Group targeting under networked synergies

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A B S T R A C T

A principal targets agents organized in a network of local complementarities, in order to increase the sum of agents' effort. We consider bilateral public contracts à la Segal (1999). The paper shows that the synergies between contracting and non-contracting agents deeply impact optimal contracts: they can lead the principal to contract with a subset of the agents, and to refrain from contracting with central agents.

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1. Introduction

Institutions often contract with agents to trade effort against transfer, exploiting synergies and positive externalities between them. These synergies not only exist between contracting agents but also, in many economic contexts, between contracting and non-contracting agents. To cite a few examples: in conditional cash transfer (called CCT thereafter) programs in education, the students who do not receive the grant still interact with those who obtain the grant; in price discrimination with network externalities, where a firm offers network-based discounts on top of a homogeneous price, a consumer who does not receive a discount still consumes and interacts with consumers accepting a discount; in organizations where the firm offers workers a bonus, a worker not receiving any bonus still interacts with other workers; in R&D networks where a public fund provider allocates subsidies, a non-subsidized firm still spends on R&D and interacts with partner firms.[1] These networked peer effects can strongly affect incentives, in which case the institution can hardly ignore them. How should the institution take social influence into account? Can it be optimal to contract with a strict subset of the society? Should the institution always contract with the agents with largest social influence?

To understand the relationship between the structure of the network of synergies among agents and optimal contracts, we build a principal – agent model, where the principal, maximizing the sum of agents' effort, trades effort against transfer

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[1] Other applications could fit. For instance, in a political economy context, a government or an elite may give transfers to citizens in order not to participate to a coup; see Naidu et al. (2015) for a framework with a similar payoff function as in the current article. We thank an anonymous referee for mentioning this application.

in a context where non contracting agents (called *outsiders* thereafter) exert effort and interact with contracting agents, so that reservation utilities are endogenous to offered contracts. We study bilateral contracts with public offers *à la* Segal (1999) and we specify linear-quadratic utilities with local synergies and positive externalities.

There are several forces shaping optimal contracts in the network. First, the concavity of utilities pushes the principal to contract with all agents in the society. Second, the agents with larger social influence may be offered contracts with higher effort and higher transfers. This being said, the synergies between contracting agents and outsiders matter too. Indeed, an agent rejecting an offer takes into account the reaction of outsiders to own effort reduction, which triggers a further decrease of own effort by the synergies. Hence, outsiders have a *disciplinary effect* on contracting agents. As a consequence, in principle it may not be optimal to contract with the whole society. It may even be not optimal to contract with central agents, as the principal may be incited to exploit the agents with a large social influence as outsiders in order to discipline contracting agents.

The analysis reveals that the disciplinary effect deeply impacts the principal's strategy. We will speak about *limited targeting* when the targeted group is a strict subset of the whole society. We obtain two main messages. First, *the disciplinary effect can lead to limited targeting*, i.e. the principal may not find it optimal to contract with all agents. Limited targeting emerges under high intensities of interaction, where outsiders have a large disciplinary effect on contracting agents. Limited targeting also emerges under low budget. In particular, with zero budget, contracting with the whole society has no impact on effort, whereas the presence of outsiders stabilizes contracts with enhanced effort. Second, *the principal can find it optimal not to contract with central agents*,[2] meaning that the principal prefers to exploit their large social influence to discipline contracting agents rather than to contract with them and exploit increased synergies with other agents.

We illustrate the emergence of limited targeting in the context of CCT programs in education. Here a public institution, like a state or a municipality, trades school attendance in education against cash transfers.[3] There are many CCT programs as, for instance, PROGRESA-Opportunidades in Mexico and Bolsa-Familia in Brazil.[4] Recent papers have identified friendship networks at school where peer effects play a significant role. We consider the model of friendship networks at school of Calvò-Armengol, Patacchini and Zenou (2009)[5] and we incorporate a public institution enforcing a CCT program in the classroom. Focusing on a real 16-student friendship network presented in their paper, we find limited targeting for reasonable parameter values.

The emergence of limited targeting obtains under three key assumptions: contracts are bilateral, the principal commits to the proposed contracts, and agents take into account outsiders reaction to deviation. We then check whether limited targeting can emerge under alternative assumptions. We first show that contingent contracting (to others' contract acceptance) forbids limited targeting. Indeed, raising contingent contracts allows the principal to extract the full surplus of each agent, so it is always optimal to contract with all agents. Second, when the principal cannot commit to offered contracts, whether the budget is exogenous or endogenous matters. An endogenous budget deters limited targeting because, from any targeted group, the principal can always create value by contracting with an additional agent. In contrast, under exogenous budget, the budget may play as a commitment device, and limited targeting is still possible. Last, we examine a simpler model without disciplinary effect, by assuming rather that agents consider outsiders' play as fixed when they reject an offer. We then show that (i) when the principal deals with a fixed number of contracts, the most central agents are always selected, exert higher effort and receive higher transfers, and (ii) enlarging the targeted group is always beneficial to the principal, implying that it is optimal to offer contracts with positive transfers to all agents.

Related literature. This paper contributes to the two strands of literature on optimal intervention in presence of synergies between agents. The first strand considers optimal targeting in presence of interacting agents.[6] Allouch (2015) considers a model of a local public good under linear substitute interactions, and explores optimal transfers to improve aggregate effort. Demange (2017) studies the optimal targeting strategies of a planner aiming to increase the aggregate action of agents embedded in a social network, allowing for non-linear interaction. Galeotti et al. (2017) study optimal targeting in networks, where a principal aims at maximizing utilitarian welfare or minimizing the volatility of aggregate activity. In the drop-out game of Calvò-Armengol and Jackson (2004), the planner subsidizes agents' entry into the labor market. There is also an auxiliary literature studying lobbying or marketing on networks; see Battaglini and Patacchini (2018) for a recent contribution in the context of linear interaction, and references therein. In our work, we take into account participation constraints, not addressed in the above papers.

The second strand of literature studies principal/multi-agent contracting in presence of synergies, taking into account participation constraints. A closely related literature considers coordination issues with binary actions. Bernstein and Winter (2012) study a costly participation game where participants receive positive and heterogeneous externalities from other

[2] The relevant centrality index is the so-called Bonacich centrality measure, which naturally emerges in games of linear interaction (see Ballester et al. (2006)).

[3] This is to the extent that school attendance is positively correlated to a productive effort.

[4] For a review, see Fiszbein et al. (2009), or Reimers et al. (2006).

[5] Calvò-Armengol, Patacchini and Zenou (2009) identify peer effects in friendship networks at school and show that, after controlling for observable individual characteristics and unobservable network specific factors, a standard deviation increase in the Katz-Bonacich centrality increases the pupil school performance by more than 7 percent of one standard deviation.

[6] See Bloch (2015) for a recent survey.

participants, and characterize the contracts inducing full participation while minimizing total subsidies. In Sakovics and Steiner (2012), a principal subsidizes agents facing a coordination problem akin to the adoption of a network technology. Optimal subsidies target agents who impose high externalities on others and on whom others impose low externalities. With respect to this literature, we introduce continuous effort, and we cover situations where contracting with a subset of the population can be optimal. Recent studies explore optimal linear pricing with interdependent consumers (Candogan et al. (2012), Bloch and Quérou (2013) and Fainmesser and Galeotti (2016)). We contribute to this literature by enriching the set of contracts.

The efficiency of contracts in presence of externalities between agents has been considered by Holmstrom (1982) and Segal (1999), but our paper differs in two main respects. First, our primary focus is the impact of the network structure on optimal contracts, and not trade efficiency. Second, we allow for synergies between non-contracting and contracting agents. Genicot and Ray (2006) – see also Galasso (2008) – study a dynamic game where outside opportunities rise with the number of non-contracting agents. Conversely, in our setting outside opportunities tend to decrease with the presence of non-contracting agents, since when an agent rejects a contract he exerts less effort.

The paper is organized as follows. The model is presented in Section 2. Section 3 presents the main results of the article. Section 4 provides a concrete illustration of limited targeting in the context of a CCT program at school. Section 5 examines whether limited targeting emerges under contingent contracting, when the principal does not commit to offers, and when agents do not take into account the reaction of outsiders to own deviation. Section 6 concludes. All proofs are presented in Appendix A. Appendix B details how to compute the performance of groups on any network, and Appendix C analyzes contingent contracting deeper.

2. Model

We develop a principal – agent model where agents are organize in a network of interaction. The principal commits to a set of contracts with a finite set of agents organized in a fixed network of local complementarities. A crucial feature of our model is that non contracting agents, that we call outsiders, still exert effort and interact with contracting agents. For instance, in the example of CCT programs in education, the principal can be a public institution like a state or a municipality, and agents represent students in the classroom organized in a friendship network.

We consider a three-stage game. In the first stage, the principal proposes bilateral contracts as in Segal (1999). Each contract is an effort-transfer pair. In the second stage, agents simultaneously decide whether to accept or reject their respective offers. In the third stage, agents exert effort and transfers are realized. Both effort, contracts and network are assumed to be publicly observable. We study a Subgame Perfect Nash Equilibrium (SPNE). In this model, coordination is an issue and equilibrium multiplicity can arise. We analyze the SPNE of the game which maximizes the principal's objective; that is, we focus on the equilibrium such that all proposed offers are accepted.

Notations. Real numbers or integers are written in lower case, matrices (including vectors) in block letters and in boldface. We denote by $\mathbf{1}_p$ the p-dimensional vector of ones, for all $p \in \mathbb{N}$. For convenience, symbol $\mathbf{1}$ will quote for $\mathbf{1}_n$. Similarly, symbol $\mathbf{0}$ represents the n-dimensional vector of zeros. We let superscript ^{T} stand for the transpose operator. For instance, we write vector $\mathbf{X} = (x_i)_{i\in\mathcal{N}}$, with x_i its ith entry, and $x = \mathbf{1}^T\mathbf{X}$ denotes the sum of entries of vector $\mathbf{X}$. Calligraphic characters describe sets; e.g. $\mathcal{S}$.

The game in the absence of a principal. We let $\mathcal{N} = \{1, 2, \cdots, n\}$ be the set of agents organized in a network of bilateral relationships. The network is undirected, i.e. it is formally represented by a symmetric adjacency matrix $\mathbf{G} = [g_{ij}]$, with binary element $g_{ij} \in \{0, 1\}$.[7] By abuse of language we will speak of network $\mathbf{G}$. The link between agents i and j exists whenever $g_{ij} = 1$, in which case we will say that agents i and j are neighbors. By convention, $g_{ii} = 0$ for all i. We let $\omega(\mathbf{G})$ denote the largest eigenvalue of the adjacency matrix $\mathbf{G}$.

We consider linear quadratic utilities of the form

$$u_i(q_i, \mathbf{Q}_{-i}) = q_i - \frac{1}{2}q_i^2 + \delta\sum_{j\in\mathcal{N}} g_{ij}q_iq_j \tag{1}$$

with $\mathbf{Q} \geq \mathbf{0}$. Parameter $\delta > 0$ measures the strength of complementarities, or intensity of interaction, between neighbors. With the above specification, utilities depend positively on neighbors' effort, and neighbors' effort levels are strategic complements.

We define Bonacich centralities, which play a prominent role in network games with linear-quadratic utilities (see Bonacich (1987)). We let the n-dimensional square matrix $\mathbf{M} = (\mathbf{I} - \delta\mathbf{G})^{-1} \geq 0$. The condition $\delta\omega(\mathbf{G}) < 1$ guarantees $\mathbf{M} \geq 0$. We let the n-dimensional vector $\mathbf{B} = \mathbf{M}.\mathbf{1}$, with entry i called b_i, denote the vector of *Bonacich centralities* of the network

[7] Our results are qualitatively robust to the introduction of non-binary and asymmetric relationships, where centralities are replaced by inward/outward centralities, and also to the introduction of a low level of strategic substitutability.

weighted by parameter δ (we avoid references to network G and parameter δ for convenience). The quantity b_i is the number of paths from agent i to others, where the weight of a path of length k from agent i to agent j is δ^k.

In the absence of contracts, agents play a unique Nash equilibrium. Any agent $i \in \mathcal{N}$ exerts an effort equal to her Bonacich centrality b_i and obtains a utility level equal to $\frac{1}{2}\, b_i^2$.

Contracts. A contract between the principal and agent i specifies an effort $x_i \in \mathbb{R}^+$ and a monetary transfer $t_i \in \mathbb{R}$ from the principal to agent i. Assume that the principal contracts with subset $\mathcal{S} \subset \mathcal{N}$, with cardinality s. We let $\mathbf{X}_{\mathcal{S}} = \{x_i\}_{i \in \mathcal{S}}$ represent the corresponding profile of effort. For convenience, we will denote $\mathbf{X} = \mathbf{X}_{\mathcal{N}}$. To determine the utilities of contracting agents and outsiders, we need to distinguish two cases.

Case 1: All agents in $\mathcal{S}$ accept their contracts. Then outsiders play a Nash equilibrium effort given $\mathbf{X}_{\mathcal{S}}$. Agent j' best-response effort is given by

$$y^{BR}_{\mathcal{N}\setminus\mathcal{S},j} = 1 + \delta \sum_{k \in \mathcal{S}} g_{jk} x_k + \delta \sum_{k \in \mathcal{N}\setminus\mathcal{S}} g_{jk} y_k$$

where y_k represents the effort of outsider k. Let $\mathbf{G}_{\mathcal{N}\setminus\mathcal{S}}$ be the $(n-s) \times (n-s)$ sub-matrix of matrix $\mathbf{G}$ representing the bilateral influences between pairs of agents in $\mathcal{N} \setminus \mathcal{S}$, and $\mathbf{G}_{\mathcal{N}\setminus\mathcal{S},\mathcal{S}}$ the $(n-s) \times s$ sub-matrix of matrix $\mathbf{G}$ representing the bilateral influences between agents in $\mathcal{S}$ and agents in $\mathcal{N} \setminus \mathcal{S}$. The Nash equilibrium effort profile of outsiders $\mathbf{Y}^*_{\mathcal{N}\setminus\mathcal{S}}$, given $\mathbf{X}_{\mathcal{S}}$, is written as:

$$\mathbf{Y}^*_{\mathcal{N}\setminus\mathcal{S}} = (\mathbf{I} - \delta \mathbf{G}_{\mathcal{N}\setminus\mathcal{S}})^{-1}(\mathbf{1}_{n-s} + \delta \mathbf{G}_{\mathcal{N}\setminus\mathcal{S},\mathcal{S}} \mathbf{X}_{\mathcal{S}}) \tag{2}$$

The utility of a contracting agent i is given by

$$v_i(\mathbf{X}_{\mathcal{S}}, \mathbf{Y}^*_{\mathcal{N}\setminus\mathcal{S}}, t_i) = u_i(\mathbf{X}_{\mathcal{S}}, \mathbf{Y}^*_{\mathcal{N}\setminus\mathcal{S}}) + t_i \tag{3}$$

Case 2: An agent i in $\mathcal{S}$ rejects the offer while all other offers are accepted. Agent i becomes an outsider and plays a Nash equilibrium effort with other outsiders. We denote by z_i her effort, which is the entry corresponding to agent i in the following Nash profile:

$$\mathbf{Y}^*_{\{i\}\cup\mathcal{N}\setminus\mathcal{S}} = (\mathbf{I} - \delta \mathbf{G}_{\{i\}\cup\mathcal{N}\setminus\mathcal{S}})^{-1}(\mathbf{1}_{n-s+1} + \delta \mathbf{G}_{\{i\}\cup\mathcal{N}\setminus\mathcal{S},\mathcal{S}\setminus\{i\}} \mathbf{X}_{\mathcal{S}\setminus\{i\}}) \tag{4}$$

The equilibrium utility of agent i is then given by

$$u_i(\mathbf{X}_{\mathcal{S}\setminus\{i\}}, \mathbf{Y}^*_{\{i\}\cup\mathcal{N}\setminus\mathcal{S}}) = \frac{1}{2} z_i^2 \tag{5}$$

Principal's program. The principal's objective is to maximize the aggregate effort of both contracting agents and outsiders, subject to both the budget constraint and individual participation constraints. The principal's program, called program $\mathcal{P}$, is written as:

$$\max_{\{(x_i,t_i)\}_{i\in\mathcal{S}}} \quad \mathbf{1}^T \mathbf{X}_{\mathcal{S}} + \mathbf{1}^T \mathbf{Y}^*_{\mathcal{N}\setminus\mathcal{S}}$$

s.t.

$$\frac{1}{2} z_i^2 \le u_i(\mathbf{X}_{\mathcal{S}}, \mathbf{Y}^*_{\mathcal{N}\setminus\mathcal{S}}) + t_i,\ \forall i \in \mathcal{S} \tag{6}$$

$$\sum_{i\in\mathcal{S}} t_i \le t \tag{7}$$

where $\mathbf{Y}^*_{\mathcal{N}\setminus\mathcal{S}}$ and z_i are functions of $\mathbf{X}_{\mathcal{S}}$ and determined respectively by equations (2) and (4).

The above program can be part of a more general program with endogenous budget, where the principal's payoff is an increasing and concave function of the sum of agents' effort net of transfers. For clarity, and because the impact of network structure is essentially captured by the restricted sub-problem with a fixed budget, we abstract from optimal budget selection considerations throughout the paper and assume that the budget is fixed and not larger than the optimal budget.

The analysis calls for intuitive preliminary observations. First, individual participation constraints are binding at optimum, otherwise the principal could save on transfers for the same objective, and use the saved budget to increase effort. Second, at optimum, transfers are non-negative; otherwise agents are better off rejecting the offer and playing their best-responses. That is, the principal should only be rewarding agents. Last, such a program should admit optimal contracts for all selected groups $S \in \mathcal{N}$. We note that the condition $\delta < \frac{1}{2\omega(\mathbf{G})}$ is sufficient.[8]

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[8] As will be shown thereafter, this condition guarantees the existence of the optimal contingent contract, and no set of bilateral contracts with any group of agents can do better.

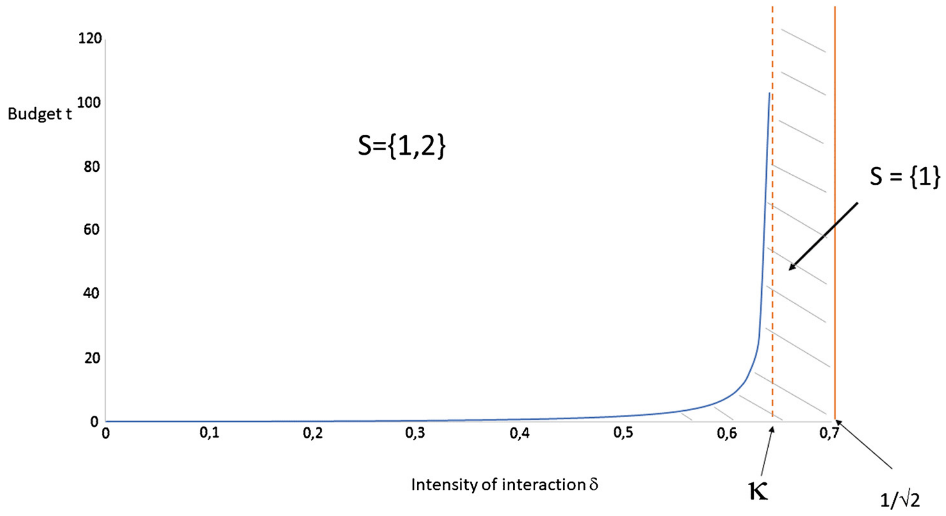

Fig. 1. $n = 2$; Optimal group as function of parameters δ, t.

3. Optimal bilateral contracts

In this section, we study the optimal bilateral contracts in presence of disciplinary effect. This means that, when agents reject an offer, they take into account that outsiders adjust their play. To get some insights, we first present theoretical results about the emergence of limited targeting, and then we explore specific network structures.

3.1. Theoretical results about limited targeting

To get some insights, we first examine the two-agent case, then we analyze optimal group selection for general networks in the polar cases of low and high intensities of interaction.

Two-agent society. We consider a society composed of two connected agents, say agents 1 and 2. If the principal contracts with both agents, the reservation utility of an agent takes as given the effort prescribed in the contract of the other agent. In contrast, when the principal contracts with, say, agent 1, agent 2 always plays a best-response to agent 1's effort; so, when agent 1 rejects her offer, agent 2's effort is lower than her effort under offer acceptance. Hence, agent 1's reservation utility is lower when the principal contracts with agent 1 than when the principal contracts with both agents. This decrease in the outside option of agent 1 allows the principal to increase agent 1's effort, and, ultimately, it may be profitable for the principal to exclude agent 2. That said, by concavity of utilities, distributing transfers among a large group is attractive. Which effect dominates is highly dependent on parameters and network structure.

We let $t_c(\delta) = \frac{2\delta^2(1+\delta)}{1-2\delta^2-\delta^4}$ for $\delta \in [0, \kappa[$ with $\kappa = \sqrt{\sqrt{2}-1}$. We obtain:

Proposition 1. *In the two-agent society, it is optimal to contract with a single agent if and only if $\delta \in [\kappa, \frac{1}{\sqrt{2}}[$, or $\delta < \kappa$ and $t < t_c(\delta)$.*

Proposition 1 essentially provides three messages. First, it may not be optimal to select the whole society, confirming that the disciplinary effect can dominate. Second, when $\delta < \kappa$, the disciplinary effect dominates under very low budget. Indeed, even with null budget, the principal can increase aggregate effort by contracting with a single agent, exploiting both the commitment feature of the contract (from the agent) and the disciplinary effect. In contrast, contracting with the two agents with null budget yields no increase in effort. Last, the disciplinary effect is prevailing for high intensities of interaction; even more, under very high intensities of interaction (i.e., for $\delta \geq \kappa$ on the figure), limited targeting emerges for all budgets. These messages are illustrated in Fig. 1, which presents the optimal group size as a function of the budget and of the intensity of interaction. The range of possible intensities of interaction is $[0, \frac{1}{\sqrt{2}}[$. The upper bound corresponds to the threshold above which the optimum no longer exists (i.e. effort would escalate to infinity):

General case. The study of optimal contracts in general network structures confirms these intuitive messages. The next proposition clarifies the issue in the limit cases of low and high intensity of interaction:

Proposition 2. *Fix $t > 0$. (i) When the intensity of interaction is high enough, the optimal group is a strict subgroup of the society. (ii) When the intensity of interaction is low enough, the optimal group is the whole society. Optimal contracts $\{(\hat{x}_i, \hat{t}_i)\}_{i\in\mathcal{N}}$ satisfy:*

$$\begin{cases} \hat{x}_i = b_i + \frac{\sqrt{2t}}{\|\mathbf{B}\|} \cdot b_{\mathbf{B},i} \\ \hat{t}_i = \frac{t}{\|\mathbf{B}\|^2} \cdot b_i^2 \end{cases} \tag{8}$$

Proposition 2 indicates that, under high intensity of interaction, the disciplinary effect dominates. In the opposite, under low intensity of interaction the concavity of utilities dominates, so that the principal contracts with the whole society and the solution of the program coincides with the optimal contract of the simplified model without disciplinary effect.

Note also that, when the optimal group is the whole society, both supplementary effort, $\hat{x}_i - b_i$, and transfer $\hat{t}_i$ are increasing with centrality measures. Increased effort is proportional to weighted Bonacich centrality, with weights themselves equal to un-weighted Bonacich centrality. Note that the two centrality measures $b_{B,i}$ and b_i may not be aligned. Transfers are positive[9] and proportional to squared centralities, meaning that there is a bonus for central agents. We also note that the network structure affects the transfer per unit of increased effort; i.e., $\frac{t_i}{\hat{x}_i - b_i}$ is proportional to $\frac{b_i^2}{b_{\mathbf{B},i}}$. Moreover, budget-sharing among agents, $\frac{\hat{t}_i}{t} = \frac{b_i^2}{\|\mathbf{B}\|}$, is independent of budget level and is concentrated in favor of central agents. Note that the optimal aggregate effort is written

$$\hat{x} = b + \sqrt{2t} \cdot \|\mathbf{B}\| \tag{9}$$

Interestingly, limited targeting not only arises under high interaction, but also under low budget level:

Proposition 3. *Fix $\delta > 0$. When the budget is small enough, the optimal group is a strict subset of the society.*

Proposition 3 suggests that the disciplinary effect dominates for low budget. The intuition is the same as in the two-agent case (commitment plus disciplinary effect).

Propositions 2 and 3 confirm the messages obtained in the two-agent case, by providing conditions under which it is optimal to concentrate the budget over a subset of the whole society. However, these propositions are silent about the best group to target. Finding the optimal group can hardly be done in the general case. We investigate the simpler case of null budget and low intensity of interaction. In this simplified setting, the principal only partitions the society but does not bear the allocation problem of transfers. Still finding the optimal group is challenging. For each group $\mathcal{S}$, let $d_i^{\mathcal{N}\setminus\mathcal{S}} = \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij}$ be the number of neighbors of agent i in the set $\mathcal{N}\setminus\mathcal{S}$, and let $\mathcal{P}(\mathcal{N})$ be the set of all parties of $\mathcal{N}$:

Proposition 4. *Under null budget and under sufficiently low intensity of interaction, the program of the principal is given by*

$$\max_{\mathcal{S}\in\mathcal{P}(\mathcal{N})} \sum_{i\in\mathcal{S}} \sqrt{d_i^{\mathcal{N}\setminus\mathcal{S}}} \tag{10}$$

In the problem defined in equation (10), the principal chooses the group that maximizes the sum of the squared roots of the number of cross-links over all agents in $\mathcal{S}$. This problem echoes the so-called MaxCut problem where the principal maximizes the number of links between the two groups — when the graph is undirected, the Maxcut problem can be written as $\max_{\mathcal{S}\in 2^{\mathcal{P}(\mathcal{N})}} \sum_{i\in\mathcal{S}} d_i^{\mathcal{N}\setminus\mathcal{S}}$ —.[10] This latter problem, which has been deeply studied in the literature of computer science and operations research, is NP-hard.

Proving that the problem defined in equation (10) is NP-hard is still an open issue. However, this problem is easily solved on simple network structures. For example, in the complete network, the optimal group size is close to $\frac{2n}{3}$; this is a larger size than under the MaxCut problem: by the concavity of squared roots, the trade off between increasing the group size and increasing the number of cross-links per member induces profitable group enlargement beyond equal split. In a circle network and in a line network of even size, the optimal group is the Maximal Independent Set with maximal number of agents (i.e., $s^* = \frac{n}{2}$). In the star network, the optimal group is the set of all peripheral agents, meaning that the principal does not contract with the central agent; in particular, contracting with the central agent only induces the same number of cross-links but its is better to lower the average number of cross-links per member of the group.

Although NP-hardness is still an open issue, we perform simulations by implementing the *greedy algorithm*[11] on random networks. In our context, the algorithm works as follows. In step 1, it determines the best singleton; then it determines

[9] It can be shown that transfers are positive for any concave utilities satisfying complementarities.

[10] In Zhou and Chen (2015), the problem of finding the optimal split into a leader group and a follower group reduces to a MaxCut problem under low intensity of interaction.

[11] The algorithm is available upon request. The so-called greedy algorithm is often used to accelerate the search of the optimal group at the price of a bounded inefficiency (Ballester et al. (2010) use such an algorithm in the key group policy context). However, the condition under which the greedy algorithm guarantees a lower bound on inefficiency (around 36 percent) does not apply in our context: the principal's objective, i.e. the performance of a group, is neither submodular neither supermodular.

Table 1
Average over 100 random networks of the relative error of approximation of the greedy algorithm in percentage of the optimal performance.

Case $n=10$				Case $n=15$			
		$t=1$	$t=10$			$t=1$	$t=10$
$p=0.25$	$\delta=0.01$	98.42	97.00	$p=0.25$	$\delta=0.01$	99.09	98.20
$p=0.25$	$\delta=0.1$	98.70	97.29	$p=0.25$	$\delta=0.066$	99.22	98.37
$p=0.5$	$\delta=0.01$	98.42	97.01	$p=0.5$	$\delta=0.01$	99.10	98.18
$p=0.5$	$\delta=0.1$	99.26	97.78	$p=0.5$	$\delta=0.066$	99.47	98.53
$p=0.75$	$\delta=0.01$	98.42	97.00	$p=0.75$	$\delta=0.01$	99.10	98.18
$p=0.75$	$\delta=0.1$	98.02	98.79	$p=0.75$	$\delta=0.066$	99.19	99.17

the best pair containing the best singleton, and if its performance is smaller than that of the best singleton, it stops and selects the best singleton as the optimal group; otherwise in step 2 it determines the best triplet containing the best pair, and if its performance is smaller than that of the best pair, it stops and selects the best pair as the optimal group; etc. This algorithm converges in at most n steps. We implement this algorithm with the following initial parameters: for network size, we initiate $n \in \{10, 15\}$ (combinatorial concerns become very important for larger values of n); for the uniform probability p of link existence, $p \in \{0.25, 0.5, 0.75\}$ representing respectively sparse, moderate, and dense networks; for the intensity of interaction, $\delta \in \{0.01, \frac{1}{n}\}$; for the budget, $t \in \{1, 10\}$. In total, we have 24 scenarios. In each scenario, we perform a simulation generating 100 random networks. For each simulation, we find the optimal group by searching through all possible groups (for instance, with $n=10$, the program searches through $2^{10}-1=32767$ subsets agents) and obtain its performance. Then, we approximate the optimal group using the greedy algorithm and we obtain its performance. Finally, we compute the relative error of approximation of the greedy algorithm in percentage of the optimal performance. Table 1 display the results of our numerical simulations. The numbers in Table 1 are the average relative error of approximation (in percentage) over the 100 networks in each scenario. Roughly speaking, the greedy algorithm performs very well, the average relative error of approximation being less than 4 percent in any case.

3.2. Optimal targeting on specific network structures

The performance of any targeted group can be computed, but in general sorting groups by their performance can hardly be done analytically, because there is no monotonic relationship between group composition and parameters t or δ. Appendix B presents the performance of any targeted group through standard Lagrangian method on any network. To illustrate further how network structure affects optimal group selection, we explore specific network structures by means of numerical computations.[12] We explore in the order the complete network, the circle, the star and line. The numerical analysis illustrates the relationship between the size of the group and parameters δ and t and the relationship between centrality and optimal targeting.

The complete network. The complete network is such that there is a link between all pairs of agents. In the complete network, all agents have same centrality. Furthermore, once the principal targets a group, all agents inside the group have the same positions and are offered the same contract. As well, all outsiders have the same positions. Hence, with this structure, we only study the link between group size and optimal targeting. Let

$$\begin{cases} A = -\frac{1}{2} + \delta(s-1) + \delta^2 \frac{s(n-s)}{1-\delta(n-s-1)} - \frac{\delta^2}{2} \frac{(s-1)^2}{(1-\delta(n-s))^2} \\ B = 1 + \delta \frac{n-s}{1-\delta(n-s-1)} - \delta \frac{s-1}{(1-\delta(n-s))^2} \\ C = \frac{t}{s} - \frac{1}{2(1-\delta(n-s))^2} \end{cases}$$

We have $A<0$ for low intensity of interaction, and we consider by δ_A the smallest intensity of interaction such that $A=0$ (this ensures the convexity of the participation constraint). Let $x(s) = \frac{B+\sqrt{B^2-4AC}}{-2A}$. We obtain:

Proposition 5. *Let $\delta < \min(\delta_A, \frac{1}{n-1})$ and $t \geq 0$. The optimal group size s^* maximizes*

$$\frac{(1+\delta)s\,x(s) + n - s}{1-\delta(n-s-1)}$$

Fig. 2 illustrates optimal group targeting on the complete network with $n=8$ for various parameters δ and t. All three

[12] We run simulations that compute the optimal group by sorting the performance over all possible groups; the performance of a group is determined according to the Lagrangian method given in Appendix B. The matlab program generating the optimal group through a systematic search over all possible groups is available upon request.

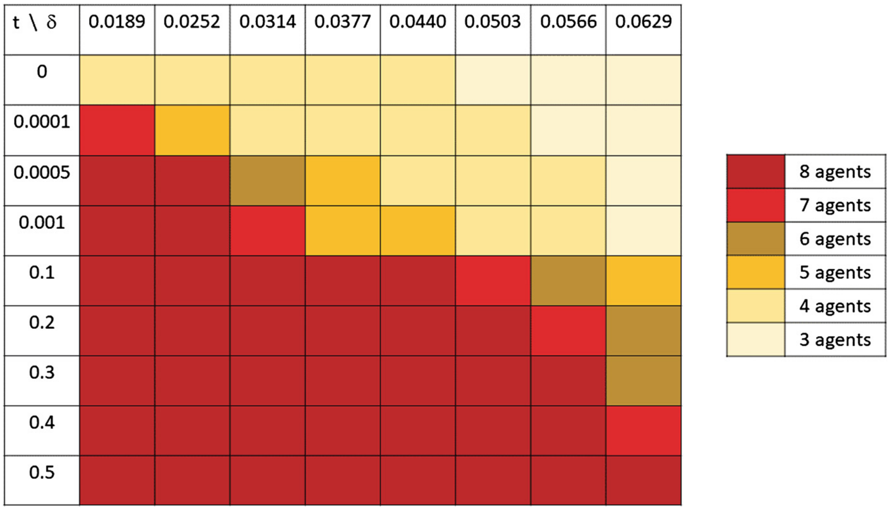

Fig. 2. Optimal group size on the complete network ($n = 8$), as a function of the budget and of the intensity of interaction. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

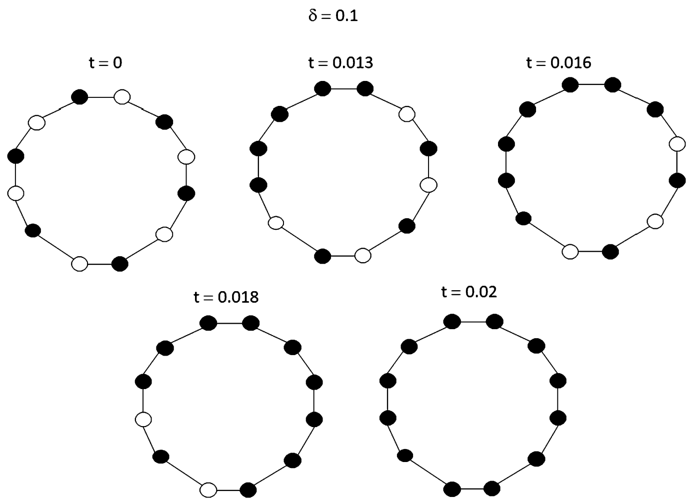

Fig. 3. Optimal group on the circle network; targets are black.

messages given for $n = 2$ are confirmed in this figure, and also emerge from a large set of numerical computations. First, the disciplinary effect can lead the principal to select a group of intermediary size. The second message is that fixing the intensity of interaction and increasing the budget can only increase the size of the optimal set. Third, fixing the budget and increasing the intensity of interaction can only reduce the size of the optimal set.

The circle network. The circle network contains n links and each agent has two neighbors. All agents have the same structural positions on the circle network. However, in contrast with the complete network, the disposition of contracting agents is a matter. Numerical computations on the circle network confirm the huge impact of the disciplinary effect. Fig. 3 presents the optimal group for a fixed intensity of interaction and varied budgets on the 12-agent circle network. It not only shows that intermediary groups can emerge at optimum, but also that both contracting agents and outsiders can be irregularly distributed on the circle, which stands in sharp contrast with the benchmark without disciplinary effect (where, by Lemma 1, all targeted groups of same size yield the same performance).

The star network. The star structure contains $n - 1$ links and one agent is involved in all links. Here, agents have heterogeneous positions on the network, and in particular the agent with $n - 1$ links is unambiguously more central than other agents. We call her the *central agent* and the other agents are called *peripheral agents*. Actually, the central agent needs not belong to the optimal group. Again, this stands in sharp contrast with the model without disciplinary effect. Fig. 4 illustrates this finding on the 10-agent star network. Fig. 4 shows that (i) the disciplinary effect can lead to exclude the central agent from the optimal group, and (ii) there is a non-monotonic effect in the composition of the optimal group with

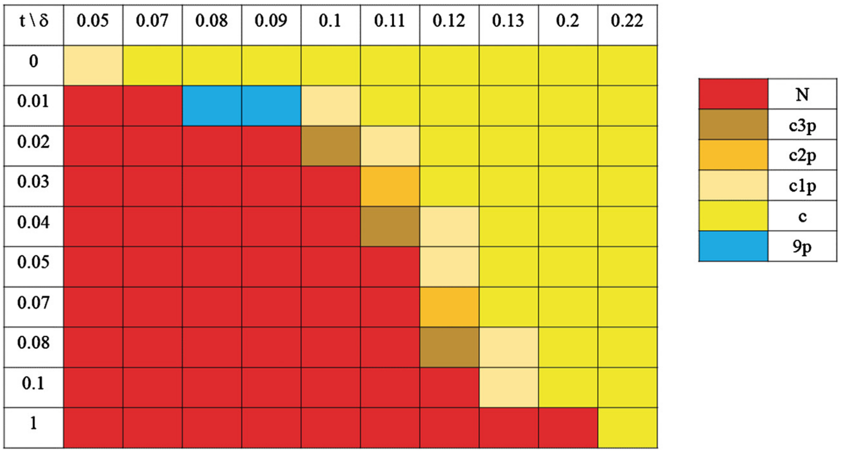

Fig. 4. Optimal group size on the 10-agent star network, as a function of the budget and of the intensity of interaction. We use the following symbols: c for the central agent; $9p$ for the nine peripheral agents; cip for the central agent and a number $i = \{1, 2, 3\}$ of peripheral agents; N for the whole society. In blue cells, the principal does not contract with the central agent.

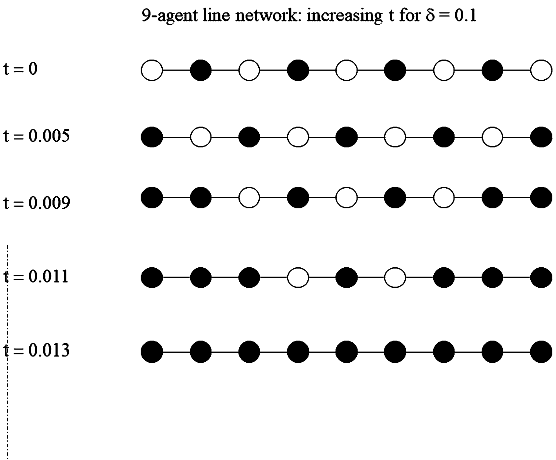

Fig. 5. Optimal group on the line network; targets are black.

respect to both budget level and intensity of interaction. I.e., fixing the intensity of interaction and increasing the budget, or fixing the budget and increasing the intensity of interaction, it can be that the central agent belongs to the optimal group, then is excluded, and then belongs again to the target.

The line network. The line network contains $n - 1$ links and no agent has more than two neighbors. Like the star network, agents closer to the middle of the line are unambiguously more central. Fig. 5 illustrates that the principal may not contract with agents of intermediary centrality on the 9-agent line network even when the most peripheral agents receive an offer. By contrast, in the model without disciplinary effect, the best target of fixed size contains the most central agents. Increasing the budget enlarges the optimal group, but does not induce a systematic concentration of targets toward the center of the line. Note that, as budget increases, a given agent can be included in the targeted group, then excluded, and then included again.

4. Limited targeting in conditional cash transfers in education

This section provides a concrete illustration of the possible emergence of limited targeting in the context of conditional cash transfers in education.

It is often argued that peer effects are prevalent among teenagers. In order to foster educational effort, public institutions such states often propose grants conditional to students attendance.[13],[14] There are many programs of cash transfers conditional on school attendance, especially in developing countries, like PROGRESA-Opportunidades in Mexico, Bolsa-Familia in Brazil, FFE, Eduque a la Niña, PRAF, GABLE, RPS to cite a few (for a review, see Fiszbein et al. (2009), and Reimers et al. (2006) for an evaluation of the efficiency of such programs).

We argue that limited targeting is a plausible outcome of CCT programs in education. There are at least two reasons. First, as suggested in Dieye et al. (2014), "Friendship data reveals that treated and untreated students interact together". This means that the main hypothesis of our model, the interaction between contracting agents and outsiders, is empirically attested. Second, the conditions of limited targeting are likely to be satisfied in many cases. Precisely, our work suggests that limited targeting can occur for rather low budget or for high intensities of interaction, and that the higher the level of interaction, the larger the budget compatible with limited targeting. There is much evidence of a limited budget devoted to such grants, especially in developing countries. In fact, the resources allocated to CCTs are variable, ranging from rather substantial to nearly negligible.[15] Furthermore, data suggest that the intensity of interaction between students at school is large. For instance, Calvò-Armengol, Patacchini and Zenou (2009) identify peer effects in friendship networks at school, with a specification of teenagers' utilities close to the present model. The average estimated intensity of interaction is around 0.56 (see Table 3, pp. 1254 in their paper – in their data there are 181 networks between 16 and 121 members).

To illustrate further the possible emergence of limited targeting, we consider the model of Calvò-Armengol, Patacchini and Zenou (2009). In this model, student i exerts effort $y_i^0 \geq 0$ absent of any peer influence, and exerts a peer effort $x_i \geq 0$ whose return depends on other's peer effort. Her utility is given by

$$U_i(\mathbf{Y}^0, \mathbf{X}) = \underbrace{\theta_i y_i^0 - \frac{1}{2}(y_i^0)^2}_{\text{Idiosyncratic component}} + \underbrace{u_i(x_i, \mathbf{X}_{-i})}_{\text{Peer component}}$$

where $\theta_i \geq 0$ is the idiosyncratic productivity of student i absent peer effects,[16] and where $u_i(x_i, \mathbf{X}_{-i})$ is given by equation (1).[17] Under the condition $\phi\omega(\mathbf{G}) < 1$, the individual equilibrium outcome is uniquely defined and given by

$$y_i^* = \theta_i + b_i(\mathbf{G}, \phi)$$

The aggregate equilibrium effort being an increasing function of the aggregate Bonacich centrality, the public institution, interested in fostering effort in education, wants to maximize the sum of Bonacich centralities. Note also that incentives to contract are not affected by the idiosyncratic part of utilities. In total, the program of the institution coincides with the principal's program of our model.

We obtain limited targeting on the 16-student network depicted in Calvò-Armengol, Patacchini and Zenou (2009) (this is the network depicted in Figure 2 pp. 1248). On this network, presented in Fig. 6, the maximal intensity of interaction for which centralities are well-defined is slightly above 0.25. The set of nodes with red circles is the optimal group for $t = 10, \delta = 0.19$. Table 2 confirms the emergence of limited targeting in the 16-student network for possible various values of t, δ.

Table 2 shows limited targeting for values of δ around 0.15 and larger. We also observe a non-monotonic relationship between the composition of the optimal group and the budget level. Indeed, agent 8 is selected for $t \in \{7, 8, 10\}$ but not for $t = 9$. Finally, we mention two features which are not shown in the table. First, in this network, for $t = 1$ limited targeting emerges as soon as δ attains approximately 0.13. Second, for $\delta = 0.15$ and larger, the optimal targeting holds for arbitrarily large budget values.

5. Discussion

One main insight of the above analysis is that contracting with the whole society may not be optimal under high intensities of interaction and under low budget level. This is because outsiders discipline contracting agents. This result obtains under key assumptions in the model: we assumed bilateral contracts, we also assumed that the principal committed

[13] The general goal behind conditional cash transfers is a mix of poverty reduction and education. So, maximizing the sum of effort may not be the sole objective of the public institution. The institution may also be interested in enhancing the education effort of the least performing students. Such an objective is not studied here. Our model still holds in cases where all students are considered poor; otherwise restricting the set of eligible students to the poorest ones in the classroom would bring additional constraints into the principal's program.

[14] School attendance is not the sole outcome from these grants. Educational programmes also attended to increase school enrollment, promotions, and ultimately aim at enhancing students' learning in a longer horizon.

[15] Fiszbein et al. (2009) report that, "in terms of budget, the costs range from about 0.50 percent of gross domestic product (gdp) in such countries as Brazil, Ecuador, and Mexico to 0.08 percent of gdp (Chile). The generosity of benefits ranges from 20 percent of mean household consumption in Mexico, to 4 percent in Honduras, and to even less for programs in Bangladesh, Cambodia, and Pakistan" (see also Figure 3.1, pp. 74 in their report).

[16] Parameter θ_i can depend on own characteristics but also on neighbors' characteristics.

[17] There is a slight simplification with respect to Calvò-Armengol, Patacchini and Zenou (2009). In their paper the private return of peer effort depends positively on own degree. To perfectly match with our model we rather assume an homogenous private return that we normalize to unity. Introducing a dependence of private return to degree modification does not change our message qualitatively, i.e. limited targeting emerges for similar parameters.

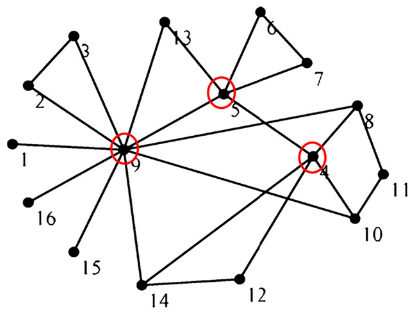

Fig. 6. The 16-student network. For $\delta = 0.19, t = 10$, the optimal group is composed of the students labeled with numbers 4, 5 and 9.

Table 2
Optimal group in the 16-student network as a function of the budget and the intensity of interaction.

$t \setminus \delta$	0.1	0.15	0.19
1	$\mathcal{N}$	{4,6,7,9,11,12}	{4,5,9}
2	$\mathcal{N}$	{4,6,7,9,11,12}	{4,5,9}
3	$\mathcal{N}$	{4,5,6,7,9,11,12}	{4,5,9}
4	$\mathcal{N}$	{4,5,6,7,9,11,12}	{4,5,9}
5	$\mathcal{N}$	{4,5,6,7,9,11,12}	{4,5,9}
6	$\mathcal{N}$	{4,5,6,7,9,11,12}	{4,5,9}
7	$\mathcal{N}$	{4,5,6,7,**8**,9,11,12}	{4,5,9}
8	$\mathcal{N}$	{4,5,6,7,**8**,9,11,12}	{4,5,9}
9	$\mathcal{N}$	{4,5,6,7,9,10,11,12}	{4,5,9}
10	$\mathcal{N}$	{4,5,6,7,**8**,9,10,11,12}	{4,5,9}

to proposed contracts, and last we assumed that an agent who rejects a contract takes into account the reaction of non contracting agents. In this section, we re-examine limited targeting under contingent contracts, when the principal does not commit to her offers, and when the agents who are offered a contract do not take into account the reaction of non contracting agents to own contract rejection.

Contingent contracts. So far, we have restricted attention to simple bilateral contracts. The principal can improve his payoff by proposing contingent contracts. A set of contingent contracts $(x_i, t_i)_{i\in\mathcal{N}}$ is defined here as a collection of individual take-it-or-leave-it offers made to all members of the society simultaneously and such that when one agent rejects the offer, no contract is executed. In this case, each agent i exerts the effort b_i corresponding to the Nash equilibrium played in the absence of contracts and obtains a utility level equal to $\frac{1}{2}b_i^2$.[18] This means that reservation utilities are exogenous to offered contracts. Since reservation utilities are exogenous, it is optimal to contract with all agents (see Appendix C for a characterization of optimal contingent contracts).

No principal's commitment. We examine whether limited targeting can emerge when the principal does not commit to her offers. Limited targeting depends on whether the budget is exogenous or endogenous. Under fixed budget, the budget may play as a commitment device (by the principal), and limited targeting is possible. For instance, in the two agent case, if limited targeting is optimal under commitment, the corresponding optimal contract will be still optimal under no commitment. Under endogenous budget, once an initial subset of agents accept their contracts, the principal always finds it profitable to propose a contract to an outsider with a transfer ϵ and increased effort with respect to the initial play equal to $\sqrt{2\epsilon}$ (this contract will be accepted by the agent – see equation (19)). This results in an increase of the principal's objective for ϵ sufficiently low. Assuming that the outsider does not anticipate a further deviation of the principal, she is better off accepting the offer.

No disciplinary effect. To assess the role of the disciplinary effect, we study optimal contracting in a model without disciplinary effect.[19] We assume that an agent, receiving an offer and contemplating the opportunity of rejecting the offer, considers outsiders' play as fixed[20] (in contrast, the agent takes into account outsiders' reactions to own deviation in the

[18] It is assumed here that the principal cannot threaten agents with a utility level below that obtained in the absence of contracts. Hence, in the principal's program, agent i's participation constraint is given by $\frac{1}{2}b_i^2 \le x_i - \frac{x_i^2}{2} + \delta \sum_{j\in\mathcal{N}} g_{ij}x_ix_j + t_i$.

[19] As for the above model, the principal cannot tax an agent, otherwise the agent would be better off rejecting the offer and playing her best-response effort.

[20] This is equivalent to assume that outsiders play before contracting agents.

model). To simplify notations, we denote by y_j be the effort of outsider j under acceptance of all contracts in $\mathcal{S}$ (see equation (2)). The best-response effort of an agent $i \in \mathcal{S}$ is given by $x_i^{BR} = 1 + \delta \sum_{j \in \mathcal{S}} g_{ij} x_j + \delta \sum_{j \in \mathcal{N} \setminus \mathcal{S}} g_{ij} y_j$. Agent i's individual participation constraint becomes:

$$\frac{1}{2}(x_i^{BR})^2 \le u_i(\mathbf{X}_{\mathcal{S}}, \mathbf{Y}^*_{\mathcal{N} \setminus \mathcal{S}}) + t_i \tag{11}$$

We obtain:

Lemma 1. *Suppose that the principal contracts with a set $\mathcal{S} \subset \mathcal{N}$, so that there is no transfer for agents in the set $\mathcal{N} \setminus \mathcal{S}$. The performance of the optimal contracts is equal to $\sum_{i \in \mathcal{N}} b_i + \sqrt{2t} \sqrt{\sum_{j \in \mathcal{S}} b_j^2}$.*

This simple lemma admits two immediate implications. First, for a fixed number of contracts, the most performing group maximizes the sum of squared Bonacich centralities, meaning that agents with the highest centralities are selected. In particular, on the star network, the central agent belongs to the group of largest performance; on the line network, all members of the best group are positioned at the center of the line; on any regular network, like the circle or the complete network, all groups of same size generate the same performance. Second, including an additional agent in a group strictly increases the principal's objective. This leads to an unambiguous prediction about the possible emergence of limited targeting. Defining $\|\mathbf{B}\|$ as the euclidian norm of vector $\mathbf{B}$, we obtain:

Proposition 6. *When there is no disciplinary effect, the optimal group is the whole society and all transfers are positive. Optimal contracts $\{(\hat{x}_i, \hat{t}_i)\}_{i \in \mathcal{N}}$ are given by equation (8).*

6. Conclusion

This paper considered agents organized in a network of local complementarities, and a principal trading effort through bilateral contracts. We found that the synergies between contracting and non-contracting agents have a strong impact on the relationship between network structure and optimal contracting. The main message of this paper was that such synergies can lead the principal to target a strict subgroup of the society, and possibly to refrain from contracting with central agents.

The issues raised in this paper merit further related investigation. First, it would be interesting to examine deeper how the framework should be applied when it is computationally intractable; i.e., answering about NP-hardness, and exploring further algorithms that can generate a good approximation of the original problem. Second, the model could also potentially fit with other applications where contracting agents interact with outsiders, as suggested in the introduction. Last, in some circumstances the principal could design the network to increase the sum of agents' effort. It would be challenging to study the optimal policy if the principal could use his budget to subsidize the formation or deletion of links as well as effort.

Appendix A. Proofs

This appendix gathers all proofs.

Proof of Proposition 1. We compare the increase of aggregate effort in the two situations.

- Performance of the target $\mathcal{S} = \{1, 2\}$:

  We have $P_{\{1,2\}} = x - b = \sqrt{2t}\|B\|$ with $b_i = \frac{1}{1-\delta}$. We find

  $$P_{\{1,2\}} = \frac{2\sqrt{t}}{1-\delta}$$

- Performance of the target $\mathcal{S} = \{1\}$:

  We have $P_{\{1\}} = 1 + (1+\delta)x - \frac{2}{1-\delta}$, with x solving the participation constraint given by $x - \frac{x^2}{2} + \delta x(1+\delta x) + t - \frac{1}{2}\frac{1}{(1-\delta)^2} = 0$. I.e., for $\delta < \frac{1}{\sqrt{2}}$, $x = \frac{1}{1-2\delta^2}\left[1 + \delta + \frac{1}{1-\delta}\sqrt{\delta^4 + 2(1-2\delta^2)(1-\delta)^2 t}\right]$. We deduce that $P_{\{1\}} = \frac{(1+\delta)}{(1-2\delta^2)(1-\delta)}\left[\delta^2 + \sqrt{\delta^4 + 2(1-2\delta^2)(1-\delta)^2 t}\right]$.

We have $P_{\{1\}} > P_{\{1,2\}}$ if and only if

$$(1+\delta)\left[\delta^2+\sqrt{\delta^4+2(1-2\delta^2)(1-\delta)^2t}\right] > 2(1-2\delta^2)\sqrt{t}$$

We do the squaring, then isolate the square root and then do the squaring again. This gives in total $at^2+bt<0$ with $a=(1-2\delta^2)^2(1-2\delta^2-\delta^4)^2$ and $b=-4\delta^4(1+\delta)^2(1-2\delta^2)^2$. Then $P_{\{1\}} > P_{\{1,2\}}$ when $t<t_c(\delta)$ such that

$$t_c(\delta)=\frac{2\delta^2(1+\delta)}{1-2\delta^2-\delta^4}$$

This is an increasing and convex function, which tends to infinity when δ tends to $\kappa=\sqrt{\sqrt{2}-1}$. When $\delta\in[\kappa,\frac{1}{\sqrt{2}}[$, we have $a\leq 0, b<0$ thus it is always true that $at+b<0$. □

Proof of Proposition 2. Assume $t>0$. We show that, when δ is sufficiently low, the optimal group is the whole society. Assume $\delta=0$. Suppose that the principal contracts with a set $\mathcal{S}$ of agents of cardinality s. Then the principal proposes each an homogeneous transfer $\frac{t}{s}$. From binding participation constraints, we get $x_i=1+\sqrt{\frac{2t}{s}}$ for all $i\in\mathcal{S}$, and the optimal aggregate effort is equal to $n+\sqrt{2st}$. This quantity being increasing in s, the principal finds profitable to contract with the whole society when $\delta=0$ and $t>0$. By continuity on parameter δ, the result follows for low enough intensities of interaction.

We show that when δ is high enough, the optimal group is a strict subgroup of the society. It is sufficient to show that targeting a unique agent is better than targeting the whole society. Indeed, by selecting a unique agent i, it is easily shown that the principal induces an increase in aggregate effort equal to

$$\frac{b_i}{m_{ii}}\cdot\frac{\sqrt{(m_{ii}-1)^2b_i^2+2m_{ii}(2-m_{ii})\,t}+(m_{ii}-1)b_i}{2-m_{ii}} \tag{12}$$

Then observe that effort goes to infinity as soon as δ tends to δ_c (which is the intensity of interaction such that $m_{ii}=2$). At this value δ_c, the increase in aggregate effort, when the principal contracts with the whole society, is finite and equal to $\sqrt{2t}\|\mathbf{B}\|$: indeed, we characterize the optimal contracts as follows. Transfers are given by applying equation (21) to the case $\mathcal{S}=\mathcal{N}$, so we get $t_i=\frac{t}{\|\mathbf{B}\|^2}b_i^2$. Remembering that $x_i=x_i^{BR}+\sqrt{2t_i}$, that $\mathbf{X}^{BR}=\mathbf{1}+\delta\mathbf{GX}$ and plugging the value of the transfer into the above expression, we obtain $\mathbf{X}=\mathbf{B}+\frac{\sqrt{2t}}{\|\mathbf{B}\|}\mathbf{B_B}$ (where by convention $\mathbf{B_B}=\mathbf{MB}$) and $x=b+\sqrt{2t}\ \|\mathbf{B}\|$. To finish, we note that the objective of the planner, $\hat{x}$, is increasing in the budget, confirming that all constraints are binding. □

Proof of Proposition 3. Fix $\delta>0$. With a null budget, the principal cannot modify effort when dealing with the whole society, as shown in equation (9). In contrast, with $t=0$, dealing with a single agent i allows the principal to strictly increase aggregate effort, as shown in equation (12) by setting $t=0$. □

Proof of Proposition 4. The performance of group $\mathcal{S}$ is given by

$$F(\mathcal{S})=\max\sum_{k\in\mathcal{S}}x_k+\sum_{j\in\mathcal{N}\setminus\mathcal{S}}y_j^*$$

Note first that a zero budget induces no transfer at all, otherwise there would be a negative transfer to an agent, but this agent would find it profitable to play her best-response and receive no transfer. Second, absent any transfer, agent i's participation constraint is given by

$$x_i^2-2\Big(1+\delta\sum_{k\in\mathcal{S}}g_{ik}x_k+\delta\sum_{j\in\mathcal{N}\setminus\mathcal{S}}g_{ij}y_j^*\Big)x_i+z_i^2=0 \tag{13}$$

where $Y^*=\bar{M}(\mathbf{1}_{n-s}+\delta G_{\mathcal{N}\setminus\mathcal{S},\mathcal{S}}X_{\mathcal{S}})$ is the vector of best-responses of agents in $\mathcal{N}\setminus\mathcal{S},\mathcal{S}$ given contracts established with agents in $\mathcal{S}$; for convenience, we denote $\bar{M}=(I_{n-s}-\delta G_{\mathcal{N}\setminus\mathcal{S},\mathcal{S}})^{-1}$, and we write $\bar{M}=(\bar{m}_{jp})$ for $j,p\in\mathcal{N}\setminus\mathcal{S},\mathcal{S}$. With such notation, we have

$$y_j^*=\bar{b}_j+\delta x_i\sum_{p\in\mathcal{N}\setminus\mathcal{S}}\bar{m}_{jp}g_{pi}+\delta\sum_{k\in\mathcal{S}\setminus\{i\}}x_k\sum_{p\in\mathcal{N}\setminus\mathcal{S}}\bar{m}_{jp}g_{pk} \tag{14}$$

Plugging equation (14) into equation (13) and rearranging, x_i is the maximal root of the equation $Ax_i^2-2Bx_i+z_i^2=0$ with

$$\begin{cases} A = 1 - 2\delta^2 \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij} \sum_{p\in\mathcal{N}\setminus\mathcal{S}} \bar{m}_{jp} g_{pi} \\ B = 1 + \delta \sum_{k\in\mathcal{S}} g_{ik} x_k + \delta \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij} \bar{b}_j + \delta^2 \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij} \sum_{k\in\mathcal{S}\setminus\{i\}} x_k \sum_{p\in\mathcal{N}\setminus\mathcal{S}} \bar{m}_{jp} g_{pk} \end{cases}$$

so that $x_i = \frac{B+\sqrt{B^2-Az_i^2}}{A}$.

We consider a Taylor development at order 1 in δ. Let us define $x_k = 1 + b_k\delta$ as δ tends to zero for all $k \in \mathcal{S}$, where b_k are given by participation constraints. We need to develop quantities A, B, z_i^2 at order 2 to deal with the square root. For all agents $l \in \mathcal{N}$, let d_l^S be the number of neighbors of agent l in S and $d_l^{\mathcal{N}\setminus\mathcal{S}}$ be the number of neighbors of agent l in $\mathcal{N}\setminus\mathcal{S}$; for all agent $i \in \mathcal{S}$, let $d^{(2)}_{i,\mathcal{N}\setminus\mathcal{S}} = \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij} d_j^{\mathcal{N}\setminus\mathcal{S}}$ be the number of paths of length 2 starting from agent i in $\mathcal{N}\setminus\mathcal{S}$ where intermediaries are in $\mathcal{N}\setminus\mathcal{S}$. Then, taking into account that $\bar{m}_{jj} = 1 + o(1)$ while $\bar{m}_{jp} = o(1)$ for all $p \in \mathcal{N}\setminus\mathcal{S}\setminus\{j\}$,

$$A = 1 - 2\delta^2 d_i^{\mathcal{N}\setminus\mathcal{S}} + o(\delta^2) \tag{15}$$

and recalling that $\sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij}\bar{b}_j = d_i^{\mathcal{N}\setminus\mathcal{S}} + \delta \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij} d_j^{\mathcal{N}\setminus\mathcal{S}} + o(\delta)$,

$$B = 1 + \delta d_i + \delta^2 \underbrace{\left[\sum_{k\in\mathcal{S}} g_{ik} b_k + \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij} d_j - d_i^{\mathcal{N}\setminus\mathcal{S}}\right]}_{=K_B} + o(\delta^2) \tag{16}$$

The best-response z_i of i, when he deviates, is given by

$$z_i = 1 + \delta d_i + \delta^2 \left[d^{(2)}_{i,\mathcal{N}\setminus\mathcal{S}} + \sum_{k\in\mathcal{S}} g_{ik} b_k + \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij} d_j^S - d_i^{\mathcal{N}\setminus\mathcal{S}}\right] + o(\delta^2)$$

Therefore,

$$z_i^2 = 1 + 2\delta d_i + \delta^2 d_i^2 + 2\delta^2 \underbrace{\left[d^{(2)}_{i,\mathcal{N}\setminus\mathcal{S}} + \sum_{k\in\mathcal{S}} g_{ik} b_k + \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij} d_j^S - d_i^{\mathcal{N}\setminus\mathcal{S}}\right]}_{=K_Z} + o(\delta^2) \tag{17}$$

We deduce x_i. Plugging equations (15), (16) and (17) into the greatest root of the participation constraint, we get $x_i = 1 + \delta d_i + \sqrt{\Delta}$, where

$$\Delta = 1 + 2\delta d_i + \delta^2 d_i^2 + 2\delta^2 K_B - (1 - 2\delta^2 d_i^{\mathcal{N}\setminus\mathcal{S}})(1 + 2\delta d_i + \delta^2 d_i^2 + 2\delta^2 K_Z) + o(\delta^2)$$

that is, $\Delta = 2\delta^2\left(K_B - K_Z + d_i^{\mathcal{N}\setminus\mathcal{S}}\right) + o(\delta^2)$. Replacing K_B, K_Z and simplifying, we get $K_B = K_Z$, and we deduce $\Delta = 2\delta^2 d_i^{\mathcal{N}\setminus\mathcal{S}} + o(\delta^2)$. In the end, we find

$$x_i = 1 + \delta d_i + \delta\sqrt{2 d_i^{\mathcal{N}\setminus\mathcal{S}}} + o(\delta) \tag{18}$$

We turn to the principal's objective. Noticing that $y_j^* = 1 + \delta d_j + o(\delta)$, and taking x_i as in equation (18) for all $i \in \mathcal{S}$, we obtain:

$$F(S) = n + 2\delta L + \delta\sqrt{2} \sum_{i\in\mathcal{S}} \sqrt{d_i^{\mathcal{N}\setminus\mathcal{S}}} + o(\delta) \quad \square$$

Proof of Proposition 5. Consider any set S selected by the principal. By symmetry of the problem, the pairs effort – transfer are identical for each agent in S. We let x denote the optimal effort of any agent in S. As well, we let y be the representative effort of any non-contracting agent in $\mathcal{N}\setminus\mathcal{S}$. Since non-contracting agents play a best-response, taking as given the effort of contracting agents, we get $y = \frac{1+\delta sx}{1-\delta(n-s-1)}$. Hence, the principal maximizes $P(s, x) = sx + (n-s)\frac{1+\delta sx}{1-\delta(n-s-1)} = \frac{(1+\delta)s\ x(s)+n-s}{1-\delta(n-s-1)}$ over individual effort $x \geq 0$ under the participation constraint:

$$x - \frac{1}{2}x^2 + \delta x(s-1)x + \delta x(n-s)\frac{1+\delta sx}{1-\delta(n-s-1)} + \frac{t}{s} = \frac{1}{2}\left(\frac{1+\delta(s-1)x}{1-\delta(n-s)}\right)^2$$

The principal has to determine the optimal group size s^*. To solve this problem, the principal has to find the maximal nonnegative number $x(s)$ solving the participation constraint, and then determine the number s^* maximizing $P(s, x(s))$. The optimal effort $x(s)$ solves a second-order polynomial equation, and it is expressed as $x(s) = \frac{-B-\sqrt{B^2-4AC}}{2A}$, where

$$\begin{cases} A = -\frac{1}{2} + \delta(s-1) + \delta^2 \frac{s(n-s)}{1-\delta(n-s-1)} - \frac{\delta^2}{2}\frac{(s-1)^2}{(1-\delta(n-s))^2} \\ B = 1 + \delta\frac{n-s}{1-\delta(n-s-1)} - \delta\frac{s-1}{(1-\delta(n-s))^2} \\ C = \frac{t}{s} - \frac{1}{2(1-\delta(n-s))^2} \end{cases}$$

The solution is well-defined when δ is sufficiently low so that the inequality $A < 0$ holds (which ensures convexity of the participation constraint). □

Proof of Lemma 1. Suppose that the principal contracts with all agents in the set $S = \{1, \cdots, s\}$ (labeling is without loss of generality). Define the n-dimensional vector $\mathbf{\Phi} = (\phi_i)_{i\in\mathcal{N}}$ such that $\phi_i = \sqrt{2t_i}$ for all $i \in \mathcal{S}$ and $\phi_i = 0$ otherwise. Then, we observe that $u_k(x_k, x_{-k}) = x_k x_k^{BR} - \frac{1}{2}x_k^2$, i.e. $\frac{1}{2}\big(1 + \delta\sum_j g_{kj}x_j\big)^2 - u_k(x_k, x_{-k}) = \frac{1}{2}\big(x_k - x_k^{BR}\big)^2$. The participation constraint of an agent $i \in \mathcal{S}$ can then be written as

$$x_i - x_i^{BR} = \phi_i \tag{19}$$

Also, the play of an outsider $j \in \mathcal{N}\setminus\mathcal{S}$ basically satisfies $y_j - y^{BR}_{\mathcal{N}\setminus\mathcal{S},j} = \phi_j (= 0)$. For convenience define the n-dimensional vector $\mathbf{V} = (v_i)_{i\in\mathcal{N}}$ where $v_i = x_i$ for $i \in \mathcal{S}$ and $v_j = y_j$ for $j \in \mathcal{N}\setminus\mathcal{S}$. The whole system of the n equations is then written as $\mathbf{V} = (\mathbf{I} - \delta\mathbf{G})^{-1}(\mathbf{1} + \mathbf{\Phi})$, from which we deduce by summation over all entries the aggregate effort in the society:

$$\sum_{i\in\mathcal{S}} x_i + \sum_{j\in\mathcal{N}\setminus\mathcal{S}} y_j = \sum_{k\in\mathcal{N}} b_k + \sum_{i\in\mathcal{S}} b_i\sqrt{2t_i} \tag{20}$$

Since $\sum_{k\in\mathcal{N}} b_k$ is independent of the set $\mathcal{S}$, the optimal contracts in the set $\mathcal{S}$ maximize the quantity $\sum_{i\in\mathcal{S}} b_i\sqrt{2t_i}$ under the budget constraint $\sum_{i\in\mathcal{S}} t_i = t$. Basic optimization through Lagrangian method yields the optimal transfer

$$t_i = \frac{t}{\sqrt{\sum_{j\in\mathcal{S}} b_j^2}} b_i^2 \tag{21}$$

Plugging all transfers in equation (20), aggregate effort reaches the value $\sum_{k\in\mathcal{N}} b_k + \sqrt{2t}\sqrt{\sum_{j\in\mathcal{S}} b_j^2}$. □

Proof of Proposition 6. By Lemma 1, enlarging a set $\mathcal{S}$ to $\mathcal{S}' = \mathcal{S} \cup \{j\}$ for any agent $j \in \mathcal{N}\setminus\mathcal{S}$ induces a strict increase in the principal's objective. This implies that the principal finds it optimal to contract with the whole society. □

Appendix B. Optimal group selection

In this appendix, we show how to find the performance of any contracting group S for a general intensity of interaction on any network. Shortly speaking, the solutions of the linear system are written as a function of the Lagrangian multiplier. Plugging these values in the budget constraint, the Lagrangian multiplier associated with the optimal contract is clearing the budget constraint.

We show here how the principal contracts with a group S of agents for a fixed budget t as defined in program (4) as exposed in the paper. We consider a group S selected by the principal. The following analysis assumes that the budget is fully spent ($\sum_i t_i = t$) and that all the participation constraints of program (4) are binding – this is checked ex post. Hence all constraints have the same weight say $\lambda > 0$ and can be aggregated in the Lagrangian.

We introduce some useful notations. We call by $\mathbf{Y} = \mathbf{Y}^*_{\mathcal{N}\setminus\mathcal{S}}$ the Nash effort profile of agents in the set $\mathcal{N}\setminus\mathcal{S}$, $\mathbf{Z}^{(i)} = \mathbf{Y}^*_{\mathcal{N}\setminus\mathcal{S}\cup\{i\}}$ the Nash profile of agents in the set $\mathcal{N}\setminus\mathcal{S}\cup\{i\}$. Here $z_i^{(i)}$ represents the entry corresponding to agent i in vector $\mathbf{Z}^{(i)}$, that is, the Nash play of agent i given $X_{S\setminus\{i\}}$ and given the Nash play of agents in $\mathcal{N}\setminus\mathcal{S}$; so we build the s-dimensional vector $\mathbf{Z} = (z_i)_{i\in\mathcal{S}}$ where $z_i = z_i^{(i)}$. Both profiles depend on the set of contracts $\mathbf{X}_S$, we omit the reference to contracts $\mathbf{X}_S$ in our notation for convenience. Remember that $\mathbf{G}_{\mathcal{N}\setminus\mathcal{S}}$ is the $(n-s)\times(n-s)$ sub-matrix of matrix $\mathbf{G}$ representing the bilateral influences between pairs of agents in $\mathcal{N}\setminus\mathcal{S}$, and $\mathbf{G}_{\mathcal{N}\setminus\mathcal{S},S}$ the $(n-s)\times s$ sub-matrix of matrix $\mathbf{G}$ representing the bilateral influences between agents in S and agents in $\mathcal{N}\setminus\mathcal{S}$. We have:

$$\mathbf{Y} = \mathbf{Y}^0 + \mathbf{A}\mathbf{X}_S \tag{22}$$

where

$$\begin{cases} \mathbf{Y}^0 = (\mathbf{I}_{\mathcal{N}\setminus\mathcal{S}} - \delta\mathbf{G}_{\mathcal{N}\setminus\mathcal{S}})^{-1}\mathbf{1}_{n-s} \\ \mathbf{A} = \delta(\mathbf{I}_{\mathcal{N}\setminus\mathcal{S}} - \delta\mathbf{G}_{\mathcal{N}\setminus\mathcal{S}})^{-1}\mathbf{G}_{\mathcal{N}\setminus\mathcal{S},S} \end{cases}$$

(matrix $\mathbf{A}$ is of dimension $\mathcal{N} \setminus \mathcal{S} \times S$). Also, the s-dimensional vector $\mathbf{Z}$ can be written as

$$\mathbf{Z} = \mathbf{Z}^0 + \mathbf{B}\mathbf{X}_S \tag{23}$$

where entry i of vector $\mathbf{Z}^0$ satisfies

$$z_i^0 = \left[(\mathbf{I}_{\mathcal{N}\setminus\mathcal{S}\cup\{i\}} - \delta \mathbf{G}_{\mathcal{N}\setminus\mathcal{S}\cup\{i\}})^{-1} \mathbf{1}_{n-s+1} \right]_i$$

and the $S \times S$ matrix $\mathbf{B} = (b_{ij})$ is built as follows:

(1) We have $b_{ii} = 0$ for all $i \in \mathcal{S}$; indeed, under offer rejection, agent i is only influenced by agents in $S \setminus \{i\}$.
(2) For off-diagonal entries (i, j) where $i \in \mathcal{S}$ and $j \in \mathcal{S} \setminus \{i\}$:

$$b_{ij} = \left[\delta (\mathbf{I}_{\mathcal{N}\setminus\mathcal{S}\cup\{i\}} - \delta \mathbf{G}_{\mathcal{N}\setminus\mathcal{S}\cup\{i\}})^{-1} \mathbf{G}_{\mathcal{N}\setminus\mathcal{S}\cup\{i\}, S\setminus\{i\}} \right]_{ij}$$

In short, the ith row of matrix $\mathbf{B}$ is given by the invert linear system describing the Nash equilibrium of agents in $\mathcal{N} \setminus \mathcal{S} \cup \{i\}$ given the profile of effort $X_{S\setminus\{i\}}$, and z_i^0 is the ith entry of the constant of this system. So, it is important to observe that each row of matrix $\mathbf{B}$ corresponds to a distinct Nash equilibrium. The Lagrangian is written:

$$L = \sum_{i\in\mathcal{S}} x_i + \sum_{j\in\mathcal{N}\setminus\mathcal{S}} y_j + \lambda \left[t + \sum_{i\in\mathcal{S}} \left(u_i(\mathbf{X}_S, \mathbf{Y}) - \frac{1}{2}(z_i)^2 \right) \right]$$

The derivative of the Lagrangian with respect to x_i entails:

$$1 + \sum_{j\in\mathcal{N}\setminus\mathcal{S}} \frac{\partial y_j}{\partial x_i} + \lambda \left[1 - x_i + \delta \sum_{k\in\mathcal{S}} g_{ik} x_k + \delta \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij} y_j + \delta x_i \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij} \frac{\partial y_j}{\partial x_i} \right.$$
$$\left. + \sum_{k\in\mathcal{S}\setminus\{i\}} \left(\delta g_{ik} x_k + \delta x_k \sum_{l\in\mathcal{N}\setminus\mathcal{S}} g_{kl} \frac{\partial y_l}{\partial x_i} - z_k \frac{\partial z_k}{\partial x_i} \right) \right] = 0$$

Taking care of the notations given by equations (22) and (23), we obtain the following system of linear interaction:

$$\mathbf{X}_S = \mathbf{H}^{-1} \mathbf{Q}(\lambda) \tag{24}$$

where, for all $i \in \mathcal{S}$,

$$q_i(\lambda) = 1 + \frac{1 + \sum_{j\in\mathcal{N}\setminus\mathcal{S}} a_{ji}}{\lambda} + \delta \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij}\, y_j^0 - \sum_{k\in\mathcal{S}\setminus\{i\}} b_{ki}\, z_k^0$$

$$h_{ii} = 1 - 2\delta \sum_{j\in\mathcal{N}\setminus\mathcal{S}} g_{ij}\, a_{ji} + \sum_{k\in\mathcal{S}\setminus\{i\}} b_{ki}^2$$

and for all pairs $i \in \mathcal{S}, k \in \mathcal{S} \setminus \{i\}$,

$$h_{ik} = -2\delta g_{ik} - \delta \sum_{j\in\mathcal{N}\setminus\mathcal{S}} \left(g_{kj} a_{ji} + g_{ij} a_{jk} \right) + \sum_{l\in\mathcal{S}\setminus\{i\}} b_{li} b_{lk}$$

The solutions $\mathbf{X}_S$ of the linear system given by equation (24) are written as a function of the weight λ. Plugging this value in the budget constraint, the weight λ associated with the optimal contract clears the budget constraint (this is the unique positive root of an order-2 polynomial expression).

Appendix C. Contingent contracts

In this appendix, we explore contingent contracts in more details. Contingent contracting allows the principal to extract the full surplus from agents. A set of contingent contracts $(x_i, t_i)_{i\in\mathcal{N}}$ is defined here as a collection of individual take-it-or-leave-it offers made to all members of the society simultaneously. Hence, when one agent rejects the offer, no contract is executed; each agent i exerts the effort corresponding to the Nash equilibrium played in the absence of contracts, i.e. b_i, and obtains a utility level equal to $\frac{1}{2} b_i^2$.[21] This means that reservation utilities are exogenous to offered contracts. Since

[21] It is assumed here that the principal cannot threaten agents with a utility level below that obtained in the absence of contracts.

reservation utilities are exogenous, optimal group targeting is not an issue here, i.e. the principal always finds it optimal to propose contracts to every agent in the society. Agent i's participation constraint is given by:

$$\frac{1}{2}b_i^2 \leq x_i - \frac{x_i^2}{2} + \delta \sum_{j\in\mathcal{N}} g_{ij}x_i x_j + t_i$$

We characterize the optimal contingent contracts. We set $X = X_N$ for convenience, and take care that both the budget constraint and participation constraints are binding at optimum. For convenience, we write $\mathbf{B}' = \mathbf{B}(\mathbf{G}, 2\delta)$ and $b' = b(\mathbf{G}, 2\delta)$. We define $\kappa(t) = \sqrt{1 + \frac{2t - \|\mathbf{B}\|^2}{b'}}$.[22] We let $\{(\tilde{x}_i, \tilde{t}_i)\}_{i\in\mathcal{N}}$ be the set of optimal contingent contracts. We obtain:

Proposition A. *The optimal contingent contract is written for all $i \in \mathcal{N}$:*

$$\begin{cases} \tilde{x}_i = \big(1 + \kappa(t)\big)b_i' \\ \tilde{t}_i = \frac{1}{2}\Big[b_i^2 + (\kappa(t)^2 - 1)b_i'\Big] \end{cases}$$

Proof of Proposition A. We first suppose that both participation constraints and the budget constraint are binding at optimum, and second, we check that these constraints are binding.

The reservation utility of every agent k is exogenous to contracts and equal to $\frac{b_k^2}{2}$, and the sum of all reservation utilities is thus equal to $\frac{\|\mathbf{B}\|^2}{2}$. The derivative of the Lagrangian with respect to x_i entails $x_i - 2\delta \sum_{j\in\mathcal{N}} g_{ij}x_j = 1 + \frac{1}{\lambda}$. Recalling that $b_i' = b_i(\mathbf{G}, 2\delta)$ (this centrality are well-defined as $2\delta < \omega(\mathbf{G})$), we get $x_i = \left(1 + \frac{1}{\lambda}\right)b_i'$. Agent i's binding participation constraint is written $t_i = \frac{b_i^2}{2} + \frac{1}{2}\left(x_i - 2\delta \sum_{j\in\mathcal{N}} g_{ij}x_j - 2\right)x_i$. Plugging effort in this latter equation, we get $t_i = \frac{b_i^2}{2} + \frac{1}{2}\left(\frac{1}{\lambda} - 1\right)\left(1 + \frac{1}{\lambda}\right)b_i'$. Summing transfers over all agents and remembering that the budget constraint is binding, we obtain $2t = \|\mathbf{B}\|^2 + \sum_i \left(\frac{1}{\lambda} - 1\right)\left(1 + \frac{1}{\lambda}\right)b_i'$. Rearranging, we obtain $\lambda = \sqrt{\frac{b'}{2t + b' - \|\mathbf{B}\|^2}}$, and we are done.

To finish, we note that the objective of the planner, $\tilde{x}$, is increasing in the budget. This implies that all constraints are binding. $\square$

Optimal contingent effort is always well-defined. Effort increases with Bonacich centrality of decay parameter equal to 2δ; that is, it takes into account both received and generated externalities. Agent i's optimal effort can be decomposed into a zero-budget component and a pure budget component:

$$\tilde{x}_i = \underbrace{\left(1 + \sqrt{1 - \frac{\|\mathbf{B}\|^2}{b'}}\right) b_i'}_{\text{zero-budget component}} + \underbrace{\left(\kappa(t) - \sqrt{1 - \frac{\|\mathbf{B}\|^2}{b'}}\right) b_i'}_{\text{pure budget component}}$$

With null budget, the optimal effort $\tilde{x}_i$ is still enhanced and proportional to the Bonacich centrality b_i'. This is in sharp contrast with bilateral contracts, where the principal cannot increase effort when $t = 0$.

The relationship between transfer and centrality, or effort, is not monotonic, and budget matters. For sufficiently large budget, i.e. for $t > \frac{\|\mathbf{B}\|^2}{2}$, transfer increases with centralities b_i and b_i'. However, for $t < \frac{\|\mathbf{B}\|^2}{2}$, i.e. when the budget does not cover the sum of utilities in the absence of contracts, the relationship between transfer and centralities is ambiguous: it increases with b_i^2 and decreases with b_i'. Furthermore, the principal can even tax the agents with the smallest indexes $\frac{b_i^2}{b_i'}$. Precisely an agent is taxed whenever $\frac{b_i^2}{b_i'} < \frac{b'}{\|\mathbf{B}\|^2 - 2t}$. In general, the emergence of taxation depends on the network structure. Taxation is more likely to occur under high dispersion in Bonacich centralities. Moreover, as budget t tends to zero, there is always a taxed agent on the network, for all positive intensities of interaction (except on a regular network where all centralities are identical).

Finally, the performance of the aggregate optimal effort is measured by $\tilde{x} = b' + \sqrt{b'(2t + b' - \|\mathbf{B}\|^2)}$. It is easily shown that the aggregate optimal effort $\tilde{x}$, as well as the gap between contingent and bilateral contract, $\tilde{x} - \hat{x}$, increase with link addition, with intensity of interaction δ, and with budget level.

[22] The member under the square root is positive. Indeed, the positiveness is equivalent to budget t being larger than the difference between aggregate initial equilibrium utilities and aggregate utilities of the efficient allocation in the absence of contracts. This latter difference is negative and the budget is nonnegative.

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