Global modes in Taylor-Couette-Poiseuille flow with a permeable inner cylinder

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Variations in the local stability of the flow in a Taylor–Couette cell can be imposed by adding an axial Poiseuille flow and a radial flow associated with the a weakly permeable inner cylinder through which the radial flow is governed by Darcy's law. At a given rotation rate of the inner cylinder, this results in adjacent regions of the flow that can be simultaneously stable, convectively unstable, and absolutely unstable, making this system fit for obtaining global, synchronized, modes of centrifugal instability.
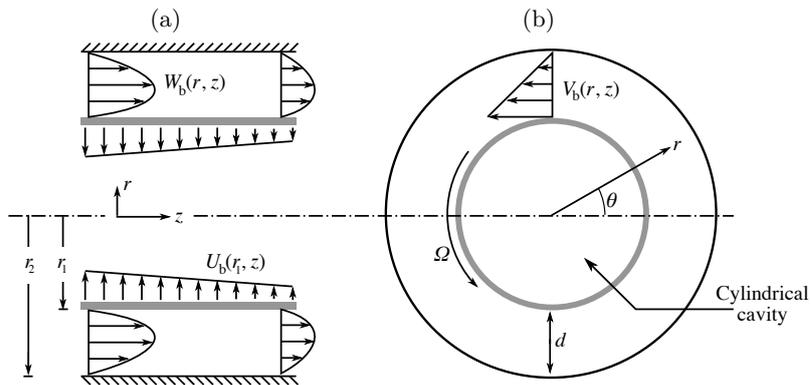
Figure 1: (a) Radial and axial velocities of the laminar Taylor–Couette–Poisuille (TCP) flow. The outer cylinder is impermeable while fluid flows across the inner permeable cylinder, according to Darcy's law, leading to a base flow varying along the axial direction. (b) shows the azimuthal velocity of the base flow in an equatorial plane.

A two-pronged approach is adopted. First, building on the existing convective/absolute stability analysis in the axially invariant TCP flow with impermeable cylinders [1], the analytical frameworks of linear and non-linear global modes are used to obtain the critical conditions and characteristics of the instabilities in the axially varying flow with a permeable inner cylinder. These analyses are based on the selection of a specific axial location that acts as a wavemaker and imposes its unstable dynamics and frequency to the rest of the perturbation. Then, Direct Numerical Simulations using a dedicated pseudospectral method implementing the Darcy's condition on the permeable cylinder [2] are performed to shed new light on the validity of the analyses in global modes. Guided by application to filtration devices, we consider a set-up where fluid extraction occurs along the full length of the inner cylinder. As one moves downstream, the mean axial flow decreases and so does the local stability of the

base flow. Close to critical conditions, the global mode is governed by a linear wavemaker located at the outlet, in agreement with the wavepacket of toroidal vortices obtained in the DNS shown in Figure 2(a). As the rotation rate is increased, the linear global mode evolves to a non-linear global mode governed by wavemaker located at the boundary between the convectively and absolutely unstable regions, in agreement with the DNS shown in Figure 2(b). The global mode analyses do not fully explain, however, that the instabilities observed in the numerical simulations take the form of axial stacks of wave-packets, as observed in Figure 2(b), characterized by step-downs of the temporal frequency.
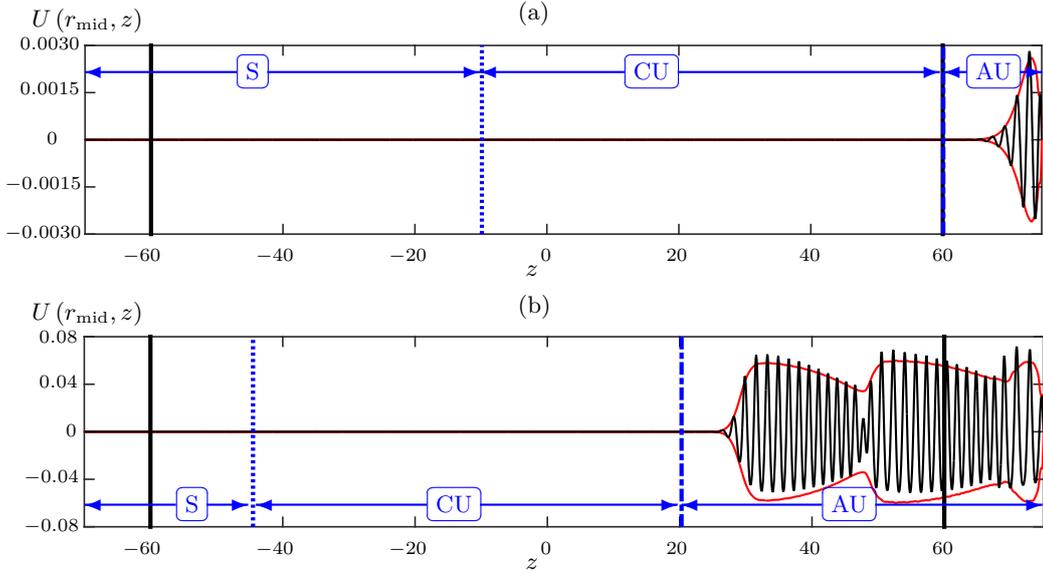

Figure 2: Direct Numerical Simulation of TCP flow with a permeable inner cylinder: radial velocity at mid-gap, as a function of the axial coordinate z. The rotation rate of the inner cylinder is 0.6% (a) and 16% (b) above its analytical critical value. The stable, convectively unstable and absolutely unstable regions are denoted by blue S, CU and AU, respectively.

References

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