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To cite this version:
Raouf Boucekkine, Rodolphe Desbordes, Paolo Melindi-Ghidi. Particularism, dominant minorities and institutional change. 2019. hal-02317663

HAL Id: hal-02317663
https://hal-amanu.archives-ouvertes.fr/hal-02317663
Submitted on 16 Oct 2019

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Particularism, dominant minorities and institutional change

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October 14, 2019

Abstract

We develop a theory of institutional transition from dictatorship to minority dominant-based regimes. We depart from the standard political transition framework à la Acemoglu-Robinson in four essential ways: (i) population is heterogeneous, there is a minority/majority split, heterogeneity being generic, simply reflecting subgroup size; (ii) there is no median voter in the post-dictatorship period, political and economic competition is favorable to the minority (fiscal particularism); (iii) (windfall) resources are introduced, and (iv) we distinguish between labor income and resources, and labor supply is endogenous. We first document empirically fiscal particularism, its connection with resource endowment, and the impact of both on revolutionary bursts. Second, we construct a full-fledged model incorporating the four characteristics outlined above. We show, among others, that polarization is a sufficient condition for revolutions, while resource rents are not: they do matter though when polarization is low. In agreement with our empirical facts, countries engaging in revolutions tend to be slightly less resource-rich than other countries. We also outline the interplay between resource rents, polarization and labor market conditions at the dawn of institutional change. Our theory is appropriate to understand the institutional dynamics in highly homogeneous resource-rich countries, which after post-independence autocratic regimes, turn to be dominated by minorities, Algeria being the paradigmatic case.

Keywords: Political transition, minority/majority, fiscal particularism, dominant minority, resources, labor market

JEL classification: D72, C73, Q32

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1 Introduction

In the two last decades, an abundant economic literature has been devoted to the mechanisms of institutional change, roughly the transition from dictatorship to democracy. Perhaps the best representative of this strong political economy stream is the book of Acemoglu and Robinson (2006); see also Acemoglu and Robinson (2000, 2001). The basic story underlying the theory of institutional change developed in this literature is typically driven by revolutions by the people and for the people to use the expressions of the political scientists Albertus and Menaldo (2018). In short, the unequal redistribution of income by a ruling elite induces the population to revolt provided the cost of revolting is low enough compared to the grievance caused by the perceived inequality. In the basic story, population is homogeneous and so is the elite. The revolution is launched by the population, it is by the people. The costs incurred are typically twofold: coordination costs as it is the case for any collective action, and all the potential costs related to the degree of vulnerability (Boucekkine et al., 2016) or entrenchment (Caselli and Tesei, 2016) of the ruling elite, typically determining the extent and ferocity of repression use. Also in the standard theory, the revolution is for the people: in particular, the fiscal policy which is in the iron hands of the elite under dictatorship (and thus, it being the source of inequality and grievance), turns to be determined by the median voter after revolution and the rise of democracy. Incidentally, this view of the democratic transition goes together with the belief that democracies do foster redistribution from the rich to the poor and end up delivering robust welfare states. This has been undoubtedly the case in Western Europe after the Second World War and it is also roughly consistent with the observed distributional impact of the more recent democratization waves in Latin America and Eastern Europe (see details in Albertus and Menaldo, 2018).

While this rosy story of the transition from authoritarianism to democracy by the people and for the people has been of course challenged and enriched in many respects (to the point that several authors have proposed convincing models in which such a transi-
tion cannot occur under some circumstances making permanent dictatorship preferable, see for example Shen, 2007), it remains the benchmark theory in the political economy literature. This is by no way the case in the political science literature, as illustrated for example the highly influential books of Haggard and Kaufman (2016) and Albertus and Menaldo (2018). For the latter distinguished political scientists, the very definition of the above mentioned transition as a benchmark can be heavily questioned nowadays, to be precise: for the late-2000s (Haggard and Kaufman, 2016, page 1). This would clearly call into question the relevance of a large number of related economic papers for this most recent period. A first series of facts questioning the benchmark theory is what Haggard and Kaufman call “backsliding from democracy”, that is the incapacity of some democracies to consolidate with reversal to certain types of autocracy. Among the countries having experienced such institutional paths, the latter authors cite Russia, Venezuela, Kenya, Turkey or Hungary. Our reading of the “backsliding from democracy” cases outlined by Haggard and Kaufman, in the light of the standard mechanisms displayed in the beginning of this introduction, is the following: while the ruling leaders have come to power democratically, that is they have been chosen by the people, they end up governing, not for the people, not only by restricting civil and political liberties, but also by taking economic advantage of their office, ultimately undermining the supposed superiority of democracies in terms of redistributive justice. Of course, the idea that democracies may not be stable is not new in Economics.\footnote{For example, in the new Lipset-like theories of modernization, the role of human capital accumulation is put forward not only to favor democratization but also to build democracies on stable bases. See for example, Glaeser et al. (2007).} However, the recent political science literature mentioned above pushes much further the distrust in the democratic model and its presumed virtues.

The picture is even more pessimistic for Albertus and Menaldo (2018). In their introductory chapter (“A deeper critic of democracy”), these authors argue: “...democracy is made from above and designed to reflect the interest of former autocratic elites. At first
blush, this might seem like a rather rash accusation. But consider the expression of the social contract at the core of every modern polity: its constitution. From 1800 to 2006, only 34 percent of new democracies began with a constitution that they created themselves or inherited from a past episode of democratic rule in their country'. A key aspect underlying the latter reasoning is that while institutional change may be indeed operated by the people through violent or non-violent revolutions, the new emerging “democratic” rules may not or even are not likely to be for the people. A typical example is South Africa, where the white minority has kept dominating the political and economic rules of this country despite the apparent equality in political and economic rights, irrespective of ethnic origins, after the end of the apartheid regime. This is a clear example of the so-called elite-led democracies as the rules of the new democratic regimes, coming after the demise of the preceding autocracies, are typically imposed by the former ruling elite.

A related more general question, which is tackled in this paper, is how the majority of population during and/or in the aftermath of a revolutionary episode, closing up an autocratic episode, can leave the economic and/or political controls to a minority, be it or not related to the ousted dictators. In this paper, we precisely seek to uncover some of the essential conditions under which dominant minorities can emerge in the course of institutional change. Of course, the inherent research question may be somehow easy when it comes to the case where these minorities are connected to the falling autocracy as they are typically the most informed about the functioning of the state and its economy. Because they hold such a valuable informational (and often technological) advantage, they typically come back to the front of the scene after a short period of distrust. This clearly happened in Tunisia a few years after the 2011 Jasmin revolution, and it is no question that many current dominant minorities in the world are of this sort.\footnote{In the case of South Africa, no waiting period was observed before involving the white minority in the governing bodies of the post-apartheid regime, due probably to the exceptional credibility of Mandela’s leadership.} In this paper, we abstract away from the political, ethnic or economic origins of the minorities,
and assume that they do not have necessarily ties with the former autocracy and no particular associated informational or skill advantage.

Precisely, mimicking the standard democratic transition model, we study the conditions under which a majority of people revolt against a dictatorship to fall into an alternative regime in which the associated economic system, including redistribution (in a sense which will be clearer here below), is driven by dominant minorities. Since we also assume, as explained above, that these minorities need not have any relationship with the former autocracy, our theory is not really a theory of elite-led democracies in the sense of Albertus and Menaldo (2018), though it shares with the latter that despite the institutional change is by the people, through revolutions, the new regime is dominated by a minority or (new in our case) elite, and is certainly not for the people. A key novel ingredient with respect to the institutional change models à la Acemoglu and Robinson is the heterogeneity of population: there are a minority and a majority. Our model is agnostic as to the origins of this heterogeneity, it can be of any sort, but in our theory, it is reducible to a number: the fraction of population which belongs to the minority group. Under autocracy, the elite treats exactly in the same way both the minority and majority: no advantage is given to the minority in particular. This is pretty much the case in a wide range of dictatorships, in particular those of the communist type, which often attempt to erase any apparent trace of group singularity. While our theory has not been built up to study specifically the latter case, it is also meaningful to understand the institutional dynamics of countries with highly homogeneous population in resource-rich countries with an initial (typically post-independence) autocratic regime. See below.

A key and original ingredient of the model is the modeling of the dominance features of the minority. In this respect, we build on an abundantly documented indicator, fiscal particularism. Section 2 below provides the necessary empirical support, taken from the V-Dem (Varieties of Democracy) database. When social and infrastructural spending in the national budget goes to a particular group (region, ethnic group, politically-
connected group,...), that is when this spending does not benefit everyone (no public
good), we refer to this situation as a case of fiscal particularism. Accordingly, the
dominant minorities in our model are precisely those which capture a fraction of public
spending which is above their demographic weight. To make our research question even
trickier, we consider the case where the latter fraction is inversely proportional to its
demographic weight. Under which conditions the majority of population would prefer
the ruling of such an extreme dominant minority (or such an extreme fiscal particularism)

Needless to say, the response to this question also depends on how income is generated
in the model and on individual preferences. Again differently from the standard demo-
ocratic transition model, we consider two income sources: windfall income (or resources)
and labor income. Moreover, individuals enjoy leisure, the theories à la Acemoglu and
Robinson do not account for this ingredient. We believe that this is an important addi-
tion to the model as political regimes do not only differ in their redistribution policies,
they can also drastically alter the incentives to work. This is documented in Section 2
here below.

Two remarks are worth mentioning at this stage. First of all, we could have also
postulated heterogeneity with respect to individual preferences. We don’t: we focus here
on a single aspect of minority dominance, that is fiscal particularism, already outlined
above. In our model, the actual extent of particularism is generically the result of a
Stackelberg game whose leader is the minority. We deliberately leave aside all types of
“psychological” differences between minority and majority members. In particular, we
do not assume any difference in the disutility from working.

Second, we introduce resources in our story, not only for the sake of generality.
As documented in Section 2, fiscal particularism is more significant in resource-rich
countries, and this is no surprise given the level of rent-seeking in these countries and
the inherent emergence of local or regional dominant groups. Within our framework, we
are able to analyze the interplay of fiscal particularism with population fractionalization (in particular, with the degree of polarization) and with level of resource windfalls and participation in the labor market as well. We show, among others, that polarization is a sufficient condition for revolutions, while resource rents are not: they do matter though when polarization is low. Consistently with the empirical regularities identified in Section 2, we also find that countries engaging in revolutions tend to be slightly less resource-rich than other countries. As mentioned above, our theory is particularly appropriate to understand the institutional dynamics in highly homogeneous resource-rich countries, which after a post-independence autocratic communist-like regime, turn to be dominated by minorities. The case of Algeria is paradigmatic in this respect.

Relation to the literature

Our paper is related to several streams of the Economic and Political Science literature. Focusing on the former, our work is obviously related to the democratization theories developed by Acemoglu and Robinson, notably in their 2006 book. Indeed, we show in this paper that our model does degenerate into the basic Acemoglu and Robinson model in the absence of population heterogeneity and the associated fiscal particularism. Because of the resource ingredient, our paper is also related to the abundant literature on conflicts and institutional change in resource-rich countries (Ross, 2001; Collier and Hoeffler, 2004; Friedman, 2006; Hodler, 2006; Alexeev and Conrad, 2009; Haber and Menaldo, 2011; Robinson, Torvik and Verdier, 2006 and 2014; Robinson, 2014 and 2016; Boucekkine et al., 2016; Caselli and Tesei, 2016, to cite a few). In terms of theoretical contribution, key differences with respect to this literature are population heterogeneity (minority/majority), fiscal particularism and work incentives. Another (subsequent) major difference is that we do not study democratic transitions in the traditional sense of median voter decision-making but transitions to democracies (in the sense that the regime change is driven by the majority) dominated by minorities.

Our paper is also related to a large literature in Political Economy and Public Eco-
nomics dealing with the political and developmental role of minorities. For example, the fiscal particularism story can be related to the “paradox of power” problem studied in the public choice literature in particular (see the seminal paper of Hirshleifer, 1985, for example). In addition, and more related to our redistribution story, it is widely admitted in the economic literature that the presence of majority and minority groups in society may be highly important in this respect. Easterly and Levine (1997), Roemer (1998), Alesina et al. (1999), Shayo (2009) find that diversity impacts public good provision. As such, societies populated by majority and minorities groups are likely to have lower levels of goods that are publicly provided. However, these works do not concentrate on the character - particularistic vs. universalistic - of the public good but on its overall provision.

Further research has shown that there exists a tendency for the groups with de facto power to transfer higher benefits to members of their group. In this vein, Besley et. al. (2004) analyze this issue, showing that sharing the politician’s group identity is a means to obtain greater proportion of publicly provided private goods. Banerjee et al. (2005) and Banerjee and Somanathan (2007) find that group identities are correlated with access to public goods in India. Kimenyi (2006) show that in African countries ethnic groups that control the government will adopt a particularistic redistribution of the public good. Bandiera and Levy (2010) find that democratic policy outcomes are closer to the preferences of the elites when there is diversity in preferences among the poor majority. Finally, the literature on conflicts in fractionalized societies (see for example, Esteban and Ray, 2011) has already stressed the role of polarization in conflicts but it is not about institutional change per se.

The paper is organized as follows. Section 2 presents the main stylized facts on which our model is based. Section 3 describes our set-up in detail. Section 4 delivers the main outcomes of our analysis concerning institutional change in the presence of dominant minorities. The roles of resources, labor market and the polarization level of population
are highlighted. Section 5 concludes.

2 Stylized Facts

2.1 Government spending and taxation

Political regimes may choose to allocate most of their expenditures to a small and selected fraction of the population. To illustrate this, we use data from the V-Dem (Varieties of Democracy) database, covering the period 1960-2018. Our indicator of particularistic spending corresponds to the answers of country experts to the following question, subsequently mapped on a linearized ordinal scale (ranging from about -3 to 3): “Considering the profile of social and infrastructural spending in the national budget, how ‘particularistic’ or ‘public goods’ are most expenditures?”; 0 (almost all of the social and infrastructure expenditures are particularistic [narrowly targeted]) to 4 (almost all social and infrastructure expenditures are public goods in character [intended to benefit everyone]). The scale of this indicator is inverted such as a larger value implies more particularistic spending.

Using the values of Western Europe and North America (WENA) as reference points, Figure 1 shows that particularistic spending has been a consistent feature of political regimes in other regions, especially in Latin America and Caribbean (LAC), Middle East and North Africa (MENA), and Sub-Saharan Africa (SSA). Nevertheless, the nineties appear to have been a turning points towards less particularistic spending, except in post-communist Eastern Europe and Central Asia (ECA) and MENA countries. One may worry that particularistic spending is the consequence of insufficient resources. However, Figure 2 shows that, even after adjusting for differences in income per capita, a similar picture emerges.

Figure 2 has shown that, for a given level of income per capita, countries can diverge

\(^{3}\)https://www.V-Dem.net/
in their particularistic spending. Figure 3 indicates that one factor associated with this divergence is natural resources abundance. Resource-rich countries tend to engage more in particularistic spending. Figure 4 highlights that public spending on health, a widely available indicator of redistribution, tends to be smaller in these countries than other countries. As pointed out by Ebeke et al (2015), for various reasons, redistribution may rather take the form of energy subsidies in resource-rich countries. Figure 5 indeed reports that energy subsidies are much larger in resource-rich countries. Turning to taxation, Figure 6 depicts the relationship between the taxation of individuals and natural resources abundance. Revenues from the taxation of the income (from labor and capital) of individuals, as a share of GDP, tend to be lower in resource-rich countries.

Natural resources abundance may lead to a fall in labor force supply. We use the share of young people (15-24 years old) who are not in education, employment, or training (NEET) as a crude indicator of the willingness of people to supply their labor. Figure 7 shows that the relationship between the NEET share and natural resources abundance is positive, but only for the most resource-abundant countries.

### 2.2 Revolutions

The V-Dem database reports 12 different types of processes that can lead to the end of a political regime (e.g. coup, loss in civil war, popular uprising, peaceful political liberalization). We consider a revolution to have taken place if a popular uprising has played a role in ending a regime. Furthermore, we only consider pro-democratic transitions: the V-Dem electoral democracy index (the existence of free and fair elections) must be higher in the following regime. Finally, given that some regimes are themselves transition regimes, we impose that the end of the new regime (which may not have occurred at the end of our sample) must occur at least three years after the fall of the previous regime. Figure 8 highlights that pro-democratic transitions have been recurring events since 1960, with a global peak in the nineties. Revolutions tend to be much less frequent.
Figure 1: Particularistic spending, by period and region


Figure 2: Particularistic spending, adjusted for income per capita

Values have been adjusted for the log of income per capita (constant 2010 US$), taken from the World Bank World Development Indicators (WDI) (http://wdi.worldbank.org/) using a locally weighted regression (‘lowess’).
Figure 3: Particularistic spending in resource-rich countries

Notes: Data come from the V-Dem database and Haber and Menaldo (2011). Natural resources abundance corresponds to total resource income (the volume of production of oil, gas, coal, metals times the price of these resources) per capita, expressed in 2007 US dollars, and transformed using an inverse hyperbolic sine transformation (IHS) (to deal with outliers and zero values). Values of both variables have been adjusted for the log of income per capita (constant 2010 US$), taken from the World Bank WDI, using a multivariate locally weighted regression.

Figure 4: Health spending in resource-rich countries

Notes: Data come from Haber and Menaldo (2011) and World Bank WDI. Health spending: public expenditure on health from domestic sources (% of GDP). Values of both variables have been adjusted for the log of income per capita (constant 2010 US$), taken from the World Bank WDI, using a multivariate locally weighted regression. IHS: inverse hyperbolic sine transformation.
Figure 5: Energy subsidies in resource-rich countries

Notes: Data come Haber and Menaldo (2011) and Coady et al (2015). Energy subsidies in 2013 correspond to existing fuel consumption multiplied by the positive gap, when such a gap exists, between supply costs and consumer prices. Most recent values for natural resources abundance: 2005. Values of both variables have been adjusted for the log of income per capita (constant 2010 US$), taken from the World Bank WDI, using a multivariate locally weighted regression. IHS: inverse hyperbolic sine transformation.

Figure 6: Individual taxation in resource-rich countries

Notes: Data come Haber and Menaldo (2011) and the ICTD/UNU-WIDER Government Revenue Dataset (https://www.wider.unu.edu/project/government-revenue-dataset). Sample is maximized for the year 2000. Values of both variables have been adjusted for the log of income per capita (constant 2010 US$), taken from the World Bank WDI, using a multivariate locally weighted regression. IHS: inverse hyperbolic sine transformation.
than other types of transitions. In our sample, we observe 30 revolutions out of 227 pro-democratic transitions (13% of total).

Revolutions, in comparison to other pro-democratic transitions, ought to require the coordination of a large number of people. The V-Dem database provides some information on the degree of anti-system opposition at a given point of time. We use the responses, subsequently mapped on a linearized ordinal scale (ranging from about -3 to 4), to the following question “Among civil society organizations, are there anti-system opposition movements?” Responses, on the original scale, can vary from 0 (no, or very minimal) to 4 (there is a very high level of anti-system movement activity, posing a real and present threat to the regime). To be considered, the movement must have a mass base and an existence separate from normal electoral competition. Figure 9 shows that revolutions are characterized by \textit{ex ante} stronger anti-system opposition than other transitions. In the same Figure, we also report the degree of natural resources abundance. Countries engaging in revolutions tend to be slightly less resource-rich than
Figure 8: Number and types of pro-democratic transitions

Notes: Data come from the V-Dem database. Revolutions: popular uprisings have played a role in ending a political regimes. Decennial periods. Regions according to World Bank classification. ASIA: East Asia and Pacific + South Asia. ECA: Eastern Europe and Central Asia. LAC: Latin America and Caribbean. MENA: Middle East and North Africa. SSA: Sub Saharan Africa. WENA: Western Europe and North America.

While a popular uprising can lead to a democratic transition, this does not mean that all social groups in the following regime have equal power. The V-Dem database provides some information on the distribution of political power across social groups at a given point of time. We use the responses, subsequently mapped on a linearized ordinal scale (ranging from about -3 to 4), to the following question “Is political power distributed according to social groups?” Responses, on the original scale, can vary from 0 (political power is monopolized by one social group comprising a minority of the population) to 4 (all social groups have roughly equal political power or there are no strong ethnic, caste, linguistic, racial, religious, or regional differences to speak of). We consider the next regime to be minority-dominated if the ordinal index takes the values of 0 or 1 (monopolization of political power by a minority of the population). 30% of the post-revolution regimes are minority-dominated. Figure 10 shows that for these regimes, minority domination was also a feature pre-revolution (in the Figure, the scale of the
Notes: Data come from the V-Dem database and Haber and Menaldo (2011). Reported values: average of values one and two years before transition. For ease of interpretation, the IHS value of natural resources abundance has been divided by ten.

Linearized political power index is inverted such as a larger value implies more unequal political power. In addition, there is a clear relationship, pre- and post-revolution between minority domination and particularism. Minority domination is associated with more particularistic public spending. Post-revolution, particularism tends to fall, but much more strongly in majority-dominated (i.e. not minority-dominated) regimes.

3 The Set-up

3.1 General specifications

As in the standard institutional change theory (Acemoglu and Robinson, 2006), we consider an economy initially populated by two groups: a non-benevolent dictator of size $x < 1$ and a group of citizens of size 1. Initially, the political regime in place is an autocracy taking all the relevant economic decisions, in particular redistribution (for example of natural resources windfalls) or taxation (say, of labor income), not speaking about the exclusive use of coercion and repression. The unique potentially powerful
Figure 10: Minority domination and particularism in revolution-led democratic transitions

Notes: Data come from the V-Dem database. Before reported values: average of values one and two years before transition. After values: median values over the life of the post-revolution regime.

lever in the hands of citizens is the possibility to revolt against the autocracy, which would pave the way to democracy or at least to a more democratic regime in a sense to be defined later. If the revolutionary threat is credible, the autocracy may adjust their economic and political decisions to curb it in order to retain power when possible. If not, a political regime change will occur. All the action takes place in one unit of time, the model is static.

As outlined in the introduction, we depart from the standard framework just described above in four essential ways. First of all, we consider that the population is not homogeneous. We stick here to the most generic heterogeneity: citizens are divided in two subgroups, a minority and majority, no matter which deep reason is behind this subdivision (ethnic, economic, religious or whatever). Here, heterogeneity is generic, it is simply reflected in the subgroup size. Second, citizens’ group membership is an essential determinant of civil conflicts and political competition, and can hardly be omitted in any comprehensive analysis of endogenous institutional change, starting with revolutions and civil wars. As explained in Section 2, this shows up neatly in the particularistic na-
ture of public expenditures, which captures the unequal distribution of political power across subgroups of citizens and may be at the onset of severe civil conflicts. While the distinctive political and economic behavior of minorities has been invoked in several theories of political competition (see again, Hirshleifer and his paradox of power, 1992, and the very long sequence of papers devoted to this paradox), their role in political regime change has not been yet treated in a full-fledged formal setting although several recent contributions in political sciences (see again Albertus and Menaldo, 2018) have documented a decisive role of such minorities in numerous regime changes, which we have already accounted for in Section 2. Here, we provide a simple theory connecting the so-called dominant minority with institutional change. Third, we introduce (windfall) resources, which plays a central role in the literature of civil conflicts (see for example, Collier and Hoeffler, 2004). More importantly, incorporating resources in the model allows to address the so-called resource curse (see again Robinson et al., 2006) in a new framework with a minority/majority structure, therefore allowing potential interactions with another powerful engine of conflicts, population polarization. Fourth, we distinguish between labor income and resources, and labor supply is endogenous. As we have mentioned in Section 2, different institutional regimes might induce different work incentives, and the presence of resource redistribution adds another driver for these incentives, the overall effect being far nontrivial.

Hereafter, we describe the general ingredients of the theory. Consistently with the discussion above, we postulate that citizens are divided in two subgroups of different size: they might belong either to the majority (M) of size $q$, or to the minority (m) of size $1 - q$. Of course $q \in (\frac{1}{2}, 1)$. There are two main sources of income in this economy: resource revenues windfalls, $R$, and the labor income, $w$, earned by citizens in both the formal and the informal sector, where $w$ is the exogenous hourly wage rate and $l$ the time worked or labor supply. Here, we use the simplest specification for resource revenues (no extraction sector, no dynamics). An interesting possible economic configuration
opened by the incorporation of resource windfalls is the possibility for citizens to choose
low or even zero labor supply without starving and for the autocrats to play on rent
redistribution not only to satisfy citizens’ consumption needs but also to increase their
utility from leisure. The inclusion of windfall resources into the theory is also consistent
with the fact that a large number of recent revolutionary and regime change episodes
have occurred in resource-rich countries, the Arab spring being the most recent one (see
Section 2). Furthermore, to avoid any source of heterogeneity except size, we assume
that productivity \( w \) is the same across groups of citizens.

The utility of the representative agent \( i = \{M, m\} \) at time \( t \) is defined over the
consumption of a private good, \( c \) and leisure, \( 1 - l \) and has the following quasi-linear
form:

\[
U^{k,i} = c^{k,i} + \gamma \ln[1 - l^{k,i}]
\]

with \( k = \{A, D\} \), the political regime at time \( t \): autocracy (A) or democracy (D)
with \( k = A \) at \( t = 0 \). Parameter \( \gamma > 0 \) defines the weight of leisure with respect to
consumption, i.e. the marginal rate of substitution of leisure in terms of consumption.\(^4\)

The assumption of a quasi-linear utility function is the first departure from the typical
model of the institutional changes literature. This function is able to capture the role
that labor market and income taxation might have on the emergence of revolutionary
movements and linearity in consumption is needed for analytical tractability. We will
highlight along the way the specific implications of the log-linear specifications and their
connection with the stylized facts highlighted in Section 2 will become apparent.

In the rest of this Section, we proceed as follows. We first characterize citizens’ deci-
sions in the autocratic regime for given redistribution policy, after a couple of important
remarks on how we deal with citizens and income heterogeneities. Then we describe the
post-revolution regime driven by a dominant minority with a special emphasis on the

\(^4\) Preferences for leisure are the same in the citizens population. Indeed, there is no reason to suppose
that they are different. This reasonable simplification allow us to concentrate on the role played by the
size heterogeneity and obtain general results that are not driven by other types of heterogeneity.
definition of particularistic public expenditures and the game-theoretic foundations of minority dominance.

3.2 The autocratic regime

Initially, the autocrats hold all the political and economic power, the country lives in full autocracy. As in the seminal democratization theory developed by Acemoglu and Robinson (2001), the autocrats are the strategic leader of the game. They set their redistribution, taxation and repression policies to cope with the revolutionary threats, anticipating the reaction of citizens to their policies. In other words, the autocrats anticipate the reaction functions of citizens to their policies, and are assumed to identify accurately the revolutionary threats, allowing them to adjust policies to remain as long as possible in office. In this subsection, we characterize the latter reaction functions.

With respect to the standard theory à la Acemoglu and Robinson, two new questions arise. One has to do with citizens’ heterogeneity. Autocrats may not treat the minority in the same way as the majority. Things can go in all ways: the dictator may treat better either the majority or the minority or may not wish to discriminate between the two groups. In the recent years, there has been a growing literature both in political science and economics on the so-called liberal democracies, that is democracies guaranteeing the civil rights of minorities. Mukand and Rodrik (2015) is an illustration of such a literature stream in economics.5 In other contexts, elite may rely on minorities to consolidate their position in office, such as in autocracies dominated by an ethnic minoritarian group (a very know example is Saddam Hussein’s long reign over Irak backed by a quite thin Sunni minority). Finally, as in former communist dictatorships, the autocrats could fiercely deny particularity to any group and treat equally (at least apparently) all the citizens. Here, we assume that the autocrats do not discriminate in any essential way between the majority and the minority; in particular, the same tax and redistribution rates are

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5Yascha Mounk’s best-seller “The People vs Democracy” is a much more visible example of this growing literature following the rise of populism in the occidental world and Eastern Europe.
applied to both subgroups. By doing so, we do not aim to specialize in communist regimes or the like but to shut down a grievance channel in the autocratic phase in order to focus on pure political competition in the post-autocratic regime as claimed in the introduction.

The second new specification question has to do with the composition of income in our model. Contrary to the traditional setting, national income is twofold, resource windfalls and labor income. The former is redistributed to citizens at a rate, say $\mu^A$, and labor income is possibly taxed at a rate $\tau^A$ so that a fraction $1 - \tau^A$ of labor income remains at the disposal of citizens. Strictly speaking, two fiscal instruments are in the hands of the dictator. We shall however assume that the autocrats redistribute resource windfalls and labor income at the same rate, that is: $\mu^A = 1 - \tau^A$. This simplification will not only ease computations without affecting the generality of our main results, it is also supported by evidence on resource-rich countries, displayed in Section 2, according to which large levels of resource revenues redistribution in these countries (when counting the so-called implicit subsidies, in particular energy subsidies) come together with a relatively low level of labor income taxation.

In what follows, we shall therefore assume that the autocrats sets a unique redistribution rate, $\mu^A$, which applies to all citizens, irrespective of their group membership and the revenue source. This in turn delivers the per capital consumption under autocracy:

$$c^A = (R + wl^A)\mu^A,$$

the upper-index $A$ being meant for autocracy. Given the choice of redistribution imposed by the elite, the representative citizen will choose a work effort, $l^A$ such that the utility function (1) is maximized under the constraint (2):

$$l^A = 1 - \frac{\gamma}{w\mu^A}$$

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First of all we need to define the threshold $\mu^A \equiv \frac{\gamma}{w}$, giving the minimum share of income redistributed to citizens for labor supply to be non-negative. Accordingly, we impose the following constraint:

**Constraint 1.** *Redistribution policy is such that $\mu^A \geq \underline{\mu}^A$ with $\gamma < w$.***

Under the constraint above, particularly the condition on labor productivity, $\gamma < w$, we ensure that $\underline{\mu}^A \in (0, 1)$ and $l^A \in [0, 1)$. It is worth pointing out that our constraint on the control set of the dictator (minimum redistribution) does prevent zero redistribution even in the absence of explicit revolutionary threat. This comes from the log-linear specification of the utility function: if redistribution tends to 0, then consumption goes to zero by equation (2) and labor supply becomes infinitely negative (so as to increase utility from leisure infinitely). We shut down this theoretical (and unrealistic) possibility by setting the minimal redistribution threshold, $\underline{\mu}^A$. Active revolutionary threat would lead the elite to raise redistribution above this level.

### 3.3 The post-revolution regime

Let us assume at the moment that a revolution took place and that a new regime has replaced the initial autocracy (which ends up leaving the country). Of course, revolutions do not occur systematically and we shall characterize accurately their occurrence in the next section. Here, we need a couple of preliminary points to properly describe citizens’ behavior in the post-revolution era. In particular, we need to specify the cost of revolutions and the role of social groups (minority/majority in our context) in this process, a new question implied by our heterogeneous population assumption.

Concerning the cost of revolutions, we follow Boucekkine et al. (2016) by assuming that citizens incur a coordination cost $\psi[q]$ when revolting against the elite. The latter cost dependent on the degree of population polarization in a precise way to be specified later. This is the single cost we account for in our setting. Needless to say, we could have easily dealt with an additional fixed cost reflecting destructions due to popular
uprising as in Acemoglu and Robinson (2006). Without loss of generality, we deliberately focus on coordination costs, which do depend on population heterogeneity, a feature we incorporate in our model with the minority/majority split. Since citizens are divided into majority and minority groups, we reasonably model this coordination cost as an increasing function of the polarization level of the society, where \( \psi[q] \) is an inverted u-shaped with a maximum in \( q = \frac{1}{2} \). The larger the polarization within citizens, the larger the cost of the revolution and, therefore, the lower the post-revolution disposal material pay-off.

Finally, we make two simplifying assumptions. First of all, we remove all sources of uncertainty from the model: in particular we postulate that the cost of revolution is deterministic, and unless the autocrats are able to avert the revolution by an appropriate and feasible policy, the autocracy will be removed provided citizens are better off (in terms of utility) after revolution. Second, we assume that the revolutionary threat is only credible if it is backed by the majority. In other words, the support of the minority, whatever its demographic weight \( (1 - q < \frac{1}{2}) \), is not a prerequisite for revolutions. Of course, the demographic weight of the minority, that is polarization, is a fundamental determinant of revolutions through the coordination cost function, \( \psi(q) \), but their direct intervention in the revolutionary process is not a prerequisite. Again here, we aim to “sterilize” the direct impact of minority before the revolutionary process. Allowing for the latter channels will of course reinforce the political and economic relevance of the minority. We shut them down. As a consequence, to characterize the situations of potential popular uprisings, we only have to check whether the representative agent of the majority is better off with a revolution given the characteristics of the autocratic and post-revolution regimes.
3.3.1 Group size heterogeneity and redistribution: particularistic redistribution

We now develop the key feature of the set-up, the particularism in public spending associated with the minority/majority partition of population. Section 2 has already documented this crucial characteristic of public expenditures for a large sample of countries, in particular for resource-rich countries. To keep things comparable with the initial autocratic regime, we assume that the total income earned by any citizen in the post-revolution regime, that is again labor income and resource rents, is taxed at the same rate, irrespective of group membership. We shall denote by \( \mu^i \in (0, 1) \), with \( i \in \{M, m\} \), the after-tax share of total income retained by citizens, which is the exact counterpart of the redistribution rate, \( \mu^A \) under autocracy.\(^6\) But while the latter is fixed by the elite in autocracy, we now assume that \( \mu^i \) is determined by the group with the \textit{de jure} power. In other words, the distinctive feature of the post-revolution regime in our setting is that policy (here fiscal policy) is not necessarily determined by the majority but by the group who owns the political power. This is entirely consistent with the analysis of Albertus and Menaldo (2018) outlined in the introduction. In following this analysis, we depart from the median voter specification, which is intensively used to represent political decision-making in the standard model of democracy.

We now get to the critical point of our model, the modeling of particularistic redistribution. Assume that institutional change does not alter the fundamentals of the labor market nor the pace of technological progress such that the wage rate remains the same, equal to \( w \). We also hypothesize the same for the size for resource revenues.\(^7\) The total tax revenue collected by the democratic government at time \( t \) writes as follows:

\[
G = (R + wt^M)(1 - \mu^i)q + (R + wt^m)(1 - \mu^i)(1 - q).
\]  

\(^6\)Notice that to have redistribution between groups we must have that \( \mu^i < 1 \).
\(^7\)This is a one-period static model, and as such, it’s only supposed to capture short-term effects.
Since our economy is populated by agents belonging to different groups, the key point is to understand how public authorities operate to share public resources between groups of citizens, here minority versus majority. Given that population is normalized to 1, we observe that the per-capita public transfer to citizens can be written as $\lambda^i G$. The parameter $\lambda^i \in [0, 1]$ determines the type of redistribution system. The standard public good universalistic scenario necessarily implies $\lambda^i = 1, \forall i \in \{M, m\}$. Put differently, in this scenario the total tax revenue is equally shared between citizens and everybody receive a net transfer equal to $G$.\(^8\)

The stylized facts provided in Section 2 do not support at all this universalistic scenario. Instead strong evidence of fiscal particularism has been put forward. We therefore consider a particularistic use of tax revenues assuming that $\lambda^i$ is a function of the group’s size in the population. As argued in the introduction, the key idea is that the latter function is inversely related to the relative size of each group. Here we assume that the total tax revenue is redistributed from the government to each group such that $\lambda^M = \frac{1}{q}$ and $\lambda^m = \frac{1}{1-q}$, with $q > \frac{1}{2}$. It follows that the redistribution level of resources for any member of the majority and minority is given by $\frac{G}{q}$ and $\frac{G}{1-q}$, respectively. Accordingly, the net transfer per-capita for any member of the majority and minority is given by $s^M = \frac{G}{q} - z^M$ and $s^m = \frac{G}{1-q} - z^m$ respectively, with $z^M = (R + w^M)(1 - \mu^i)$ and $z^m = (R + w^M)(1 - \mu^i)$. Finally, the budget constraints for the representative agent of the majority ($M$) and minority ($m$) group write as:

\[
c^M = \left[ (R + w^M \mu^i + s^M \right] (1 - \psi[q])
\]

\[
c^m = \left[ (R + w^M \mu^i + s^m \right] (1 - \psi[q])
\]

\(^8\)Since heterogeneity drops as $\lambda^i G = G$, the model predicts a tax rate of zero, or $\mu^i = 1$, independently from the group who holds the political power. The proof of this claim is in appendix A.1.
Equations (5) and (6) define the disposable income and therefore consumption of a member of the majority (M) and minority (m) respectively under group size-based particularistic redistribution. Note that the total resource constraint of the economy when a revolution takes place always check that total income is equal to total consumption: 

\[ [R + w l^M q + w l^m (1-q)](1-\psi[q]) = q c^M + (1-q) c^m. \]

In other words the resource constraint of the economy is always binding, since the total transfers, i.e. \( S \equiv s^M q + s^m (1-q) \) equals the total tax revenue collected by the government \( G \).

Before getting to the second crucial aspect of our theory, that is how competition sets in between the minority and majority in the post-revolution regime, it is important to provide a more accurate characterization of particularism in order to identify precisely how it is connected with citizens’ decisions. As a preliminary remark, it is worth pointing out that, as outlined in the introduction, our simple particularistic fiscal scheme is extreme in the sense, for example, that as the minority size decreases, its share of fiscal revenues keeps on strictly increasing.\(^9\) 

Our scheme implies that redistribution goes from the majority to the minority, and the smaller the minority, the more it gets favored by this scheme. As we have argued above, this characteristic of the problem makes the main research question even trickier.

It’s possible to characterize more finely the particularistic process. Let us work with the following indicator of particularism:

\[
P = \frac{(1-q) S^m}{(1-q) S^m + q S^M}.
\]

\(^9\)In our simple specification, the share \( \lambda^m(q) = \frac{1}{1-q} \) goes to infinity when the size of the minority goes to zero. Replacing this function with bounded \( \lambda^m(q) \) functions when \( q \) tends to 1 does not change the results qualitatively provided the functions are decreasing in the size of the minority, \( 1-q \).
Notice that $P$ is entirely consistent with the definition of particularism used in the database V-Dem exploited in Section 2. Particularism is the share in total transfers earned by the minority. Using equations (5) and (6) above, it is readily seen that $P$ is fundamentally endogenous in that it intimately depends on the decisions variables $l^m$ and $l^M$, that is:

$$P = \frac{q (R + w l^M)}{q (R + w l^M)+(1-q)(R + w l^m)}.$$ (7)

It is worth pointing out that while the policy variable $\mu^i$, does not show up directly in the expression above, it does matter via the labor supply functions, which are heavily dependent on policy (as it is already the case in the autocratic regime, see equation (3)). Note also that our particularism indicator $P$ depends directly and indirectly (via labor supplies a priori) on all the parameters of the model ($w$, $q$ and $R$). The link between particularism and polarization (as captured by $q$) is particularly interesting. Finally, it can be highlighted that our indicator $P$ may also be used under the alternative political regime, that is autocracy. Tracking the evolution of particularism with institutional change is another key question that we can touch here. The following proposition might well be useful in this respect.

**Proposition 1.** The particularism indicator $P$ checks the following properties:

1. When $l^m = l^M$, one gets $P = q$.
2. If $l^m > l^M$, then $P < q$.

The proof is trivial after substituting the net transfers per capital $s^m$ and $s^M$ (given above) in the expression of the indicator $P$. Proposition 1 is far from surprising: group size-based particularistic redistribution works as a redistribution scheme from the majority to the minority for equal labor supplies. When one group works more than the other, its share of the total fiscal revenue goes down. It can be noticed that when labor supplies are equalized, the particularism indicator is exactly equal to the size of the
majority. This allows to connect our fiscal particularism story to the polarization-based theories of conflicts (see for example, Esteban and Ray, 2011). Notice that when society tends to be fully polarized (that is \(q\) tends to \(\frac{1}{2}\)), the indicator \(P\) tends to \(\frac{1}{2}\): the two groups tend to have the same share of total fiscal revenue. Polarization has a double role in our theory: on one hand, it makes coordination costs larger and thus it discourages rebellions, but on the other hand, it increases the prospects of income extraction for the majority under the particularistic scheme, which might push the majority to revolt against the initial elite and foster transition to a minority-based post-revolution regime.

Full analysis of this configuration is provided in the rest of this paper. To this end, we need to specify how economic and political competition sets in the post-revolution regime.

### 3.3.2 Political competition in the post-revolution period

We now depict political competition in the post-revolution period. Obviously, this model should be compatible with the particularistic public spending component of the model. As clearly outlined in Section 2, based on evidence from V-Dem database, the latter type of public spending is compatible with the presence of dominant minorities. Therefore, we shall model political competition in the post-revolution period giving strategic advantage to the minority. As commented in the introduction, this advantage might derive from past proximity to the ousted incumbent, but we do not give preference to such an advantage here for reasons already outlined above. We rather stick to the Algerian case where during the dictatorship period (under Boumediene presidency, 1965-1978), minority membership was overlooked (or supposed to be so). Hereafter, we depart from the median voter specification in the typical models of democratic transition (Acemoglu and Robinson, 2006) and move to a simple Stackelberg game of political competition where the minority is the leader.\(^{10}\) Clearly, this modeling is a shortcut. In reality, mi-
nority dominance takes much more sophisticated lobbying and manipulation forms. However, our Stackelberg basic modeling captures the two essential features of the minority dominant-based post-revolution regimes we target: departure from the median voter and strategic advantage to the minority.

Let us now sketch the game and in particular the timing of the politico-economic decisions. In the spirit of Stackelberg, the leader, here the minority, is the player who moves conditionally to the best responses of the other player. This means that the follower chooses its strategy without knowing the optimal decisions of the leader. Here comes the timing in the post-revolution period if any. First, the representative agent of the majority $M$ chooses her work effort $l^M$ such that the utility function defined by (1) is maximized under the budget constraint (5), yielding

$$l^M[\mu^m] = 1 - \frac{\gamma}{w\mu^m(1 - \psi[q])}$$  \hspace{1cm} (8)

Next, the representative agent of the minority chooses her labor supply and the redistribution policy, i.e. $\mu^m$, such that the utility function (1) is maximized under the budget constraint (6), given the reaction function (8). Deriving the two corresponding first-order conditions of this problem and after a few trivial algebraic operations, one gets:

$$l^m[\mu^m] = 1 - \frac{\gamma}{w\mu^m(1 - \psi[q])}$$  \hspace{1cm} (9)

and

$$\mu^m = \frac{2\gamma q}{\gamma(1 - q) + \sqrt{\phi^m[q, \gamma, R, w]}}$$  \hspace{1cm} (10)

to $\mu^M = 1$. Because particularistic spending implies transfers from the majority to the minority, if the majority were the strategic leader, it would minimize such transfers. Actually, by setting $\mu^M = 1$, there is no redistribution, and therefore no particularistic spending. As it transpires from the empirical Section 2, the latter is inherent to the presence of dominant minorities.

The recent political turmoil and inherent (ongoing) legal processes in Algeria have uncovered a vast body of such practices that typically bypass the parliament in an otherwise formally democratic multiparty regime. See https://www.arab-reform.net/publication/algeria-inventing-new-political-rules/.
with $\phi^m[q, \gamma, R, w] = \gamma^2(q - 1)^2 + 4\gamma q(2q - 1)(1 - \psi[q])(R + w) > 0$. One should already notice that given the first-order conditions (8) and (9), the labor supplies of the minority and majority member are equal. This is no surprise as they share the same log-linear utility function and face the same taxation rate, $\mu^m$. As repeatedly commented above, the unique effective difference between the two groups derives from particularistic spending. We can go a small step further and get all the variables of the game in closed-form. Indeed, using $\mu^m$ as defined by (10), it is straightforward to derive the optimal labor supply of citizens in a regime with dominant minorities:

$$l^m = 1 - \gamma(1 - q) + \sqrt{\phi^m[q, \gamma, R, w]} \over 2qw(1 - \psi[q])$$

(11)

To guarantee that the labor supply of citizens and the redistribution policy are in their admissible intervals, we need one crucial restriction on parameters:

**Constraint 2.** For given $q$, preferences for leisure are such that $\gamma < w(1 - \psi[q])$.

Constraint 2 guarantees that the voted policy is such that $\mu^m \in (0, 1)$. Note that if $q$ is given, as we suppose here, then $\psi[q]$ measures the upper bound of the coordination cost of the revolution, that is when $q = \frac{1}{2}$ and the citizens’ population is perfectly polarized. Constraint 2 also implies that the labor supply is always positive and lower than 1 for any $\mu^m \in (0, 1)$. Given the above parameters’ restrictions to guarantee a well-specified economic problem, the model necessarily predicts that $c^m > c^M$. Indeed, since labor supplies are the same across groups, fiscal particularism implies that $P = q$, with $q > \frac{1}{2}$, by Proposition 1. In other words, given constant wage rate, minorities are not only dominant in terms on de jure power, but also in terms of de facto power: their members consume more goods than the majority group, and enjoy larger welfare (given equality of leisures across groups).

\[\text{We should note that since the problem is quadratic, we do get two solutions for } \mu^m. \text{ However, the second order conditions for a maximum are only met for one solution. A proof of this statement is given in Appendix A.2.}\]
A few more comparative statics are useful to understand better the determinants of labor decisions and policy choices.

**Proposition 2.** Under constraint 2, the following comparative statics hold:

(i) Minority Policy: \( \frac{\partial \mu^m}{\partial w} < 0, \frac{\partial \mu^m}{\partial R} < 0, \frac{\partial \mu^m}{\partial \psi[q]} > 0, \frac{\partial \mu^m}{\partial q} < 0; \)

(ii) Labor supply: \( \frac{\partial l^m}{\partial w} > 0, \frac{\partial l^m}{\partial R} < 0, \frac{\partial l^m}{\partial \psi[q]} < 0, \frac{\partial l^m}{\partial q} \leq 0; \)

(iii) Consumption of the minority members: \( \frac{\partial c^m}{\partial w} > 0, \frac{\partial c^m}{\partial R} > 0, \frac{\partial c^m}{\partial \psi[q]} < 0, \frac{\partial c^m}{\partial q} \leq 0; \)

(iv) Consumption of the majority members: \( \frac{\partial c^M}{\partial w} > 0, \frac{\partial c^M}{\partial R} > 0, \frac{\partial c^M}{\partial \psi[q]} < 0, \frac{\partial c^M}{\partial q} \leq 0. \)

**Proof.** The proof is given in appendix B.2. \(\Box\)

Some comments are in order here. First of all, the role of coordination costs can be readily understood. Indeed, the labor supply of both groups negatively depends on coordination costs as it should be: \( \frac{\partial l^m}{\partial \psi[q]} < 0. \) This outcome is quite intuitive because the cost of revolution enters multiplicatively and not additively in the budget constraint of the representative agent. It therefore plays the same role as a proportional income tax. The same results are obtained on consumption of both population groups for the very same reason. As \( \mu^m \) is chosen by the minority, it is reasonable to get that overall that a rising (multiplicative) cost, \( \psi[q] \), is compensated by an increase in the after-tax share of total income retained by citizens.

Now we move to the comparative statics with respect to the two components of wealth, resource revenues and labor income. Concerning the comparative statics with respect to the wage rate \( w \), one gets the usual property: \( \frac{\partial l^m}{\partial w} > 0. \) If people work more, then they get less welfare from leisure, and thus they would tend to prefer having more consumption through inter-group redistribution. Therefore, the minority members would choose \( \mu^m \) such that \( \frac{\partial \mu^m}{\partial w} < 0. \) The overall effect of the consumption of both groups is positive as it should be. Concerning the resource revenues \( R \), we have the typical wealth effects on the consumption of both groups (positive) and on labor supply (negative). More interesting, the overall effect of larger resource rents is negative on
the after-tax share of total income retained by citizens, $\mu^m$ (or equivalently, positive on the inter-group redistribution). This is one of the stylized fact highlighted in Section 2, and we shall use this property intensively in Section 4 when it comes to understand the determinants of the institutional transition.

Finally, let us examine the comparative statics with respect to polarization. Note that under constraint 2 we get $\frac{\partial \mu^m}{\partial q} < 0$. The sign of this derivative implies that polarization guarantees a higher level of non-taxed resources, i.e. $\mu^m$, to citizens. The intuition behind this result is simple: since minorities are dominant, when they are small in size (no polarization) they push for a high level of public amenities, i.e. $1 - \mu^m$ because $P = q$; when they are large in size (polarization) they prefer do not redistribute their important material pay-off because particularism reduces when $q \rightarrow \frac{1}{2}$. Put differently, government activities increase with particularism and decrease with polarization in presence of dominant minorities. However, the sign of the derivative $\frac{\partial \mu^m}{\partial q}$ remains ambiguous. An increase in $q$ brings down both the levels of the redistribution rate $\mu$ and the labor supply. Nonetheless, when $q$ increases, the coordination cost $\psi[q]$ drops which pushes up labor supply. Which effect dominates depends on how the function $\psi[q]$ is made.

4 The emergence of dominant minority-based regimes

We now move to the transition from dictatorship to dominant minority-based regimes at equilibrium. We start with some considerations on the optimal behavior of dictators, then we move to characterize the latter transitions when they occur.

4.1 The dictator/population game

We shall consider the following simple timing. First, the dictator sets its optimal redistribution policy given the labor supply function of the citizens (3). To be precise, we assume that the optimal choice of the dictator would be to set a level of redistribution $\mu^A$ such that her expected pay-off, $(R + wL^A)(1 - \mu^A)$ provided that the labor supply
and the redistribution rate are in their admissible intervals. As we will see the positivity of labor supply, will turn out to be problematic. Call $\mu^*_A$ the optimal redistribution rate of the dictator. Second, given $\mu^*_A$ and $\mu^m$, the representative member of the majority, the unique group whose rebellion is essential in revolting against the dictator, compares her payoff in the two alternative regimes and decides whether to revolt or not. If she revolts, game over: the dictator is expelled outside the country and a new regime sets in, a dominant minority-based regime. If not, the dictatorship continues.

It is important to note that by doing so, we do not allow the dictator to “strike back” in our scheme. In other words, we do not award her a strategic advantage allowing her to avert revolutions by revising her initial redistribution offer. This is common in the intertemporal democratization games à la Acemoglu and Robinson, see for example Boucekkine et al. (2016). Here, we have a one-period game and assuming our one-shot game is acceptable. The main reason however behind our simplification is, as one can guess, purely computational: if we go along the Boucekkine et al. (2016) line, we will have plenty of cases studies, some not tractable, without changing the main results qualitatively. We therefore stick to this simple dictator/population game.

We now devote a few sentences to the simple dictator’s optimal decision-making. It can be trivially established that the maximization of dictator’s payoff $(R + w\mu^A)(1 - \mu^A)$ with respect to $\mu^A$ given the labor supply function (3) delivers $\mu^i_A = \sqrt{\frac{\gamma}{R+w}}$ as interior solution. Now, again trivially, this interior solution cannot always satisfy the non-negativity of labor supply constraint as stipulated in Constraint 1: $\mu^A \geq \mu^L$ with $\gamma < w$, where $\mu^L_A$ is the redistribution level under dictatorship assuring zero labor supply. Clearly, there exists a threshold value for resources, say $R^A \equiv w\left(\frac{w}{\gamma} - 1\right)$ such that $\mu^{in} < \mu^L_A$ for all $R > R^A$. In such a case, the optimal solution for the dictator is to set $\mu^*_A = \mu^L_A$. From now on, we assume the following:

**Assumption 1.** The country owns a level of natural resources such that $R > R^A$.

Beside simplifying computations, the assumption above has two other virtues. First,
the reference dictator’s policy (which turns to be optimal under the assumption) implies a reference labor supply equal to zero, which is a quite appealing and transparent basis to study the trade-off between dictatorship and dominant-minority based regimes. Clearly, the latter have already a disadvantage with respect to the former in terms of leisure. Would the fiscal particularistic scheme compensate for this deficit in leisure by allowing for more consumption of majority member, thus fostering revolts against dictators? Second, as illustrated in Section 2, labor supply might decrease with resources, especially above a certain level of resources. Our assumption is from this point of view particularly appealing. We now move to the study of transitions.

4.2 Transitions

As previously discussed, the dictator chooses a level of redistribution \( \mu^A = \frac{\gamma}{w} \) which sets to zero the labor supply of citizens if assumption 1 holds. To explain revolutionary movements in resources-rich labor-poor countries, we only concentrate on the pay-off of the majority since successful revolution can only be launched if they are at least backed by the majority. Whether the latter will rebel at time \( t \) or not depends on the following comparison:

\[
 u^{A,M}[\mu^A] \leq u^M[\mu^m]
\]

where \( u^{A,M}[\mu^A] \) is the utility accruing to the majority under autocracy and the associated optimal redistribution, \( \mu^A \), and \( u^M[\mu^m] \) designates the utility of the majority under the post-revolution regime. In such a case \( \mu^m \) corresponds to the preferred policy of the dominant minority group, as defined by equation (10). Clearly, there will be no revolution whatever the level of redistribution if \( u^{A,M}[\mu^A] > u^M[\mu^m] \). Define with \( f[q, \gamma, R, w] = u^{A,M}[\mu^A] - u^M[\mu^m] \). Using equations (1)-(6), we get the fundamental equation the revolution condition in countries owning a sufficiently high level of natural
resources:

\[
f[q, \gamma, R, w] = \frac{\gamma R}{w} \left( \frac{(1 - \mu^m)(1 - q)(l^m w + R)}{q} + \mu^m (l^m w + R) \right) (1 - \psi[q]) - \gamma \log(1 - l^m) \tag{12}
\]

with \( \mu^m \) and \( l^m \) defined by equations (10) and (11), respectively.

The sign of the above equation heavily depends on parameters \( R, \gamma, w \) but also on the composition of the civil society, \( q \). In our particular setting the parameter \( q \) has a crucial role. Indeed, it represents the particularistic character of the public redistribution system as well as the level of polarization into the civil society. The latter also determines the level of coordination costs when citizens revolt against the autocrat. For the sake of simplicity and economic insight, we start separately studying the two extreme cases of polarized societies \((q = \frac{1}{2})\) and quasi-homogeneous societies \((q \text{ tends to } 1)\). We conclude with the appraisal with the general heterogeneous case where \( q \in [\frac{1}{2}, 1] \).

### 4.2.1 Polarized Societies

First of all, note that when the society is perfectly polarized, i.e. \( q = \frac{1}{2} \), coordination costs are maximal. The root in equation (10), that is \( \phi^m[q, \gamma, R, w] = \gamma^2 (q - 1)^2 + 4 \gamma q(2q - 1)(1 - \psi[q])(R + w) \), reduces to \( \frac{\gamma^2}{4} \). When \( q = \frac{1}{2} \) equations (10) and (11) reduces to \( \mu^m = 1 \) and \( l^m = 1 - \frac{\gamma w}{w(1 - \psi[\frac{1}{2}])} \), respectively. We can therefore rewrite the revolution condition (12) as follows:

\[
f\left[\frac{1}{2}, \gamma, R, w\right] = \frac{(R + w)(\gamma - w\left(1 - \psi\left[\frac{1}{2}\right]\right))}{w} - \gamma \log \left[ \frac{\gamma}{w(1 - \psi[\frac{1}{2}])} \right] \tag{13}
\]

**Proposition 3.** Under Assumption 1, the majority always revolts when \( q = \frac{1}{2} \).

**Proof.** The proof is given in appendix B.3. \( \square \)

Proposition 3 tells us that perfect polarization leads people to revolt independently of preferences for leisure, level of resources and wage rate. This might look strange at
first glance since the cost of revolting is larger when citizens are increasingly polarized. However, the degree of polarization, as captured by $q$, is also a determinant of the redistribution in the post-revolution period as it affects the right-hand side of the equation (12). It follows that full polarization is a sufficient condition for rebelling against the autocratic elite. This is an important result which sheds lights on the role that population composition might have on institutional changes when labor choices, political decisions and heterogeneity in population composition are all taken into account. In the traditional democratization model with homogeneous population (Acemoglu and Robinson, 2006) and its extensions to resource-rich countries (see Boucekkine et al., 2016), the role of revolution cost is decisive. In our setting with heterogeneous population, revolution occurs systematically under full polarization even though the cost of revolution is maximal. The role of polarization in social conflicts has already been outlined in the economic and political science literatures (see a survey in Esteban and Schneider, 2008). Quite naturally, it plays a central role in our theory, it even dominates the revolution cost effect, at least in the fully polarized societies case studied here. We shall see that it keeps on playing a crucial role in the more general case studied in Section 4.2.3.

We can dig deeper in the current case under scrutiny and extract further interesting results. First, note that when societies are perfectly polarized, inter-groups inequalities tend to disappear and particularism collapses to its minimum level $P = \frac{1}{2}$. Second, when $q = \frac{1}{2}$, the policy $\mu^m = 1$, meaning that redistribution between groups will not take place. The following corollary clarifies this claim.

**Corollary 1.** When $q = \frac{1}{2}$ then $c^M = c^m > c^A$.

*Proof.* The proof is given in appendix B.4.

Majority and minority members get all the same amount of consumptions, which is larger enough than consumption under autocracy to compensate the loss in leisure.
4.2.2 Quasi-homogeneous Societies

Consider now the other possible extreme case, that is, \( q \) tends to 1. In contrast to the previous case (\( q = \frac{1}{2} \)), \( q \) cannot be equal to 1 in our setting as this would mean that the division minority/majority vanishes. However, most of the magnitudes involved in the model are continuous in \( q \) at \( q = 1 \), and we can take the limit of these magnitudes when \( q \) tends to 1. In this scenario, the society is close to homogeneous. Thus, coordination costs tend to their minimal level \( \psi[1] \). When \( q \) tends to 1, we get \( \varphi^m[q, \gamma, R, w] = 4(R + w)(1 - \psi[1]) \). Using equations (10) and (11) we can easily derive that \( l^m = 1 - \frac{(R + w)\gamma}{w\sqrt{(R + w)\gamma(1 - \psi[1])}} \) and \( \mu^m = \frac{\gamma}{\sqrt{(R + w)\gamma(1 - \psi[1])}} \in (0, 1) \). However, note that when \( q \) tends to 1, labor supply is positive if and only if \( R < w\left(\frac{w(1 - \psi[1])}{\gamma} - 1\right) \). Since, \( R_A > w\left(\frac{w(1 - \psi[1])}{\gamma} - 1\right) \), when countries own sufficiently high levels of resources, that is \( R > R_A \), we observe \( l^m = 0 \) in the post-revolution regime with dominant minorities. We can therefore rewrite the revolution condition (12) as follows:

\[
f[1, \gamma, R, w] = R\gamma \left(\frac{1}{w} - \frac{1 - \psi[1]}{\sqrt{(R + w)\gamma(1 - \psi[1])}}\right)
\]

(14)

**Proposition 4.** Under assumption 1, the majority never revolts when \( q \) tends to 1.

**Proof.** The proof is given in appendix B.5.

Proposition 4 states that in quasi-homogeneous societies revolutions against the dictator do not occur. This is symmetric to the full polarization configuration analyzed just above: revolution will not occur even though the cost of revolution is minimal. Again, the composition of population is key. The intuition behind this extreme outcome is quite simple. When \( q \to 1 \), the large majority of citizens belong to the same group and particularism tends to its maximum level, \( P \to 1 \). This can be hardly acceptable by the majority, which ends up preferring autocracy. Proposition 4 shows clearly why this is so: in this extreme case, majority members will consume definitely less than under
autocracy, and will never accept the dominance of such a small minority. Corollary 2 clarifies this result.

**Corollary 2.** When \( q \to 1 \) then \( c^M < c^A \).

**Proof.** The proof is given in appendix B.6.

This is an interesting complementary result to the former extreme case: for revolutions to be launched and transitions from autocracy to minority dominant-based regimes to occur, the minority has to be large enough. In other terms, some level of polarization should prevail.

The extreme results obtained for \( q = \frac{1}{2} \) and \( q \) tends to 1 emphasize one crucial issue that we can only take into account by considering intermediate scenarios: the role that polarization might have in explaining coordination costs and revolutionary movements against autocracy. We shall therefore consider the more general case, that is an heterogeneous society in which \( q \in (\frac{1}{2}, 1) \) to check the role that both productivity, polarization and rents from natural resources might have to explain institutional changes and the emergence of elite-biased democracy.

### 4.2.3 Heterogeneous Societies

When societies are composed of different groups of citizens with given sizes, the transition problem from autocracy to dominant minority-based regime becomes quite involved. A probable key determinant of the problem is the magnitude of natural resources.\(^\text{13}\) One obvious way to view this role is to come back to the comparative statics results, that is Proposition 2. The picture is quite involved. According to the latter proposition, as the level of windfall resources goes up, labor supply of individuals under dominant minority, irrespective of their group membership, goes down. This means that labor income drops and it is unclear what the total impact on wealth would be, and this

\(^{13}\)We shall focus here on this wealth component for the connection with the resource curse literature. As we shall see just below, this does not mean at all that the labor market does not matter.
said without having incorporated yet the impact on redistribution of the initial rise in resources. It results that the impact of natural resources income is highly nontrivial in our framework. The other highly probable key determinant of institutional change is the level of polarization, as this transpires from the analysis displayed just above for extreme population composition configurations. Indeed, under full polarization, revolution will take place whatever the level of resources, as explained in Section 4.2.1. Again the comparative statics proposition shows however how analytically involved the problem outside these extreme cases is, with a general cost function $\psi[q]$.

In this section, we show that the transition problem can be deeply characterized with both the level of resources and the extent of polarization at least for generic specifications of the latter cost function. To this end, we will assume that the cost function takes the tent form, that is $\psi[q] = 1 - q$ for $q \in \left(\frac{1}{2}, 1\right)$. The tent function has been chosen for the sake of analytical tractability, but it is highly useful to isolate the main economic implications of our model.

Still, even in the tent case, the results are highly nontrivial and a few steps have to be taken. Consider the two following preliminary properties.

**Lemma 1.** Set $\bar{R}[q] = \frac{qw(\gamma w - \gamma)}{(2q - 1)\gamma}$. If $q \in \left(\frac{1}{2}, 1\right)$ and $R = \bar{R}[q] > R^A$, then $l^m = 0$ and $f[q, \gamma, \bar{R}[q], w] < 0$: people always revolt against the elites when assumption 1 holds.

*Proof.* The proof is given in appendix B.7.

**Lemma 2.** Set $\bar{q} = 1 - \frac{\gamma}{w}$. If $q \in \left(\frac{1}{2}, 1 - \frac{\gamma}{w}\right)$ then $R^A < \bar{R}[q]$, while if $q \in \left(1 - \frac{\gamma}{w}, 1\right)$ then $R^A > \bar{R}[q]$.

*Proof.* The proof is given in appendix B.8.

Lemma 1 highlights the role that rents from natural resources might have in explaining revolutionary movements. Given $q$, when countries own a level of natural resources $R = \bar{R}[q]$, citizens choose in a dominant minority regime the same labor supply as in the
autocracy but they ask for a higher level of redistribution. Since \( \mu^m = \frac{\gamma}{wq} > \mu^A = \frac{\gamma}{w} \), citizens will always revolt against the elites when \( R = \bar{R}[q] \). This result implies that for different levels of rents from natural resources if the autocrat redistributes the minimum share \( \mu^A = \frac{\gamma}{w} \), the majority of citizens may prefer support a dominant minority regime characterized by a redistribution policy \( \mu^m \). However, this decision will crucially depend on both rents from resources and polarization, as suggested by Lemma 2. The following propositions clarify this latter claim.

**Proposition 5.** Under assumption 1, if \( \frac{1}{2} < q < \tilde{q} \) citizens always revolt against the elites.

*Proof.* The proof is given in appendix B.9.

**Proposition 6.** Under assumption 1, if \( q \in (\tilde{q}, 1) \) there exists a threshold \( \bar{R}[q] \geq R^A \) such that: (i) if \( \bar{R}[q] > R^A \) then people revolt for \( R \in (R^A, \bar{R}[q]) \) and do not revolt for \( R > \bar{R}[q] \); (ii) people never revolt for \( \bar{R}[q] = R^A \).

*Proof.* The proof is given in appendix B.10.

The picture is now more complete. In a setting with dominant minorities, a different composition of the population is one of the main drivers of income distribution after a revolution is launched and a new political regime sets in, since the redistribution rate \( \mu^m \) is determined by the dominant minority. The size of the majority, \( q \), and therefore the level of polarization into the society, is one of the main determinants of the size of the cake accruing to each group under democracy. When \( \psi[q] = 1 - q \), polarization increases the coordination costs of citizens. However, the degree of polarization, as captured by \( q \), is also a determinant of the particularistic redistribution under dominant minority regime, that is \( P = q \). In particular, when polarization is sufficiently high, that is \( q < \tilde{q} \), the majority will always revolt against the elites, as suggested by Proposition 5. When, the societies is more homogeneous, that is \( q > \tilde{q} \), rents from natural resources are crucial.
Figure 11: Heterogeneous Societies when $q \in \left(\frac{1}{2}, \bar{q}\right)$: $R^\Lambda < \bar{R}[q]$

$\begin{align*}
0 & \quad R
\\
R[\bar{q}] & \quad \text{lm}
\\
\bar{R}[q] & \quad f[q, \gamma, R, w]
\\
lm & = 0
\\
lm & \quad \text{lm}
\\
lm & \quad \bar{R}[q]
\\
R & \quad \bar{R}[q]
\\
\end{align*}$

Figure 12: Heterogeneous Societies when $q \in (\bar{q}, 1)$: $R^\Lambda > \bar{R}[q]$

$\begin{align*}
0 & \quad R
\\
R[\bar{q}] & \quad \text{lm}
\\
\bar{R}[q] & \quad f[q, \gamma, R, w]
\\
lm & = 0
\\
lm & \quad \text{lm}
\\
lm & \quad \bar{R}[q]
\\
R & \quad \bar{R}[q]
\\
\end{align*}$
to determine protests against the autocratic regime. Proposition 6 shows that countries engaging in revolutions tend to be slightly less resource-rich than other countries, as suggested by stylized facts in Section 2, figure 9.

As in Boucekkine et al. (2016), two opposite forces are in action: the extent of coordination costs and the size of the cake effect outlined just before. When \( q \to \frac{1}{2} \), coordination costs are maximal but the size of the cake effect (for given coordination costs) is also maximal. The policy is therefore the most favorable for majority members when polarization tends to be complete because particularism \( P = q \) under dominant minority regimes. In contrast, the size of the cake effect, defined above, tends to diminishing with increasing majorities, that is when \( q \to 1 \). When \( q \to \frac{1}{2} \), the size of the cake effect dominates the coordination cost effect, and the majority rebels as depicted in Proposition 3. When \( q \to 1 \), the size of the cake effect tends to zero, while consumption is strictly positive under dictatorship and zero redistribution, which offsets the advantage of low coordination costs. Therefore citizens never revolt, as suggested by Proposition 4. More importantly, Proposition 5 suggests that, independently from the level of rents from natural resources \( R \), if polarization is sufficiently high \( q < \tilde{q} \), citizens will always revolt against the elites. However, natural resources play a key role in low polarized countries, i.e. \( q > \tilde{q} \). When resources are abundant, autocrats are able to redistribute a minimum share of rents from natural resources and maintain the political power, as suggested by Proposition 6.

Finally, one has to outline a crucial implication of our theory to explain real world phenomena: the interplay between resource rents, polarization and labor market conditions at the dawn of institutional change. Indeed it should be noted that an active participation of citizens to labor market under dominant minority regime is a sufficient condition to activate protests against the elite, as suggested by Figure 11. This result emerges because we have assumed a benchmark case where minimum redistribution is optimal under autocracy, i.e. \( \mu^A = \mu^d \), and, consequently, labor supply is nil. This is
an extreme scenario that simplifies and exemplifies the analysis and allows for sensible economic interpretations.

First, when polarization is sufficiently low and, consequently, fiscal particularism under dominant minority regime is sufficiently high, we do not observe an active participation of citizens in the labor market. This particular scenario is strongly useful to study resource-rich countries which, in general, show a high degree of fiscal particularism and poor labor market development, as shown in Section 2. Second, even though our benchmark theory suggests that positive labor supply is crucial to activate protests against elites, this is only part of the story since labor productivity is not a sufficient nor a necessary condition for institutional changes when societies tend to be homogeneous, as shown by Figure 12. Nevertheless, our theory highlights that the extent of polarization is less stringent when labor productivity is poor. This claim follows from the observation that \( \frac{\partial \tilde{q}}{\partial w} > 0 \): in poor-labor countries the threshold \( \tilde{q} \) tends to be located close to \( q = \frac{1}{2} \). Put differently, given \( q \) and \( R \), revolutions will be less likely. When, in contrast, labor market is strongly developed and productivity is very high, citizens are willing to participate to labor market under a dominant minority regime and they will never accept taxation and minimum redistribution from the elites.\(^{14}\)

4.3 A case study: Algeria

The theoretical results provided in the previous section put forward three main channels to explain the multiplicity of political outcomes that MENA countries have experienced during the last decades. These countries diverge in their economic, social and religious characteristics, indicating that institutional changes are based on a combination of deep socio-economic factors and not solely on differences in resource windfalls. Even though our theory suggests that polarization, and therefore fiscal particularism, have a prominent role in explaining institutional changes from autocracies to dominant minor-

\(^{14}\)Note that in the extreme case where \( w \to \infty \), we can observe that \( \tilde{q} \to 1 \): citizens will always revolt.
ity regimes, we can also show how other national factors are decisive to trigger, or not, political regime switching.

More precisely, a major implication of our theory is the following. For a given level of resource windfalls, $R$, and polarization, $q$, the threshold $\tilde{q}$, which is critical for institutional switch (see Propositions 5 and 6 just above), is increasing in the wage rate $w$, a proxy for a country’s labor market conditions and economic development. When parameter $w$ goes up, then the analytical condition $q < \tilde{q}$ is met with larger probability. Institutional changes may thus emerge independently from resource rents or polarization of the population. Conversely, when parameter $w$ is low, the level of rents from natural resources and the degree of polarization are both important and interacting factors to explain institutional change. For example, in highly homogeneous countries, that is $q > \tilde{q}$, natural resources revenues play a key role.

The case of Algeria provides an interesting and dynamic application of our theory. After the initial dictatorship of the socialist President Boumediene (1965-1978), political institutions shifted from a Marxist economic and political regime (with any kind of minority singularity repressed) to the minority dominant-based regime of the just-ousted President Bouteflika (1999-2019). Figure 13 summarizes Algerian political economy during this period. In the mid-80s, the world oil price collapsed, weakening significantly the oil-dependent Algerian economy. In 1988, violent riots erupted, leading to the breakdown of the single-party regime. The sharp fall in natural resources rents, clearly visible in Figure 13, facilitated the satisfaction of condition i) in Proposition 6. Our theory can also explain the institutional dynamics observed after the civil war. Three key developments can be highlighted. First, the rise of an Algerian dominant minority in the last three decades led to more polarization, captured in Figure 13 by stronger minority domination and greater fiscal particularism. This trend accelerated during the third term of President Bouteflika (2009-2014), considered in V-Dem as a regime change. Second, Figure 13 shows that growth performance was unimpressive,
still oil-dependent, and often associated with negative total factor productivity. Third, in 2014, the start of a new negative oil shock, which revealed itself to be persistent in subsequent years, can be observed in Figure 13. Together, these events made, again, the fulfillment of condition i) in Proposition 6 more likely. Polarization \( q \) increased, \( w \) fell bringing down \( \tilde{q} \), and \( R \) decreased. Indeed, in line with the predictions of our theory, major public demonstrations led to the resignation of President Bouteflika in 2019.

Figure 13: The case of Algeria

Notes: Data on regime change, (political) minority domination and fiscal particularism come from V-Dem. The two indexes range from -0.35 to 0.35 (less to more). Data on GDP (constant 2010 US$), GDP per capita (GDPPC) and rents from fossil fuels and minerals come from WDI. Data on total factor productivity growth rate (TFP gr. rat.) come from the Conference Board Total Economy Database and are available from 1990 only.

5 Conclusion

Based on the illuminating work of Haggard and Kaufman (2016) and Albertus and Menaldo (2018), we have developed a theory of transitions from autocracies to dominant minority-based regimes. While our concept of dominant minority-based regime is not rigorously the same as the elite-based democracies highlighted in these two important books, our approach shares some essential ingredients with the latter, in particular...
the role of dominant minorities in undermining the egalitarian foundations of the democratic project. While these minorities are connected with the former elites in Albertus and Menaldo (2018) for example, ours are not. In our view, this makes our problem more generic. Even from a more applied point of view, our theory has some interest: it is able to explain institutional changes from autocracies to dominant minority regimes in both polarized and non-polarized societies, resource-poor and resource-rich countries, as well as labor-poor and labor-rich countries. In particular, it seems reasonably applicable to track the institutional dynamics in resource-rich countries, which after post-independence autocratic communist-like regimes, turn to be dominated by minorities. The case of Algeria exemplifies such a class of countries (see Boucekkine and Boukilia-Hassane (2011), for more details).

Clearly, fiscal particularism is a key ingredient of our theory. We have provided a solid empirical basis for it, and also made clear its connection with dominant minorities in practice. Our theory does not provide any theoretical foundation for this fiscal particularism. A natural candidate for that is to strengthen the connection with Albertus and Menaldo, and give minorities an active pro-dictator role in the initial autocratic period. Other channels, along with the theories around the paradox of power, are worth exploring. In this paper, we have given priority to a more generic problem which in our view is equally challenging.

References


Appendix

A Useful checks

A.1 The Model with Universalistic Public Good

Assume that the total tax revenue (4) is equally shared between majority and minority members. The budget constraints (5) and (6) can be respectively rewritten as follow:

\[ c_M = [(R + w l^M)\mu^i + G] (1 - \psi[q]) \]

\[ c_m = [(R + w l^m)\mu^i + G] (1 - \psi[q]) \]

with \( i \in \{m, M\} \). As for the particularistic redistribution system the government budget constraint is always binding since \( c_M q + c_m (1 - q) = [R + (l^M q + l^m (1 - q))(1 - \psi[q])]. \)

Without loss of generality assume that \( \psi[q] = 0 \). Consider first that the representative agent of the minority is the strategic leader of the game. Therefore, the representative agent of the majority moves first and maximizes utility (1) under the budget constraint \( c^M \) defined above and chooses her optimal labor supply:

\[ l^M = 1 - \frac{\gamma}{qw(1 - \mu^m) + w \mu^m} \]

Given majority labor supply, the representative agent of the dominant minority maximizes her utility under the budget constraint \( c^m \) to derive:

\[ l^m = 1 - \frac{\gamma}{qw(1 - \mu^m) + w} \]

Since minority members have de jure power, they will set the fiscal policy. Deriving their utility with respect to \( \mu^m \) one gets two solutions: \( \mu^m = 1 \) and \( \mu^m = \frac{q^2 + q - 1}{(1 - q)^2} \). The first solution is compatible with local concavity if and only if \( q < \frac{2}{3} \). The second solution,
if and only if \( q > \frac{2}{3} \). However, in the latter case, we observe that \( \mu^m = \frac{q^2 + q - 1}{(1 - q)^2} > 1 \).

Therefore the only acceptable solution is \( \mu^m = 1 \). When \( \mu^m \to 1 \) we get that:

\[
l^M = l^m = 1 - \frac{\gamma}{w}
\]

Therefore, the model with pure public good degenerates to the standard model of institutional changes without redistribution across groups of citizens.

Assume now that the strategic leader of the game is the representative agent of the majority. Minority members move first and choose the following labor supply:

\[
l^m = 1 - \frac{\gamma}{w - qw(1 - \mu^M)}
\]

Given minority labor supply, the representative agent of the dominant majority maximizes utility under the budget constraint \( c^m \) to derive:

\[
l^M = 1 - \frac{\gamma}{qw(1 - \mu^M) + w\mu^M}
\]

Deriving utility (1) with respect to policy, we get \( \mu^M = \{1 + \frac{1}{q^2} - \frac{3}{q}; 1\} \). The first solution is not admissible since it is always negative. Again, the model with majority strategic leader predicts that \( \mu^M = 1 \) and \( l^m = l^M = 1 - \frac{2}{w} \). As for the dominant minority scenario, the model degenerates to the benchmark case with only one group of citizens.

**A.2 Second Order Conditions for a Maximum**

Given the reaction function (8), minority members maximize (1) under the budget constraint (6). Solving both first order conditions for a maximum in terms of labor supply, one gets:

\[
l^m[\mu^m] = 1 - \frac{\gamma}{w\mu^m(1 - \psi[q])}
\]
and
\[ l^m[\mu^m] = \frac{q((\mu^m)^2(1-\psi[q])(2R+w)-\gamma)-(\mu^m)^2R(1-\psi[q])}{(\mu^m)^2(1-q)w(1-\psi[q])} \]

Since the problem is quadratic, we derive two solutions for \( \mu^m \):

\[ \mu^{m,1} = \frac{2\gamma q}{\gamma(1-q) + \sqrt{\gamma(1-q)^2 + 4q(2q-1)(1-\psi[q])(R+w)}} \]

and

\[ \mu^{m,2} = -\frac{2\gamma q}{\gamma(1-q) + \sqrt{\gamma(1-q)^2 + 4q(2q-1)(1-\psi[q])(R+w)}} \]

Using the above solutions and (9) we can derive two solutions for labor supply (say \( l^m,1 \) and \( l^m,2 \)). To check if these solutions are stable we should the sign of the determinant.

We define the Hessian matrix of second partial derivatives as follows:

\[
\text{Hess}(f[q,g,R,w]) = \begin{bmatrix}
-\frac{2\gamma}{(m-1)^2} & w(1-\psi[q]) \\
w(1-\psi[q]) & \frac{2\gamma}{(q-1)(\mu^m)^3}
\end{bmatrix}
\] (15)

The determinant of the Hessian is therefore given by:

\[ \text{det} = -\frac{2\gamma^2}{(m-1)^2(q-1)(\mu^m)^3} - w^2(\psi[q] - 1)^2 \]

Since the sign of the determinant is ambiguous, we should check local stability when solutions are given by the following two couples of policy/labor supply: (i): \( \mu^m = \mu^{m,1} \) and \( l^m = l^{m,1} \); (ii): \( \mu^m = \mu^{m,2} \) and \( l^m = l^{m,2} \). For the first solution, (i), we derive:

\[ \text{det} = \frac{w^2(\psi[q] - 1)^2\sqrt{\gamma(\gamma(1-q)^2 + 4q(2q-1)(1-\psi[q])(R+w))}}{\gamma(1-q)} > 0 \]

while for the second, (ii):

\[ \text{det} = -\frac{w^2(\psi[q] - 1)^2\sqrt{\gamma(\gamma(1-q)^2 + 4q(2q-1)(1-\psi[q])(R+w))}}{\gamma(1-q)} < 0. \]
Since the second solution is negative, the problem is convex at local level. Therefore we exclude this solution.

B Proofs

B.1 Proof of Proposition 1

The proof is trivial. If labor supplies are equal, then particularism is inversely related to polarization: as polarization decreases (that is $q$ goes to 1), the larger the share of national transfers accruing to the minority.

B.2 Proof of Proposition 2

Define for simplicity $\phi^m_0[q, \gamma, R, w] = \gamma^2(q-1)^2 + 4\gamma q(2q-1)(1-\psi[q])(R+w) \equiv \phi^m$. First of all, we check the sign of its partial derivatives: \[ \frac{\partial \phi^m}{\partial w} = \frac{\partial \phi^m}{\partial R} = 4\gamma q(2q-1)(1-\psi[q]) > 0, \] \[ \frac{\partial \phi^m}{\partial \psi[q]} = -4q(2q-1)(R+w)\gamma < 0. \] The derivative with respect to group size \[ \frac{\partial \phi^m}{\partial q} = \gamma \left( 4(4q-1)(R+w) - 2\gamma(1-q) + 4(R+w)((1-2q)q\partial \psi[q]/\partial q + (1-4q)\psi[q]) \right). \] We can show, after some tedious algebra, that \[ \frac{\partial \phi^m}{\partial q} > 0. \]

First, consider the policy $\mu^m$. The partial derivatives \[ \frac{\partial \mu^m}{\partial w} = -\frac{q\gamma \partial \phi^m}{\sqrt{\phi^m(\gamma(1-q)+\sqrt{\phi^m})}^2} < \] \[ \frac{\partial \mu^m}{\partial R} = -\frac{q\gamma \partial \phi^m}{\sqrt{\phi^m(\gamma(1-q)+\sqrt{\phi^m})}^2} > 0, \] because \[ \frac{\partial \phi^m}{\partial w} > 0, \] \[ \frac{\partial \phi^m}{\partial \psi[q]} > 0 \text{ and } \frac{\partial \phi^m}{\partial \psi[q]} < 0, \] defined above. The partial derivative \[ \frac{\partial \mu^m}{\partial q} = \frac{2\gamma \phi^m}{\sqrt{\phi^m((1-q)+\sqrt{\phi^m})}^2} \] requires more algebra. The sign depends on the numerator because \[ \frac{\partial \phi^m}{\partial \psi[q]} > 0. \] Using \[ \frac{\partial \phi^m}{\partial \psi[q]} \] defined above, we get \[ \frac{\partial \mu^m}{\partial q} = \frac{2\gamma (1-q)+\sqrt{\phi^m}+2q(R+w)(\psi[q]-1+q(2q-1)\frac{\partial \psi[q]}{\partial q})}{\sqrt{\phi^m((1-q)+\sqrt{\phi^m})}^2}, \] with \[ \frac{\partial \psi[q]}{\partial q} < 0 \] for all $q \in (\frac{1}{2}, 1)$. Using $\phi^m$ it can be shown that the numerator is always negative. It follows that, \[ \frac{\partial \mu^m}{\partial q} < 0. \]

Now consider the labor supply under dominant minority regime. The derivative with respect to the wage rate, \[ \frac{d\mu^m}{dw} = \frac{(2q-1)\gamma(2(q-1)R\gamma\sqrt{\phi^m}+2R\phi^m+w(R+w)\frac{d\phi^m}{dw})}{\sqrt{\phi^m((1-q)+\sqrt{\phi^m})}^2} > 0. \] Indeed,

15The full algebra of the latter claim is not reported here but is available upon request.
16As for $\frac{\partial \phi^m}{\partial q}$, the algebra is not reported here but is available upon request.
using \( \frac{\partial \phi}{\partial w} = 4q(2q - 1)\gamma(1 - \psi[q]) \), we can easily rewrite \( \frac{\partial c}{\partial w} = 4q(2q - 1)w(R + w)\gamma(1 - \psi[q]) + \sqrt{\phi m} + 2\phi m R > 0 \). The derivative with respect to \( R \) is given by \( \frac{\partial c}{\partial R} = \frac{(2q - 1)\gamma(2q(1 - \psi[q]) + \sqrt{\phi m}R - (R + w)\frac{\partial \phi}{\partial R})}{\sqrt{\phi m}(\gamma(1 - q) + \sqrt{\phi m})^2} \). Using \( \frac{\partial c}{\partial R} = 4q(2q - 1)\gamma(1 - \psi[q]) + \phi m \), we find that: \( \frac{\partial c}{\partial R} = \frac{\gamma(1 - 2q)w}{\sqrt{\gamma(q(1 - 2q)(1 - \psi[q]))(R + w)}} < 0 \). The derivative with respect to coordination costs \( \frac{\partial c}{\partial \phi m} = \frac{(2q - 1)(R + w)\gamma(\phi m)}{\sqrt{\gamma(1 - q) + \sqrt{\phi m}}} < 0 \) since \( \frac{\partial \phi}{\partial \psi} < 0 \). The sign of the derivative \( \frac{\partial c}{\partial \phi m} = \frac{(R + w)\gamma(2\gamma\sqrt{\phi m} - 4\phi m + (2q - 1)\frac{\partial \phi}{\partial \phi m})}{\sqrt{\phi m}(\gamma(1 - q) + \sqrt{\phi m})^2} \) is, however, ambiguous.

Finally, consider the consumption level of the minority. The derivative with respect to \( R \) is given by \( \frac{\partial c}{\partial R} = \frac{q(1 - \psi[q]) - \frac{\partial \phi}{\partial \phi m}}{1 - q} \). Using \( \frac{\partial c}{\partial \phi m} = 4q(2q - 1)(1 - \psi[q]) \), we can rewrite \( \frac{\partial c}{\partial R} = \frac{q(1 - \psi[q])(2\gamma(1 - 2q) + \sqrt{\phi m})}{(1 - q)\sqrt{\phi m}} \). The sign depends on \( 2\gamma(1 - 2q) + \sqrt{\phi m} \). Using the definition of \( \phi m \), after some algebra, we find that the latter is strictly positive. Thus, \( \frac{\partial c}{\partial R} > 0 \). The derivative \( \frac{\partial c}{\partial \phi m} = \frac{q(1 - \psi[q]) - \frac{\partial \phi}{\partial \phi m}}{1 - q} \). Proceeding as above, using \( \frac{\partial c}{\partial \phi m} \), we derive \( \frac{\partial c}{\partial \phi m} = \frac{q(1 - \psi[q])(2\gamma(1 - 2q) + \sqrt{\phi m})}{(1 - q)\sqrt{\phi m}} \). Thus, as for \( R \), \( \frac{\partial c}{\partial \phi m} > 0 \). The derivative derivative \( \frac{\partial c}{\partial \phi m} = \frac{q(R + w) + \frac{\partial \phi}{\partial \phi m}}{1 - q} \). Using the definition of \( \phi \), it can be proven that the numerator is strictly negative. Therefore, we get that \( \frac{\partial c}{\partial \phi m} < 0 \). Finally, the derivative \( \frac{\partial c}{\partial \phi m} \) is ambiguous as for labor supply.

As to the consumption of the majority members, the following results can be easily obtained. The derivative \( \frac{\partial c}{\partial R} = \frac{(2q - 1)\gamma(\phi m + 4(1 - q)\psi(1 - \psi[q]))}{\sqrt{\phi m}R^2} > 0 \), since \( \frac{\partial \phi}{\partial \phi m} > 0 \). The derivative \( \frac{\partial c}{\partial w} = \frac{(2q - 1)\gamma(\phi m + 4(1 - q)\psi(1 - \psi[q]))}{\sqrt{\phi m}R^2} > 0 \), since \( \frac{\partial \phi}{\partial \phi m} > 0 \). The derivative \( \frac{\partial c}{\partial \phi m} \) is again ambiguous. Finally, one gets: \( \frac{\partial c}{\partial \phi m} < 0 \). Of course, \( \frac{\partial c}{\partial \phi m} < 0 \) and \( \frac{\partial c}{\partial \phi m} > 0 \), given \( c^A = \frac{R^2}{w} \).

### B.3 Proof of Proposition 3

Assume \( q = \frac{1}{2} \). When a society is perfectly polarised and \( \gamma = 0 \), one can easily derive that \( f(\frac{1}{2}, 0, R, w) = -((1 - \psi[\frac{1}{2}])(R + w)) < 0 \). When preferences for leisure are at the maximum level compatible with constraint 2, i.e. \( \gamma = w(1 - \psi[\frac{1}{2}]) \), we observe from equation

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that \( f \left[ \frac{1}{2}, 1 - \psi \left[ \frac{1}{2} \right] \right], R, w \right] = 0 \). The function \( f_{\left[ \frac{1}{2}, \gamma, R, w \right]} \) is continuous on \( \gamma \in [0, 1 - \psi \left[ \frac{1}{2} \right]] \). Notice that when \( q = \frac{1}{2} \), the derivative \( \frac{\partial f_{\left[ \frac{1}{2}, \gamma, R, w \right]}}{\partial \gamma} \) = \( \frac{R}{w} - \log \left[ \frac{\gamma}{w(1 - \psi \left[ \frac{1}{2} \right]} \right] \).

Under constraint 2, the argument of the log is lower than 1. It follows that, under Assumption 1, \( \frac{\partial f_{\left[ \frac{1}{2}, \gamma, R, w \right]}}{\partial \gamma} > 0 \) for any \( R > R_A \) and \( w \) and \( \gamma \in [0, w(1 - \psi \left[ \frac{1}{2} \right]] \).

Since \( f \left[ \frac{1}{2}, 0, R, w \right] < 0 \) and \( f \left[ \frac{1}{2}, w(1 - \psi \left[ \frac{1}{2} \right]), R, w \right] = 0 \), Proposition 3 holds trivially.

### B.4 Proof of Corollary 1

Using equations (5) and (6) it is easy to verify that the limit of the difference \( c^m - c^M \) goes to 0 when \( q \to \frac{1}{2} \). Since labor supply is the same for all citizens and \( \frac{1 - q}{q} = \frac{q}{1 - q} \) when \( q = \frac{1}{2} \), it follows directly from equations (5) and (6) that \( c^m = c^M \). Note also that \( c^A = \frac{Rw}{w} \). When \( q = \frac{1}{2} \), we derive \( c^M = c^m = (R + w)(1 - \psi[q]) \). Under constraint (2) it is always true that \( c^M = c^m > c^A \).

### B.5 Proof of Proposition 4

To prove Proposition 4 consider the following. When \( q \to 1 \) the labor supply \( l^m \) is positive if and only if \( R < w \left( \frac{w(1 - \psi[1])}{\gamma} - 1 \right) \). Under assumption 1, we derive that \( l^m = 0 \), since \( R_A > w \left( \frac{w(1 - \psi[1])}{\gamma} - 1 \right) \). Note that the limit of equation (14) when \( R \to w \left( \frac{w(1 - \psi[1])}{\gamma} - 1 \right) = 0 \), while the limit when \( R \to \infty \) goes to \( \infty \). Moreover, the partial derivative of equation (14) when \( q \to 1 \) gives \( \frac{\partial f_{\left[ 1, \gamma, R, w \right]}}{\partial R} = \frac{\gamma}{w} - \frac{(R + 2w)\sqrt{(R + w)\gamma(1 - \psi[1])}}{2(R + w)^2} \).

This derivative is positive when \( 2\gamma(R + w)^2 - w(R + 2w)\sqrt{(R + w)\gamma(1 - \psi[1])} > 0 \).

Under assumption 1 and constraint 2, the latter equation is always positive. Therefore, given that for \( q \to 1 \) equation (14) is a strictly increasing function of \( R \), Proposition 4 holds trivially.

### B.6 Proof of Corollary 2

Note that when \( q \to 1 \), we get \( c^M = \sqrt{(R + w)\gamma(1 - \psi[q])} - \gamma \). Since \( c^M - c^A = \sqrt{(R + w)\gamma(1 - \psi[q])} - \gamma \frac{(R + w)}{w} < 0 \) under assumption 1, Corollary 2 holds trivially.
B.7 Proof of Lemma 1

Using (11), we derive that \( l^m = 0 \) when \( R = \bar{R}[q] \), with \( \bar{R}[q] = \frac{qw(qw-\gamma)}{(2q-1)^2} \). Note that \( \bar{R}[q] > 0 \) under constraint 2. Using equation (9) when \( R = \bar{R}[q] \) and \( l^m = 0 \), we get that \( \mu^m = \frac{\gamma}{wq} > \frac{\gamma}{q} = \mu^A \) and \( f[q,\gamma,\bar{R}[q],w] = \frac{(1-q)R(\gamma-qw)}{qw} < 0 \), under constraint 2. Therefore, under assumption 1, when \( R = \bar{R}[q] > R^A \) the function (12) is strictly negative and people always revolt for all \( q \in (\frac{1}{2},1) \).

\[ \square \]

B.8 Proof of Lemma 2

The comparison between \( R^A = w \left( \frac{w}{q} - 1 \right) \) and \( \bar{R}[q] = \frac{qw(qw-\gamma)}{(2q-1)^2} \), gives directly that \( R^A < \bar{R}[q] \) when \( q \in (\frac{1}{2},1) \), and \( R^A > \bar{R}[q] \) when \( q \in (1 - \frac{\gamma}{w},1) \).

\[ \square \]

B.9 Proof of Proposition 5

By Lemma 2 we know that if \( q \in (\frac{1}{2},\bar{q}) \), then \( R^A < \bar{R}[q] \). Using Lemma 1 we also know that when \( R = \bar{R}[q] \), \( l^m = 0 \) and \( f[q,\gamma,\bar{R}[q],w] < 0 \). Observe now that \( l^m = 0 \) for all \( R > \bar{R}[q] \), while \( l^m > 0 \) for all \( R < \bar{R}[q] \), since \( \frac{\partial l^m}{\partial R} < 0 \) by Proposition 2.

Solving equation (12) when \( R > \bar{R}[q] \), gives that \( f[q,\gamma,R,w] = 0 \) when \( R = \bar{R}[q] \), with \( \bar{R}[q] = \frac{w(qw-\gamma)(\gamma-(1-q)^2w)}{q((1-q)w-\gamma)^2} \). Note also that \( \lim_{R \to \infty} f[q,\gamma,R,w] = \infty \left( q - 1 + \frac{\gamma}{w} \right) \). Since the limit is strictly negative \( \forall q \in (\frac{1}{2},\bar{q}) \), it follows that for all \( R > \bar{R}[q] \), \( f[q,\gamma,R,w] < 0 \) because the function (12) can take the value of zero once in the point \( \bar{R}[q] \) that cannot be larger than \( \bar{R}[q] \). When \( R \in (R^A,\bar{R}[q]) \), \( l^m > 0 \). We know by Proposition 3 that when \( q = \frac{1}{2} \) the function \( f[\frac{1}{2},\gamma,R,w] < 0 \) for all \( R > R^A \). Therefore, when \( R = R^A \), \( f[\frac{1}{2},\gamma,R,w] \leq 0 \). Note also that when \( q = \bar{q} \) and \( R = R^A \) we get \( f[\bar{q},\gamma,R^A,w] = -(w-2\gamma) < 0 \). To prove that the function \( f[q,\gamma,R^A,w] < 0 \) for all \( R \in (R^A,\bar{R}[q]) \), consider the general form\( f[q,\gamma,R^A,w] = \frac{R^2 - q}{w} - q \left( \frac{1-q^m(1-q)(l^m w + R)}{q} + \mu^m (l^m w + R) \right) - \gamma (\log(1-l^m)) \).

The latter function is zero when \( l^m = 1 - \frac{\Omega[\chi[q]+R(\chi[q]+\frac{\gamma}{w})]/\chi[q]}{\chi[q]} \), with \( \Omega[.] \) defining the Lambert function and \( \chi[q] = q - 1 + \mu^m(1-2q) < 0 \). Since the argument of the Lambert function is strictly negative we have that \( \Omega[.] < 0 \). Therefore, we should necessarily have

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that \( l^m < 1 \). However, note that if \( \frac{\Omega[e^{w_x[q]} + R(x[q] + \frac{\gamma}{w_x[q]})w_x[q]]}{w_x[q]} \geq 1 \), the function \( f(q, \gamma, R^A, w) \) can be zero if and only if \( l^m \leq 0 \), that is excluded when \( R \in (R^A, \bar{R}(q)) \), by proposition 2 and by non-negativity of labor supply. Using the properties of the Lambert function, we can observe that \( \Omega[e^{w_x[q]} + R(x[q] + \frac{\gamma}{w_x[q]})w_x[q]] > w_x[q] \). Therefore, it necessarily follows that \( f(q, \gamma, R^A, w) < 0 \) when \( l^m > 0 \), that is when \( R \in (R^A, \bar{R}(q)) \) (see Figure 11).

\[ \square \]

### B.10 Proof of Proposition 6

When \( q \in (\tilde{q}, 1) \), we know from Lemma 2 that \( R^A > \bar{R}(q) \). Therefore, \( l^m = 0 \) \( \forall \) \( R > R^A \).

First, note that \( \lim_{R \to \infty} f(q, \gamma, R, w) = 0 \) is strictly positive when \( q \in (\tilde{q}, 1) \).

From the proof of Proposition 5 we also know the function \( f(q, \gamma, R, w) \) when \( l^m = 0 \) is zero when \( R = \bar{R}(q) \), with \( \bar{R}(q) = \frac{w(qw - \gamma)(\gamma - (1 - q)^2w)}{q(1 - q)^w - \gamma^2} \). Since for \( R < R^A \) the function (12) is no longer valid, two scenarios are possible when \( q \in (\tilde{q}, 1) \): (i) the function \( f(q, \gamma, R, w) \) is negative close to \( R^A \). In this case, given that \( \lim_{R \to \infty} f(q, \gamma, R, w) > 0 \), it must necessarily be that \( \bar{R}(q) > R^A \), since the function \( f(q, \gamma, R, w) \) when \( l^m = 0 \) can take the value of zero only in the point \( R = \bar{R}(q) \). It follows that \( f(q, \gamma, R, w) \leq 0 \) for all \( R \in (R^A, \bar{R}(q)) \) and that \( f(q, \gamma, R, w) > 0 \) for \( R > \bar{R}(q) \). (ii) the function \( f(q, \gamma, R, w) \) is non-negative close to \( R^A \). In this case, it must necessarily be that \( \bar{R}(q) \leq R^A \) because \( \lim_{R \to \infty} f(q, \gamma, R, w) > 0 \). However, under assumption 1, it must be that \( R > R^A \).

Therefore \( f(q, \gamma, R, w) > 0 \) for all \( R > R^A \). Indeed, note that this result confirms previous propositions: the \( \lim_{q \to 1} \bar{R}(q) = R^A \), that is that people never revolt for all \( R > R^A \), as suggested by Proposition 4; the \( \lim_{q \to \tilde{q}} \bar{R}(q) = +\infty \), and people always revolt for all \( R > R^A \), as suggested by Proposition 5 (see Figure 12).

\[ \square \]