p_i is greater than the output batch flow ($\phi_i^{out} > \varphi_r(\tau)$).
- $C\beta_r(\tau)$ is a congested batch in a downstream contact with $C\beta_h(\tau)$ that has a greater speed ($v_h(\tau) > v_r(\tau)$).
- $C\beta_r(\tau)$ is a congested batch without contact with a downstream batch that has a lower speed than $v_i(\tau)$. ■

Let us now focus on the dynamics of controllable batches in decongestion behavior, in compliance with the previous definition.

Definition 3.10: (*Exit in decongestion behavior*) A congested output controllable batch $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), S_i, v_r(\tau))$ of batch place p_i, which is in a decongestion behavior at time τ (i.e., $\phi_i^{out} > \varphi_r(\tau)$), is split into two batches as follow:

- $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), S_i, v_r(\tau))$ and
- $C\beta_{r'}(\tau) = (0, d_{r'}(\tau), x_{r'}(\tau), v_{r'}(\tau))$ with $d_{r'}(\tau) = \frac{\phi_i^{out}}{v_i(\tau)}$, $v_{r'}(\tau) = v_i(\tau)$and $x_{r'}(\tau) = S_i$

From time τ on, the dynamics of both batches, $C\beta_r(\tau)$ and $C\beta_{r'}(\tau)$, are governed by eq. (11) and eq. (12). ■

Two cases are considered for the moving dynamics in decongestion behavior: the batch moves with or without contact with a downstream batch.

Definition 3.11: (*Move in decongestion behavior with a downstream contact*) Congested controllable batch, $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ of batch place p_i without contact and which is in decongestion behavior, is split into two batches as follows:

- $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ and
- $C\beta_{r'}(\tau) = (0, d_{r'}(\tau), x_{r'}(\tau), v_{r'}(\tau))$ with $v_{r'}(\tau) = v_i(\tau)$, $d_{r'}(\tau) = d_i^{cri}(v_i)$ and $x_{r'}(\tau) = x_r(\tau)$.

From time τ on, the dynamics of each batch $C\beta_r(\tau)$ and $C\beta_{r'}(\tau)$ are governed by equation (9) and equation (10).

Definition 3.12: (*Move in decongestion behavior without contact*) Congested controllable batch, $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ of batch place p_i in contact with a downstream batch $C\beta_h(\tau) = (l_h(\tau), d_h(\tau), x_h(\tau), v_h(\tau))$, and which is in decongestion behavior, is split into two batches as follows:

- $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ and
- $C\beta_{r'}(\tau) = (0, d_{r'}(\tau), x_{r'}(\tau), v_{r'}(\tau))$ with $v_{r'}(\tau) = v_h(\tau)$, $d_{r'}(\tau) = d_h(\tau)$ and $x_{r'}(\tau) = x_r(\tau)$.

From time τ on, the dynamics of each batch $C\beta_r(\tau)$ and $C\beta_{r'}(\tau)$ are governed by equation (9) and equation (10).

IV. EXAMPLE

For illustrating the proposed dynamics of controllable batches inside TB-places, we model a road section based on an example presented in [2] where we can observe congestion and decongestion phenomena. We consider a road section S with a length $L = 12\ km$ with two lanes in one direction and without on/off ramp. The maximum flow Q^{max} of the road is equal to 4080 veh/h, the jam density k is 320 veh/km, its free speed v^{free} is limit to 120 km/h and its inflow ϕ^{in} is equal to 3060 veh/h. The CTBPN that represents such a road section is shown in Figure 6, where the input flow ϕ^{in}, output flow ϕ^{out} are represented by the maximum firing flows associated with batch transitions t_1 and t_2, respectively $\Phi_1 = 3060$, $\Phi_2 = 4080$. TB-place p_2 represents the road section and continuous place p_1 limits the capacity of the section ($k \times L$).

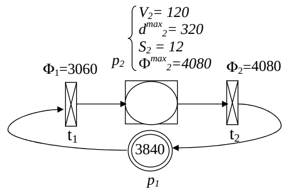

Fig. 6. CTBPN model of road section S

A. Simulation of a road section

In order to illustrate the dynamics of controllable batches inside TB-places when a congestion/decongestion phenomena traffic road occurs, we consider in the behavior the following traffic events:

- one lane is blocked by an accident at the end of the road section 15 minutes after the simulation starts to run, the maximum flow in this point is reduced to 2040;
- 2 minutes after the occurrence of the accident, VSL control is applied and the free speed v^{free} of the road section is reduced to 80 km/h;
- the accident lasts for 10 minutes;
- after the accident is over, the maximum flow is recovered to 4080 and the free speed v^{free} is returned to 120 km/h.

The evolution graph, given in Figure 7 represents the behavior of the CTBPN model in Figure 6 (see [6] for more details on this graph). As we can see in Figure 8 the variation of congestion length is stopped in 0.22 km when we apply VSL control.

V. Conclusion

We have presented in this paper a continuous-time and discrete events dynamics of controllable batches inside triangular batch place. This dynamics is based on some controlled events and respects the fundamental diagram, shockwave theory and the conservation law of traffic road systems. As we can see the definition of controlled events in the CTBPN imply in different cases of congestion/decongestion behaviors. The equations that govern each case, represent the new dynamics of the CTBPN. To validate our results, an example was presented in Section IV studying VSL control on a road section. Applicability of CTBPN to a real system has been shown in [10] from experimental data of a highway.

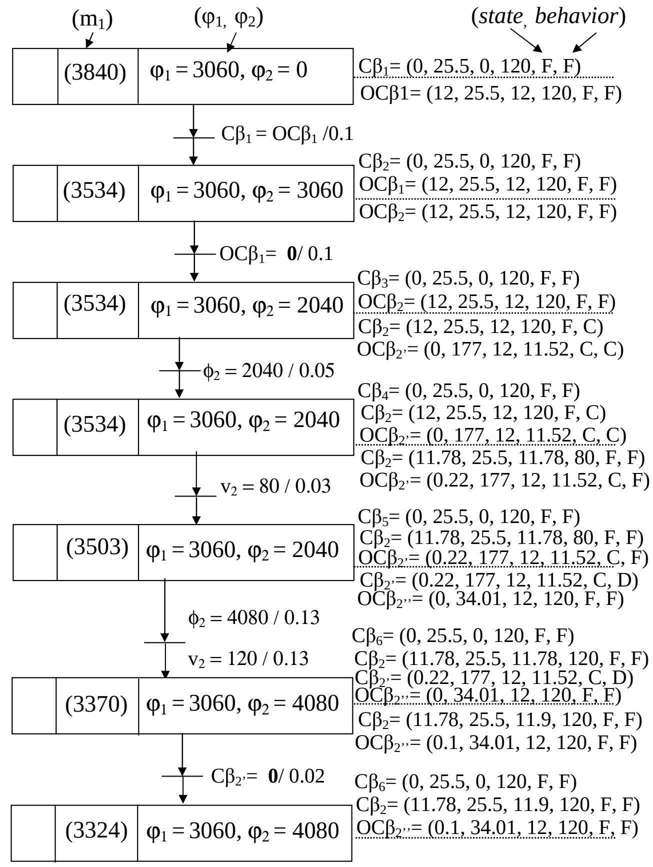

Fig. 7. Evolution graph of the CTBPN model

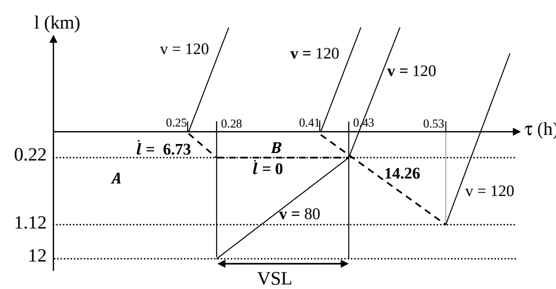

Fig. 8. Variation of congestion length with maximum speed variation

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