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▶ **To cite this version:**

Guillemant Philippe, Medale Marc. A unique but flexible space–time could challenge multiverse theories. Annals of Physics, 2019, 409, pp.167907. 10.1016/j.aop.2019.05.005 . hal-02437160

HAL Id: hal-02437160
https://amu.hal.science/hal-02437160

Submitted on 13 Jan 2020

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A unique but flexible space–time could challenge multiverse theories

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- The growing law of a billiard multiverse is independent of the discretization scale E.
- The time for any final state to be accessible from any initial one is around 3E shocks.
- The paths of billiard balls can change without changing initial or final states.
- Space-time flexibility can be achieved through paths commutations and entanglement.

A B S T R A C T

In a previous paper (Guillemant et al., 2018), we have shown using a 2D billiard toy model that the propagation of uncertainties resulting from space–time discretization leads to a classical multiverse. In this one we estimate the growing law of the number of its branches, thanks to an original shock indexation method that permits a very fast comparison of trajectories. For different involved parameters we use random sets of infinitesimally perturbed initial conditions. From each set, we compute a collection up to hundred millions of different billiard histories. Our main result is the calculation of a *Brownian saturation time* when the number of branches begins to exceed the total number of billiard states. We discuss the possibility suggested by this result to change locally the paths of objects without changing their past and future histories and then subverting their global system. We conclude that a unique but flexible space–time could compete with the current many-worlds interpretation claiming that each branch is a different universe.

Keywords:
Space–time
Multiverse
Block universe
Data-mining
Brownian
Diffusion time

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0. Introduction

From the black holes properties described by Jacob Beckenstein [1] to the holographic principle of Juan Maldacena [2], along with the quantum gravity requirements to quantify all physical quantities [3], theoretical physics have to face the necessity to define a bound limit to the density of physical information in the universe. As a consequence various discrete models of space–time have been proposed [4,5], even mixed ones [6], but they do not solve the incompleteness of known physical laws resulting from the intrinsic uncertainty at quantum level. One of the ways to restore an hypothetic determinism in this context is to appeal to a multiverse theory, for example the many worlds of Everett [7] which is now widely recognized [8,9], but the complementary laws that would be able to determine in which universe we are living in are still lacking.

In a previous paper [10] we have shown, thanks to numerical simulations of billiards, that to obtain a deterministic description of a discrete classical space–time it could be necessary to add six extra dimensions. These new dimensions are required to identify a unique billiard among the huge number of billiard branches that result from the macroscopic amplification of initial uncertainty related to discretization. Indeed, to proceed to this identification it is necessary to define, after a lot of interactions, not only the final conditions but even the paths to reach them! We think that it has interesting perspectives that first require questioning the relation between the so created multiverse and the other well known ones, especially the quantum multiverse which also emerges from quantum uncertainties.

For this purpose we intend in this paper to quantify the different parameters that are conditioning the emergence of a discrete classical multiverse, so as to characterize more precisely its specificity and possible link with quantum mechanics.

Different types of approaches are possible to calculate such a multiverse: (i) stochastic ones such as random walks [11] or Markov chains and processes [12] or based on kinetic theory [13]; (ii) deterministic ones such as Hamiltonian approaches [14] or dynamical system equations [15]. For the purpose of our paper only the last one is relevant, but as we use conservative equations (no dissipation), the time integration of dynamical equations is not required.

The well known apparent frontier between classical and quantum mechanics (or multiverse) is particularly characterized by a fundamental distinction in the interpretation of uncertainty. Unlike the quantum ones, the classical uncertainties with which statistical physics deals are generally considered as coming from our ignorance of existing details. But recently, it has been shown [16] that using deterministic equations does not prevent the laws of mechanics from settling a Brownian regime after a "long time", when balls or particles seem to have got a random course that is independent of their initial conditions. Thanks to a numerical billiard model that we investigated, we have shown that this can be imputed to the loss of information that is due to the amplification of phase uncertainty during interactions [10]. But in this precedent study we neither quantified the growing law of the number of branches that appears after a critical time T_C, nor the different phases and conditions of the growth, in absence of which our conclusions remain somehow weak. In this following paper we really proceed to these quantifications that make us focus on three specific dimensionless times, i.e. based on specific numbers of shock events:

- The critical time T_C (already statistically estimated in [10]) after which multiple histories appear for the first time and then a diffusion stage is beginning;
- The Brownian time T_B after which all the growing histories have become independent of initial conditions, meaning that the Brownian regime of [16] is really settled;
- The Brownian saturation time T_S after which *all the possible final conditions of the billiard have been statistically reached from well defined unique initial conditions.*

As the highly chaotic behavior of a billiard makes its local and discrete multiverse quantification out of reach of relevant theoretical approaches [17], we have again used computations with basic shock equations so as to calculate as much as possible different branches (until 10^8) from the same initial conditions.

We were then confronted to a data mining problem that consisted in the comparison of massive numbers of time series issued from different ball trajectories. Our challenge was then to count the time variations of the number of branches, that is multiplied at each bifurcation.

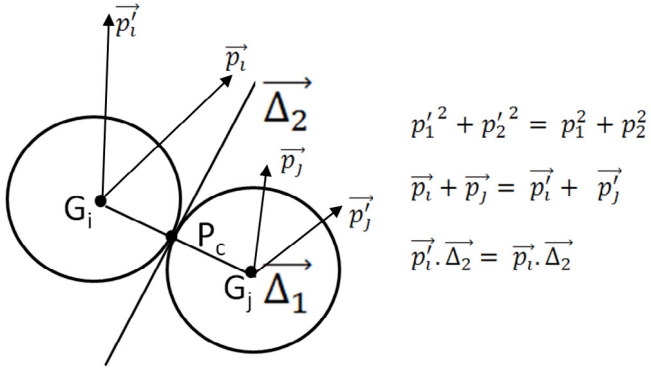


Fig. 1. Sketch of a contact between two balls, where p_i and p_i' designate the momentum of ball i at instant just prior to the (i, j) shock and just after, respectively (see [10] for more details).

A lot of methods have been proposed to compare massive time series [18]. Though they are today most often related to big data [19] or artificial intelligence [20] they are more and more interesting the future of physics [21]. The approach we propose in this paper is based on an original method of time series indexation and chaining that come from our own contribution to artificial intelligence in the past [22]. Without this method the discrimination between millions of branches we present here would have been out of reach. Its principle is to identify branches thanks to a 128 bits floating point indexation of their time series and to chain the corresponding indexes in such a way that avoids massive useless comparisons, resulting in an ultra-fast branch counting algorithm.

1. Theoretical framework

We have used the same billiard model as in the previous paper [10] in which we presented calculations of ball's uncertainties propagation. This model concerns the frictionless 2D hard-sphere model with elastic shocks of equal mass and diameter balls, whose governing equations are recalled in Fig. 1.

In the previous study we have shown that however small the uncertainties affecting the initial positions or velocities of balls, that will range in this work between 10^{-2} and 10^{-20} pixels (or 10^{-5} and $10^{-23} \times L$, where $L = 1024$ pixels is the square billiard width), there always exists a critical time where different shock histories begin to appear. In Fig. 2 this critical time occurs when it exists for the first time one ball A_i whose trajectories become to differ in two distinct histories A_{i1} and A_{i2} involving different balls C_k and C_l after a shock with B_j.

After critical step, an increasing number of different shock histories appears, corresponding to as many branches of a billiard's multiverse. We assumed at the end of our previous paper [10] that the growing law of the number of branches N_U could vary exponentially with the number of shocks, after a Brownian diffusion has been settled:

$$N_U \sim Nhcs\ D^{(N-Ncs)} \tag{1}$$

Nhcs is the total number of branches accumulated at the end of a diffusion stage, where any ball may have reached any position into the billiard, and *Ncs* the number of shocks until this moment. We then assumed the existence of a D factor for which the subsequent growth of the number of branches would satisfy (1), but we did not verify this assumption neither calculated the Brownian time nor the value of D: it is the purpose of the present paper to do so.

To make these calculations and study the influence of different involved parameters, we have computed many collections of large sets of multiple branches resulting from slightly noised initial conditions, using a maximum value of noise equal to the uncertainty ε, where $10^{-1} < \varepsilon < 10^{-20}$ depending on the collection. Thanks to an original algorithm described in the following section,

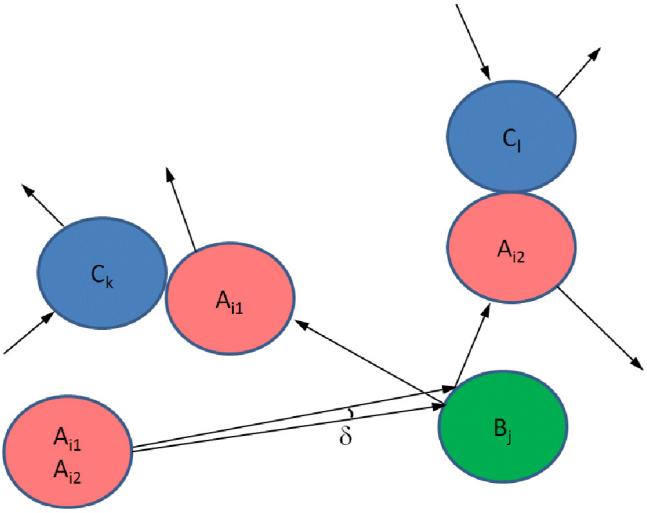


Fig. 2. First occurrence of multiple billiard histories at critical step.

we have calculated a sufficiently high number of histories $N_S = max(N_U)$ so as to draw robust conclusions for each set, the number of shots N_S varying between 10^5 and 10^8 depending on the computation. The calculations of each set of histories were pursued until a minimum time (or mean number of shocks per ball) corresponding to the situation where all the calculated billiards were diagnosed as different ones ($N_U = N_S$).

2. Computations

2.1. Fast comparison algorithm

The balls trajectories are calculated using the same algorithm as previously published in [10]. Unlike this precedent work, we do not need to synchronize billiards so as to calculate the propagation of uncertainties, and then we can pursue calculations long after the critical step until we reach a large enough set of billiard histories constituted of unique branches.

Each branch is distinguished from others using a branch identification principle based on the series $S(n, m)$ constituted by all the successive shock identifiers $Idc(n)$ of the nth shock of the mth branch, where each identifier is calculated with the two ball numbers involved at each shock, so that:

$$S(n,m) = \{Idc\,(1)\,, Idc\,(2)\,, \ldots, Idc\,(n)\} \quad with \quad 0 \leq Idc\,(k) < \frac{N_b\,(N_b-1)}{2} = N_{cp} \tag{2}$$

The last term is the number N_{cp} of couples of balls, N_b being the number of balls. The interest of this identification procedure is that it does not need any arbitrary threshold, it is perfectly adapted to count bifurcations and it furnishes the most economic identifier in terms of bits.

To be able to constitute large enough sets m of branches, we have to solve the problem of the huge computing times resulting from the comparison of all branches. For example, when we compute a set of ten million branches, we have to compare the square that is hundred billion couples of billiards, each one being characterized by N_b trajectories formed by the succession of up to hundreds of couples of balls. We have evaluated that to make a good study of the growth of the different branches, we need to reach statistically established states that impose us to use an alternative approximate method.

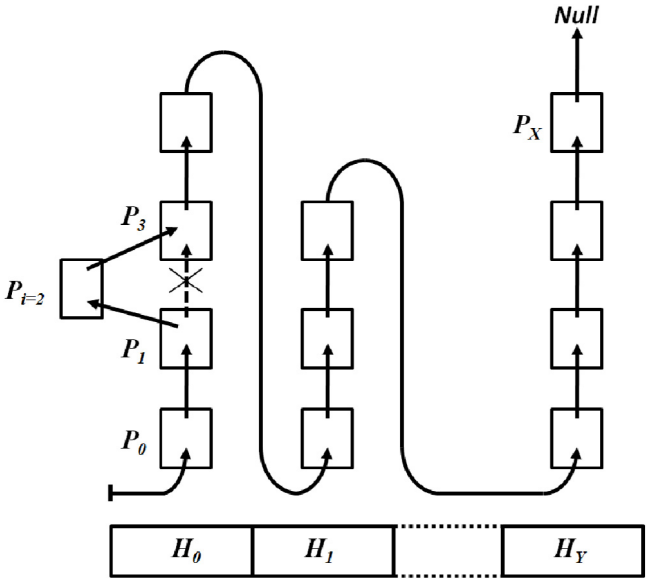


Fig. 3. Chaining algorithm of P_j using truncated values $H_j = P_i \ mod \ (N_S)$.

We were already confronted in the past to this kind of problem, when we had to compare high numbers of successive pixel's records of smoke in departures of fires, so as to develop an automatic forest fires detector [22]. For this we used a temporal embedding algorithm with a fractal indexing of pixel values that permitted to characterize each smoke evolution by only one index in the form of a 32 bits integer. In the present case we made our calculations with a 128 bits precision compiler, so we propose here a very similar method which consists in characterizing each billiard branch with a 128 bits P_i integer value (carried by a floating point) that is coding only the last part of $S(n, m)$:

$$P_i\,(n, m) = \sum_{k=n-l+1}^{k=n} Idc\,(k, m)\,N_{cp}^{k} \quad \text{where} \quad l = (int)\,(128\frac{\log\,(2)}{\log\,(N_{cp})}) \tag{3}$$

The value l is the number of successive couple identifiers that it is possible to code into a 128 bits integer, so that it is the maximum value for which $N_{cp}^{l} < 2^{128}$. In practice we do not use an integer but a floating point value that is calculated from the precedent one with the equivalent recurrent relation:

$$P_i\,(n, m) = \frac{P_i\,(n-1, m)}{N_{cp}} + \frac{2^{128}}{N_{cp}}(Idc\,(n) + 1) \tag{4}$$

The power of this algorithm is then to use a chaining of all integers $P_i(n, m)$ that is using a truncated index H_j so as to give a direct access to a very limited part of the chain, thus permitting to highly reduce the number of comparisons and also the computing time in the same proportion. (See Fig. 3.)

The dimension of the truncated index array H_j is chosen to be the same as the number of shots N_S. Our chaining algorithm then avoids us to implement a number of comparisons that is approximately equal to $n\ N_U^2$, while the number of comparisons we really make is equal to $x\ N_U$ where x is the mean number of integers P_i having the same truncated index H_j.

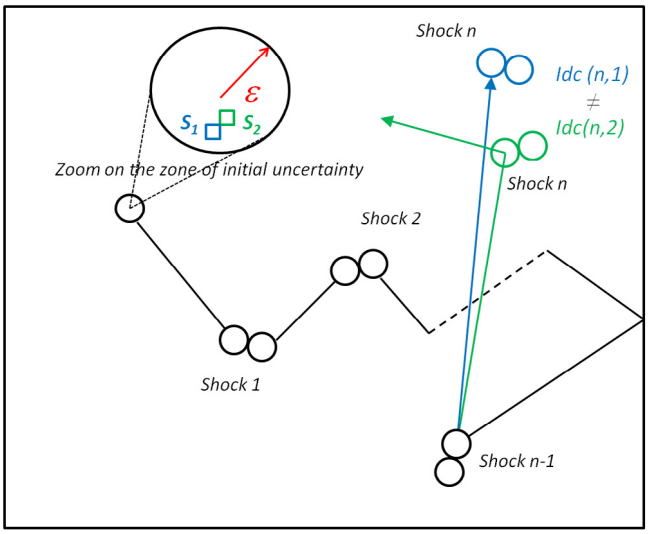


Fig. 4. Illustration of the division in two different branches at shock n (in blue and green) involving distinct couples of balls $Idc\,(n,1) \neq Idc(n,2)$ as a result of slightly noised initial positions of balls S_1 and S_2.. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2.2. Implementation and validation

For different sets of involved parameters: the value of maximum uncertainty ε, the number of balls N_b and their radius R, we associate as many randomly chosen initial conditions of balls positions and velocities. For each set of these conditions, we repeat a collection of N_S computations which differs in their branch bifurcations as a result of infinitesimal differences in starting positions, illustrated in Fig. 4 by the two squares S_1 and S_2 located inside a zone of radius ε, varying from 10^{-1} to 10^{-20} depending on the study. In practice we implement it by the addition of initial uncertainties dx and dy to the initial positions, that we randomly calculate to be always below the maximum uncertaintyε:

$$dx = r\cos(\alpha) \quad \text{and} \quad dy = r\sin(\alpha) \quad \text{with } r = \varepsilon\frac{rand(10000)}{10000} \quad \text{and } \alpha = 2\pi\frac{rand(36000)}{36000} \tag{5}$$

Note that the *rand(N)* random function returns an integer between *0* and $N-1$ and has to be called $2 \times N_b \times N_S$ times during a computation of a new collection of branches.

For each new billiard branch series, we stop the computations after a fixed number of successive shocks (generally comprised between 20 and 200) for which the number of different branches has reached its maximum value, so that all the branches are unique ones. The validation of our computation method requires the two checks (a) and (b):

(a) Comparing computing times of exact calculations and approximate indexation method.
We have got these computing times $Ct(N_S)$ for low values of N_S, varying by powers of two from $N_S = 1000$ to $N_S = 64\,000$. For this latter value the approximate method was roughly 10 times faster. Now, from the different couples $(N_S, Ct(N_S))$ that we obtained we have extrapolated that for N_S values varying from 10^5 to 10^8, which are the most often needed, the computing time ratio should vary between 100 to more than 1000, making then too prohibitive the use of the exact algorithm when N_S is equal or above one million. Note that to calculate a collection of only one million branches with our approximate algorithm we often need a computing time which is around one hour (depending on parameters), and that for the whole study we have computed hundreds of collections, some of them with hundred millions shots (series).

(b) Estimating the range of parameters for which the indexation algorithm is correct

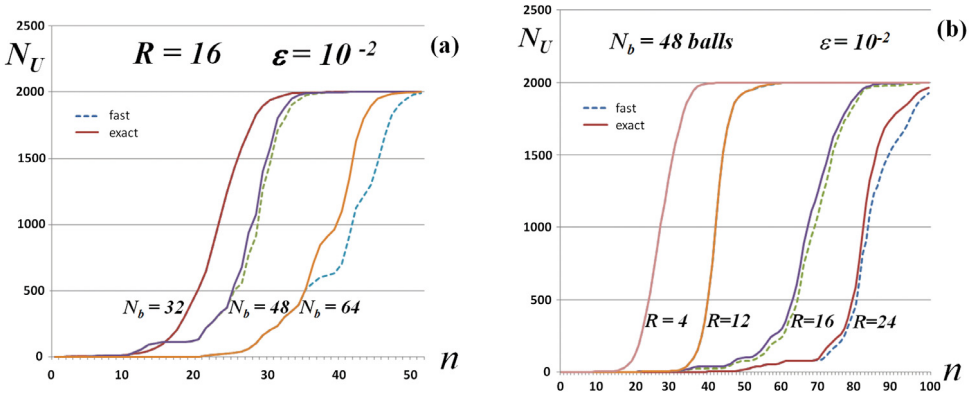


Fig. 5. Variation of N_u versus n for different values of N_b (a) and R (b). The fast algorithm results are plotted in dotted lines.

From the principle of indexation we anticipate that our fast but approximate method is only valid for low values of N_b, because the number of couples of balls is varying with the square of N_b and then requiring much more bits to encode the couple, yet the maximum number of successive shocks that we can identify is decreasing as much. For example, with only 3 balls we can encode 80 successive shocks in a 128 bits integer, while for 100 balls we can only encode 10 successive shocks, which is not enough when the number of shocks is high, because we take the risk to diagnose as a unique branch, either different ones or the same branch but at different times or values of n. However, note that it is better for the trust in our conclusions that our method tends to minimize the number of branches.

Figs. 5a and 5b compare for low numbers of branches N_U with a maximum $N_S = 2000$ the exact and approximate results of the variation of N_U with the number of shocks n for different values of N_b and R, respectively. Note that N_U varies from $N_U = 1$ during a few shocks to $N_U = N_S$ when all calculated branches are diagnosed as unique.

We notice in Fig. 5a that fast calculations are correct until $N_b = 32$ balls, but begin to diverge as predicted with growing values of N_b. This is also due to the fact that when N_b is increased the number of shocks n is also increased, yet the probability of a false identification is growing with both numbers N_b and n. In Fig. 5b, we notice that when R is increased from 4 to 24 the critical time is delayed and then the number of shocks is increased, so that for the same reason the approximate curve begins to differ from the exact one.

As a consequence, in the following we limit our study to low values of N_b ranging from 3 to 24 and also to low values of R ranging from 2 to 32. Note that low values of R correspond to high void ratios ($V_r = 1 - \frac{\pi N_b R^2}{L^2}$), which are much more interesting conditions to test than lower ones, for which we already presented a statistical study in our previous paper.

Let us recall that the present study is neither a statistical nor a stochastic one. All our calculations of billiard shocks and trajectories are purely deterministic (made at the double precision of 128 bits) and in particular, for each set m of initial conditions we calculate only one branch or set $S(n,m)$ that we identify with the integers series $P_i\,(n, m)$.

3. Results

3.1. Influence of initial and intrinsic uncertainties

So as not to confuse the effects of the initial uncertainty that depends on ε in relation (5) and the intrinsic uncertainty that depends on the compiler maximum precision of our calculations, we tested two different 64 bits and 128 bits compilers. The 64 bits one was a GCC with 52 bits of

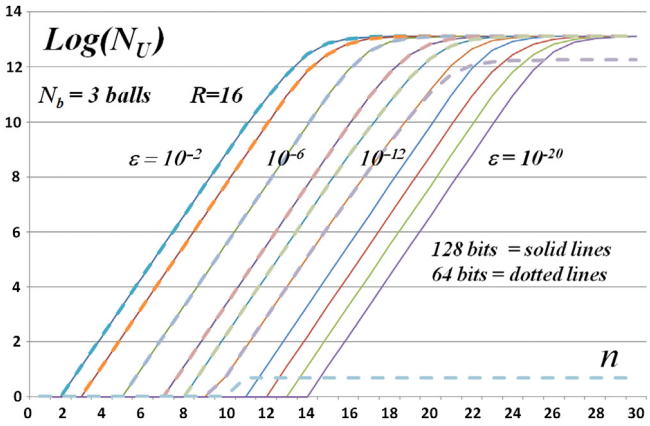


Fig. 6. Variation of $\log(N_U)$ versus n for $N_b = 3$ with different values of ε ranging from 10^{-2} to 10^{-20} and for two different compilers 64 bits (dotted lines) and 128 bits (solid lines).

effective resolution mantissa, from which we must subtract the resolution in power of two that we need so as to perturb initial positions via Eqs. (5) with always distinct uncertainties, that requires 16 bits. It is then not correct to use a 64 bits compiler with ε factors less than 2^{-36} i.e. ranging from 10^{-10} to 10^{-11}. Yet we observe in Fig. 6 that it is actually the ε interval into which the number of branches begins to fall, while the 128 bits compilers go on giving consistent results for lower values of ε (10^{-14} to 10^{-20}).

We also observe in Fig. 6 the effect of decreasing ε toward zero that is to increase approximately linearly in function of $\log(\varepsilon)$ the critical time for which multiple branches appear. This is what we already established in our previous paper, the only difference being the present roughly linearity due to the exact calculation with only one set of initial conditions that impose the influence of its specific geometry. Now, what more clearly appears in Fig. 6 is the perfect linearity of the relation between $\log(N_U)$ and n, which is due to a perfect geometric amplification of N_U by a factor $D = 3$ at each shock, that is the number of couples of balls. This result is maintained whatever initial conditions are, except for shocks occurring just after the critical step for which we can sometimes observe $D = 2$, depending on the initial geometry.

For higher values of N_b we confirm the linear increase of the critical time with $log(\varepsilon)$ but we do not illustrate this here as we already dealt with this issue in [10]. In the following we focus on the relation between $\log(N_U)$ and the number of shocks, with values of ε ranging from 10^{-4} to 10^{-1} so as to reduce the critical time and then optimize computing time.

3.2. Influence of ball number N_b

We expect that the linear increase of $\log(N_U)$ with n that we observed for $N_b = 3$ should be repeated for higher N_b values, knowing that after a critical step, the peculiar geometry of initial conditions cannot impose any more durable restrictions on the collection of different paths, because they become quickly dispersive at 360°. However it does not imply that no restriction subsists and it is neither trivial nor deprived of surprises to anticipate what can occur for $N_b > 3$ with various geometries. It is precisely the main interest of our work to explore this issue. We could for example imagine that a specific geometry of initial conditions would maintain some kind of unsuspected determinism.

The main parameter we have to explore for this purpose is the derivative of $\log(N_U)$ which depends on the ratio D between the successive numbers N_U of branches:

$$D = \frac{\log(N_U(n+1))}{\log(N_U(n))} \leq \frac{N_b(N_b-1)}{2} = N_{cp} \tag{6}$$

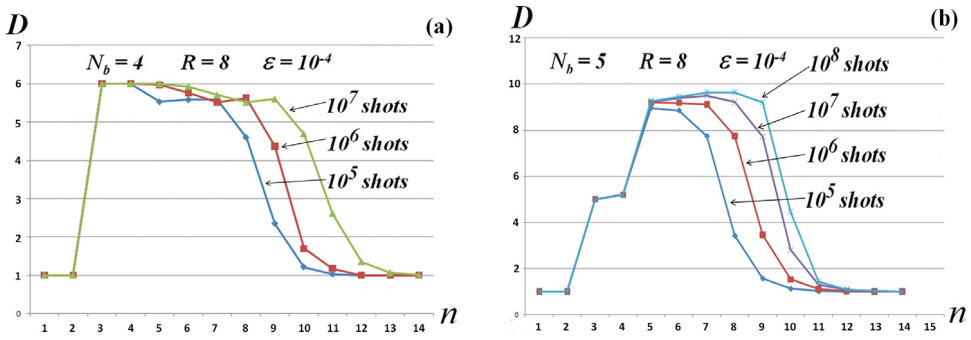


Fig. 7. Variations of D versus n for $N_b = 4$ (a) and $N_b = 5$ (b) with different numbers of shots growing by powers of 10 from 10^5 to 10^8.

Figs. 7a and 7b show the different values of D obtained with $\varepsilon = 10^{-4}$ and $R = 8$ for $N_b = 4$ and $N_b = 5$ balls, respectively.

We used in Figs. 7a and 7b different numbers of shots varying by powers of 10 from 10^5 to 10^8 so as to highlight the existence of an asymptotic maximum that is progressively reached. This maximum is reached exactly for $N_b = 4$ and approximately for $N_b = 5$, the value of it being equal to 6 and 10, respectively, so confirming that for $N_b = 3, 4$ or5 we have the equality:

$$D = N_{cp}\ (number\ of\ couples\ of\ balls) \tag{7}$$

It seems to imply that there is no constraint issued from initial conditions that impose restrictive paths that would prevent any ball to shock any other. Note that we obtained this result for $\varepsilon = 10^{-4}$ and that it would have been the same for any other value down to $\varepsilon = 10^{-20}$ and lower, or else we expect that for $N_b = 3$ we would have observed on Fig. 6 some anomaly when ε tends to zero.

Now, when we increase the number of balls with $N_b \geq 6$ we do not succeed any more in finding a maximum asymptotic value for D, one hundred million shots being the maximum permitted by the memory of our computer. Indeed, in Fig. 8a we observe that for $N_b = 6$ the number of couples $N_{cp} = 15$ is far from being reached (observed maximum = 11) and we can wonder if it is really possible for higher values of N_U. This gives rise to the question of the influence of the void ratio, because when the balls have a large radius we can expect a shield effect that prevents any ball from being reached by any other.

3.3. Influence of void ratio

We use here the radius R of balls instead of the void ratio, which remains the same. What is to be noticed about the influence of R when we look for an asymptotic value of D is the existence of an intermediary radius for which we reach a maximum for D, which corresponds to values $D_{max} = 14$ and $R = 8$ in Fig. 8b.

This intermediary radius can be explained from Fig. 4 and then noticing that when R is small, the paths permitting a shock n involving two specific different couples are reduced along with the probability of this shock, which occurs through more restrictive initial conditions requiring a more precise infinitesimal square Si. We then need much more squares S_i and then a higher number of shots N_S to be able to sample the required conditions randomly.

This also explains why we cannot calculate an asymptotic value of D for $N_b \geq 6$. The maximum value of N_S we can use being limited to 10^8, the maximum value of D we could calculate was equal to 14 in Fig. 8b, so we can guess in this particular case that D should reach its asymptote $N_{cp} = 15$ for higher values of N_S, but it is not sure. As limiting N_S is equivalent to define a threshold of branch probability $1/N_S$ below which we ignore that branch, the existence of an asymptotic value of D that does not reach the maximum value N_{cp} for $N_b \geq 6$ depends on the existence of billiard states that

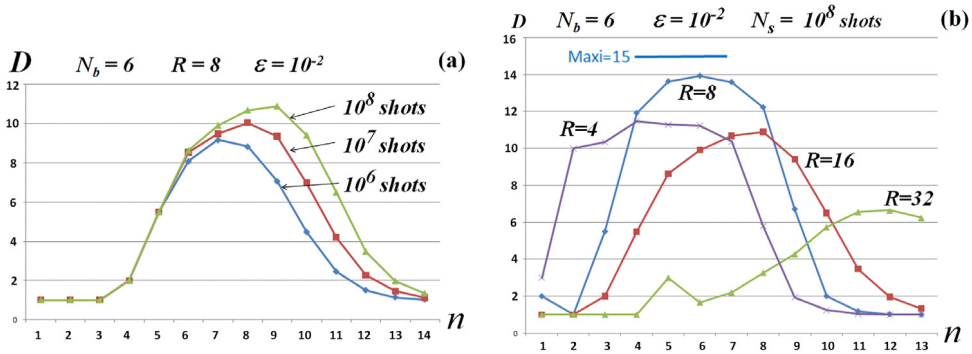


Fig. 8. Variations of D versus n for $N_b = 6$ with different numbers of shots N_S (a) and different values of R (b).

are impossible to reach. Yet this really occurs either if there is still a dependence of billiard states with the geometry of initial conditions, or if there is a shield effect due to the insufficiency of the vacuum between the balls. The possibility for D to be equal to its asymptote N_{cp} should then be satisfied through two conditions:

(a) A Brownian regime is established, so that all phases of balls have become independent of their initial conditions and then any phase state configuration becomes possible.

(b) R is small enough, so that whatever the number of balls is, there is always a configuration of ball phases for which the subsequent shock of any ball can involve any other one.

As we cannot make calculations that imply branch probabilities less than $1/max(N_S) = 10^{-8}$, it is not possible for us to estimate with our simulations the corresponding R domain values versus N_b. But from the moment where (a) is satisfied, we can assume that it is always possible to adjust for any ball its subsequent shock with any other one, provided that we can also adjust R to prevent shocks with other balls. If it is true, for any value of N_b there would be a non-zero value of R for which (b) is also satisfied.

3.4. Diffusion regime

A Brownian regime is settled after a Brownian time during which all the billiard ball phases become progressively completely independent from initial conditions. In our discrete billiard toy model, this is achieved after a certain time when, for any ball of any initial phase state, there is at least one of the multiple branches for which this ball has crossed the maximum distance (the billiard diagonal). Even if the space distribution of phases is not still uniform at this stage (states are not still equiprobable), we can then be sure that a particular geometry of initial conditions cannot any more cause a lack of presence of a ball in any region, which could be responsible of a zero-probability local phase configuration. But for the same reasons as before we cannot verify this point with our calculations.

However it is still possible to verify that the independence from initial geometry is achieved for the value of D, even if we cannot make it reach its true asymptote. This is what we can observe in Figs. 9a and 9b for $N_b = 12$ and 24 balls, respectively.

We made the calculations of Fig. 9 for different numbers of shots varying from 10^5 to 10^8. Note that when the number of shots is multiplied by 10, D is linearly increased with $\log(N_U)$, a result that let us think that D could reach its asymptote for a sufficient number of shots, but with an infinitesimal branch probability. This result is consistent with the linear correspondence between the number of shots and the distance to be covered so as to detect bifurcations with a resolution that is maintained as a constant (see Fig. 4).

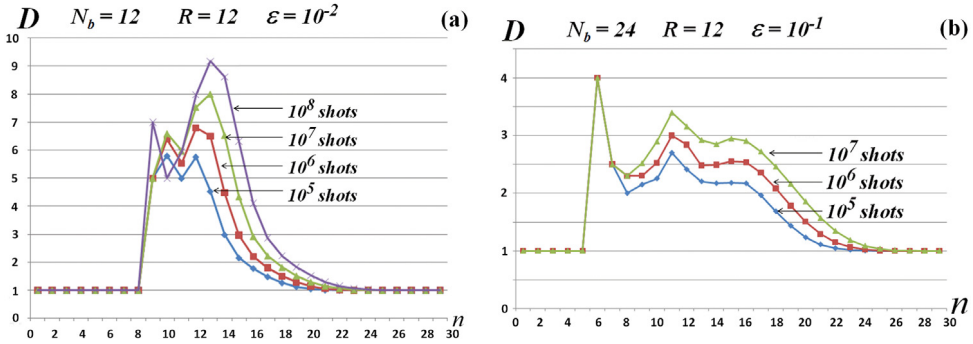


Fig. 9. Variations of D versus n for $N_b = 12$ (a) and 24 (b) with different numbers of shots from 10^5 to 10^8.

The main difference between Figs. 9a and 9b is the very low value of D (maximum 3 for a maximum asymptote 276) when $N_b = 24$ compared to the value obtained for $N_b = 12$ (maximum 9 for a maximum asymptote 66). Considering also the maximum of 14 that we obtained for $N_b = 6$, this clearly manifests the unexpected result that D decreases when N_b increases, while the opposite was expected.

This can be explained by the fact that we make these comparisons for a constant number N_S of configurations of successive shocks, while it would be necessary to adapt it to different numbers depending on N_{cp}, because the probability to sample a given configuration during a given number of shocks N varies as the inverse of the total number of different configurations emerging during the corresponding time. If we consider for example two branch configurations obtained with different N_b during N successive shocks, it would mean dealing with very distinct numbers of configurations D_1^N and D_2^N, where D_1 and D_2 are the associated growth factors. Now, as configurations are not equiprobable their probabilities are out of reach of our calculations, so that we cannot take them into account to make a correction. However, we have already given practical results and made necessary clarifications to consider that if R is low enough, there is no reason to think that we would not calculate a D factor tending toward its asymptote N_{cp} if we had no limitation on N_S.

Another aspect of Figs. 9a and 9b is the heterogeneous variation of D for low values of the number of shocks, which we attribute to the peculiar geometry of initial conditions. In Fig. 9b we observe that D is stabilized during 4 successive shocks, after which we do not have enough shots to sample more data and then D tends to 0. So as to confirm the diagnostic of the initial geometry influence, we have repeated the calculations with exactly the same parameters but for different initial conditions (Fig. 10).

Figs. 10a and 10b illustrate the variations of $\log(N_U)$ and D, respectively, for various initial conditions with $N_b = 24$ and $R = 12$ (same parameters as on 8b). To keep a clear Fig. 10b we only reported 3 of the 9 different geometries of Fig. 10a.

Fig. 10a shows discordances between the different variations of $\log(N_U)$ when we change the initial geometry, that disappear when the number of shocks increases. This can be explained by the fact that during the diffusion phase, there are still shocks of balls for which the critical step is not reached yet locally, meaning that the angular dispersion of their velocities has not reached 360° yet and then some subsequent shocks with neighboring balls are forbidden, resulting in low values of D. On the contrary, high values of D appear ephemerally after critical step as a result of a better sampling of branches when the covered distance and then the number of shocks are still low.

The parallelism of Fig. 10a curves demonstrates the convergence of $\log(N_U)$ slope toward a constant when n increases. This also results in a similar convergence of D values that we can notice on Fig. 10b along 3 or 4 successive shocks when $Nc > 12$. Below this value there are abrupt variations of D and disordered evolutions of $\log(N_U)$ caused by the differences in initial geometry. Above this value, the reason why there is just a parallelism but not an identity of $\log(N_U)$ curves is that there is a persistent effect of initial geometry that results in a slight delay in the explosion of branches, but without affecting the slope and the D growth factor.

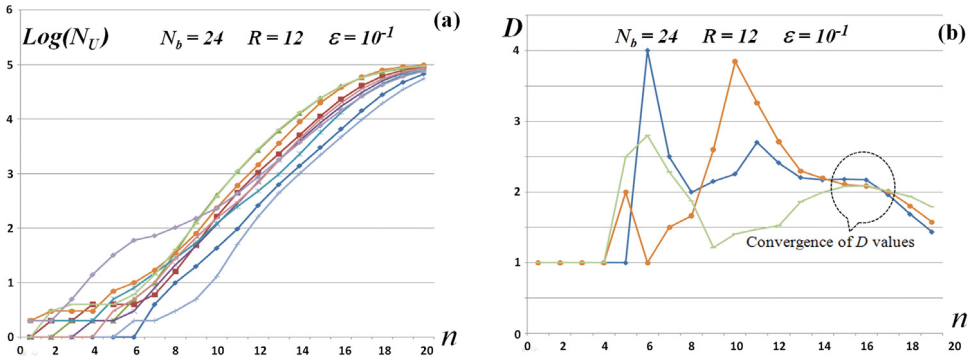


Fig. 10. Variations of $\log(N_U)$ (a) and D (b) versus n for various initial conditions and with the same parameters as on 8b: $N_b = 24$ and $R = 12$.

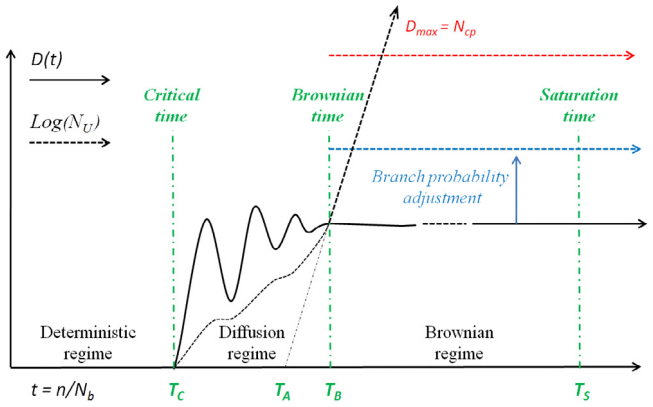


Fig. 11. Schematic time evolutions of growth coefficient $D(t)$ of a billiard's multiverse (solid lines) and number of branches N_U in logarithmic scale (dotted lines).

3.5. Brownian and saturation times

The diffusion regime is achieved at a Brownian time (see Fig. 11) reached when the accumulated uncertainty makes it possible for any ball to be located at any position into the billiard, so that its phase has become completely independent of initial conditions. For low values of N_b, specular reflections are important and the duration of the diffusion regime is short, if any. But for a higher number of balls the diffusion regime becomes dominant and the important number of shocks until the Brownian time, conjugated to the high N_b, makes it impossible for us to estimate this time with our simulations. But we can consider, as Fig. 11 illustrates, that the Brownian time T_B corresponds to the time of convergence of D values, which we interpret as the time for which all the possible positions of balls are reachable and all their phases have a 360° uncertainty. As the equations of diffusion are well known, there is no special interest to recall them to calculate T_B, and furthermore, it is not needed for the purpose of this paper. So we will focus on the much more important Brownian saturation time that we also call the saturation time T_S.

To calculate the saturation time that we have defined in the introduction, we have to count the total number N_{PS} of possible phase states of the billiard. Each phase has 4 coordinates and each coordinate has 10^E states, where E is the decimal precision of our discrete space–time (for example $E \sim 35$ for a billiard of one square meter and a precision down to the Planck scale). Then the total

number of billiard states is 10^{4EN_b}. Now, the total number of possible states is not so high because we have to consider only states with the same energy, which makes one coordinate of velocity depend on the other one, so that the number of coordinates is reduced to 3.

The saturation of states is then reached when the number of shocks N_{cc} is such that N_{PS} is equal to the accumulated number of branches, as we approximately suppose to have statistically reached uniformly distributed states:

$$N_{PS} = 10^{3EN_b} = D^{(N_{cc}-N_a)} \quad \text{with } T_C < T_A = \frac{N_a}{N_b} < T_B \tag{8}$$

We have reformulated in (8) the D growing law of relation (1) by introducing a number of shocks N_a which corresponds to the intersection with the n axis of the stabilized slope of $\log(N_U)$ versus n curve, which occurs at a time T_A for which $T_C < T_A < T_B$.

The saturation time T_S is then equal to:

$$T_S = \frac{N_{cc}}{N_b} = \frac{N_a}{N_b} + 3E\frac{\log(10)}{\log(D)} \tag{9}$$

What is remarkable in this result is that T_S is a limited value that could even tend toward the time T_A when the number of balls is high, but on the condition that D has indeed the asymptote N_{cp}. This remains contestable in the sense that it could require to accept infinitesimal probabilities that could be not possible in a discrete space–time, as we have to consider the possibility of discrete probabilities.

Now, our most solid result is obtained for low values of N_b for which the diffusion regime is negligible and $\log(D)$ is around $\log(10)$, so that the order of size of the second term of (9) is around $3E$ shocks per ball after the critical time $T_C \sim T_A$. If we also consider small balls (or high void ratios) for which we know that T_C is very low, we can neglect it and then the order of size of the saturation time is equal to $3E$, that is to say around hundred shocks per ball in the case of the Planck limit resolution ($E = 35$).

4. Discussion

The main result of this paper is to highlight that there exists a Brownian saturation time with a magnitude of order of $3E$ for the present billiard toy model (or $5E$ for a 3D dynamics), which is interesting as an estimation of the delay we would have to extend our calculations if we want to find multiple paths so as to join arbitrary given initial and final conditions. Though it involves calculating a considerable number of branches that is still out of reach of our current computers, we must notice that this delay condition could be realized for a lot of natural systems. That is why this result has a fundamental interest that we discuss in the following.

But first of all, it is noteworthy that the work we have presented in this paper has the practical outcome of our powerful indexation-chaining method to unbolt the data mining problem of millions of branches comparison, which induce unaffordable computing times. Without such a method, it would not have been possible for us to repeat numerous calculations involving comparisons of several millions of billiard histories, which enable our analysis of the growing law of billiard branches.

Yet the main focus of our paper is a theoretical one, that is to discuss the hypothesis that we have raised in its title, which we can reformulate as follows: Is it realistic to consider the possibility that some paths of objects in a local system of a discrete space–time could be replaced by other ones, while maintaining well determined final conditions?

The interest to keep fixed final conditions is here to highlight a hypothetic space–time property that could authorize changes of its local spatio-temporal structure without affecting its global or higher scale structure. Note that this is an already known property of diffusive systems for which equations are not differentiating low scale detailed trajectories. Yet we have shown that this indetermination is not just an empirical approximation but a real possibility in a discrete space–time, which enables individual paths to be not uniquely defined at low scales even though the global history remains well determined at a higher scale. Would not it be the connecting key between local

discrete and global continuous physics? Let us recall that it was recently highlighted in [16] that using deterministic equations does not prevent the basic laws from settling a Brownian regime after a "long time". However, this time was not quantified by the authors as it just results from an asymptotic mathematical property. Thus a second practical interest of our work is to show how to estimate this "long time" in a discrete space–time.

Now, let us consider the theoretical interest of our work, which is to give some realism to a possible indetermination of paths that could link well determined initial and final conditions of a local system. This possibility can be interpreted in various ways.

The first one is to propose that physical laws are still incomplete and say that when physicists will complete them they will be able to restore a perfect determinism and eventually characterize the particular parameters of the universe we live in within the classical discrete multiverse. This assumes that these parameters do not vary along time and then it is somewhat equivalent to having complementary data to add to initial conditions so as to inform in advance all choices made at all bifurcations, from the beginning (or big-bang) to now. We could then wonder about the existence of these complementary laws, while they are only justified by the necessity to supplement data to initial conditions: why not just add more precision and keep the same laws? A good answer is that adding enough precision to the initial conditions of early cosmology would require a tremendous quantity of physical information in the universe, as it should grow linearly in function of time to quasi infinitely small scales, so depending on the square of time. In the context of a finite density of information in the universe, this is proscribed. Fortunately, there is a much better alternative (also satisfying Occam's razor principle) which consists to add a very limited complementary data at all interactions. We calculated for example in our previous studies that only 3 or 4 bits per shock are generally enough to restore determinism, while the required information to make the same in the context of a continuous space–time should be a quasi-infinite one!

A second possibility is then to say that the additional but very limited data required to restore determinism needs other laws and extra dimensions, not only because it is more economic in terms of physical information, but also because it could vary along time. However we could first prefer extra dimensions without postulating the evolution of additional data. This is in particular the case in string theory where the 6D Calabi–Yau modes [23] are not supposed to vary along time, giving rise to the interpretation of a string theory multiverse which would have 10^{150} well separated branches which are as many different universes. More generally, this is also the case with other versions of the multiverse like the bubbles of early cosmology and even the Everett many worlds [7], which are all considered as deterministic, specifically because restoring determinism is the essential reason to appeal a multiverse, though it is still lacking the "hidden variables" and associated laws or rules that are necessary to make choices at bifurcations.

Yet, there are already good reasons to consider differently the problem with the multiverse, whatever the version we speak about. Both the fundamental principle of general relativity and the quantum gravity premises show that space–time has to be considered as something flexible, as it is bending and warping. Indeed, if we look at the timeless configuration of the block universe, there is something incoherent in the idea that these folds and deformations would not vary out of time and then leave the space–time exactly in the same gravitational frozen state from the big-bang to nowadays. One of the reasons it is still considered so is that our understanding of time is still vague and largely debated. But the main reason is that nobody knows how extra dimensional data could cause, by varying in time, a variation of the timeless gravitational field without destabilizing space–time as a whole. In brief, it is implicitly considered that space–time would collapse if such a variation was permitted in the equations.

The main theoretical interest of our work is therefore to demonstrate that it could not be the case, because we have shown that changing some choices at bifurcations may not affect subsequent relative final conditions, as it changes only very locally the space–time structure without collapsing it at higher scales. To say it in a more revealing way, we have shown that the opposite of the butterfly effect could also exist, meaning that even somewhat erratic changes in the complex evolution of a local system could change nothing at all to the global system in which it is immersed, thus maintaining its final state.

Such a proposition could be criticized saying that it is insane to generalize our results, as they are just calculated for a billiard with a few balls, to a real discrete space–time because the number of

interacting objects therein is incomparably higher. Yet we have shown that increasing the number of interacting objects does not increase the saturation time for which multiple paths could appear. On the contrary, while the number of branches grows as a geometrical series, the saturation time stays in the worst case independent of the number of objects and in the best case decreases with it, or even tends toward zero with T_C when the ball diameter is decreased. It is noteworthy that a high void ratio is a usual case in a lot of situations at different scales, for example from quantum particles like electrons to air molecules and planets, etc.

However, some objections could be raised against such a generalization, for example the fact that we worked with a billiard isolated by borders, but it is not the purpose of this paper to discuss them. We rather prefer to limit the scope of our finding by saying that it simply remains to affirm that in particular cases, changing a few individual histories could not change the collective one, which has already be described in other domains (for example a shoal of fish or a swarm of bees does not strictly forbid individual unconformity).

More fundamental, even if it is still speculative the possibility for the paths of objects in a local system to be not uniquely determined although the same future is maintained deserves to be mentioned as it stands because it is offering very interesting perspectives, in particular new highlights to the eternal debate about the question of time.

In physics, the current prevailing conception of space–time which is the block universe model is the object of a large debate [24] because a lot of physicists do not agree with a future that already exists with the same status as the past. If it is so, time could not exist physically as our intuitive frontier between a well determined past and an undetermined future, but as an emergent thermodynamic phenomenon [25]. But many physicists as for example Lee Smolin [26] or Georges Ellis [27,28] reject the block universe because it does not give a real physical and creative sense to the present and leaves no possibility, at best for free will, at worst for an undetermined future that seems yet promised by quantum fate.

In Fig. 12 we have condensed in the simplest possible way the basic concept of the new flexible model of space–time that we suggest. This model is reviving the function of time in a rather different way of Smolin and partially restoring the indefiniteness of the future in the same sense of Ellis. The big interest of this proposition is that though it is dynamic, it remains compatible with the mainstream static block universe model by making it at all times a well defined space–time in the past and the future. The objective sense of present time may then be to allow to define locally a succession of particular instants t_2 when each time line commutation occurs, thus permitting the block universe to evolve out of time. Such commutations should be controlled by timeless parameters, involving a new physics of time lines but which is still unknown. We cannot say at this stage if this new physics would imply or not some influence from the future that we mention as stable in Fig. 12. Though the possibility of retrocausality has long ago been suggested [29] before coming back strong recently with the work of Huw Price [30,31] and is now claimed as compatible with quantum mechanics [32], it cannot be confirmed without knowing the complementary laws of this new physics. We suggest here that it could possibly be carried out via hypothetic equations of evolution of the extra dimensions that we have proposed in our previous paper.

5. Conclusion

We have proposed a relevant response to the data-mining problem raised by quantifying the growth of the number of branches of a billiard classical multiverse, which have led us to results that are likely to question the very concept of a multiverse as a set of separate branches. Indeed, another interpretation is emerging, using space–time flexibility, which has the great interest to provide new insight on the issue of time that we felt it was important to discuss.

So, the essence of our paper is to highlight the counterintuitive fact that the concept of time determinism (or of many worlds) could be an illusion, probably due to the impossibility for our brain to conceptualize the ability for multiple objects to change their paths while joining the same past and future, because it involves an incommensurable complexity of interactions.

More concretely, we think that our result is likely to question the usual frontier between classical and quantum states of objects, that could be in the same sense somewhat illusory because we

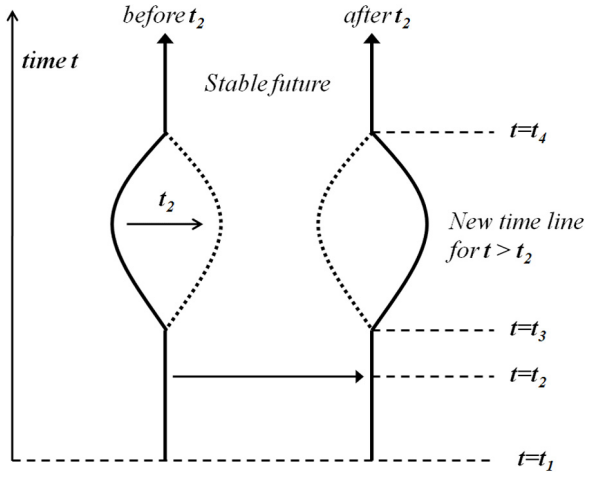


Fig. 12. In a flexible block universe, space–time could safely change its configuration by evolving via local commutations of different time lines. Note that the commutation time t_2 may be different from the bifurcation time t_3.

have shown that in a discrete classical space–time, the possibility for the path of one object to be locally not unique can be a macroscopic property. The difference between the classical and quantum ontologies could then be founded on the time delay T_C that is necessary for the first ontology to become a multi-paths one, while in quantum physics T_C would be reduced to zero as a result of the very small size of particles. In other words, our classical reality at our macroscopic scale could be similar to quantum but only in the future, though at different scales of time that depends on mass, distance and interactions between objects.

Now, the flexible universe hypothesis is challenging the multiverse theories as another candidate to report the existence of multiple paths. It is resting on the condition of accepting the possibility for extra dimensions [10,23] or quantum gravity [33] to bring the additional data that is necessary to coordinate the commutations of the time lines implied by this flexibility. The entanglement between paths would then be due to the impossibility to change one particular path of a given object without changing other involved paths and objects.

This is also raising rather more philosophical questions about the nature of time, that we have addressed in [34], and also about the difference between brain and consciousness, that has been for example deeply addressed in physics by Penrose [35] and in philosophy by Bitbol [36]. This difference could be founded on the capability of the latter to grasp multiple paths, resulting in an intuitive and perhaps not always illusory sensation of free will.

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