Low Mach Number Modeling of Stratorotational Instability in a water-filled Taylor-Couette cavity

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The Stratorotational Instability (SRI) is a purely hydrodynamic instability with distinctive local features occuring in centrifugally stable flows. It is suspected to contribute to the outward angular momentum transport in accretion discs, which is a central problem of planet formation. Only turbulence can achieve such a large angular momentum transport. SRI can be studied from specifically designed laboratory experiments and numerical simulations in an axially-stratified Taylor-Couette setup [1, 2]. Recent studies [3, 4] mentioned that, at fixed values of the Froude number, $Fr = \Omega_i/N$ (with N the buoyancy frequency and Ω_i the angular velocity of the inner cylinder), and of the angular velocity ratio $\mu = \Omega_o/\Omega_i$ (with Ω_o the angular velocity of the outer cylinder), there exists an upper limit of the Reynolds number $Re = \Omega_i R_i(R_o - R_i)/\nu$ for the occurrence of SRI. These authors also showed that the value of Re where the flow becomes stable again increases as Fr decreases. Therefore, at high Reynolds number values relevant to transition to turbulence regimes, SRI can only be obtained at small Froude number values and, consequently, for large temperature difference ΔT between the top and bottom endplates. When liquid water is used in simulations, the validity of the Boussinesq approximation may be questioned under such conditions. Indeed, Shalybkov and Rüdiger [5] suspected non-Boussinesq effects to be responsible for large discrepancies between numerical and experimental results for strong stratifications, e.g small values of Fr. Medale and Haddad [6] reported non-Boussinesq effects in a mixed convection problem using liquid water. The Boussinesq approximation can theoretically be applied with liquid water at 298.15K for temperature differences ΔT less than 5.2K [7]. We investigated the influence of non-Boussinesq effects on the characteristics of SRI by performing 3D simulations with both Low Mach number and Boussinesq approximations based on spectral methods in the configuration of experiments at the BTU Cottbus-Senftenberg, Germany [4]. For the Low Mach number modeling, the specific equation of state proposed for water by Pátek *et al.* [8] is implemented, which allows for an accurate representation of the density and other thermodynamic properties of water, with a mean deviation of 0.01% for the density compared with data provided by NIST. The simulations show that non-Boussinesq effects modify the value of the Froude number for the threshold of occurrence of SRI. Significant changes are also observed for the azimuthal wavenumber, as illustrated in figure 1, and for the fundamental frequency of the instability. The present studies will serve to guide future simulations based on High Performance Computing in order to reach higher values of governing parameters relevant to turbulence regimes [9].

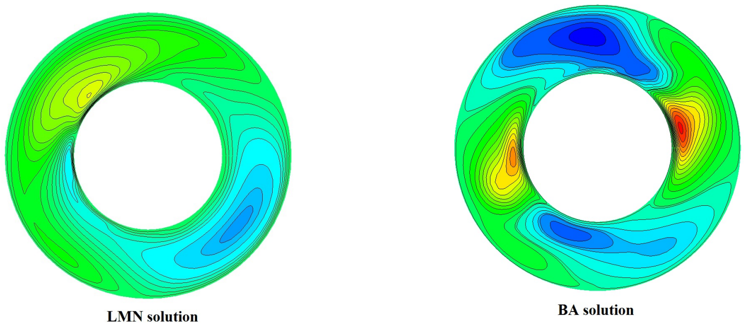

Figure 1: Iso-contours of radial velocity at mid-height of the cavity for $Re = 1000$, $Fr = 0.58$ ($\Delta T = 24$K) and $\mu = 0.35$. Left: Low Mach number approximation (azimuthal wavenumber $m = 1$, frequency ω/Ω_i=0.54). Right: Boussinesq approximation (m=2, ω/Ω_i=1.12).

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