

# PATH TRACKING FOR THE CONVOY OF AUTONOMOUS VEHICLES BASED ON A NON-LINEAR PREDICTIVE CONTROL

M-M Mohamed-Ahmed, Aziz Naamane, Nacer M&apos;sirdi

► **To cite this version:**

M-M Mohamed-Ahmed, Aziz Naamane, Nacer M&apos;sirdi. PATH TRACKING FOR THE CONVOY OF AUTONOMOUS VEHICLES BASED ON A NON-LINEAR PREDICTIVE CONTROL. THE 12TH INTERNATIONAL CONFERENCE ON INTEGRATED MODELING AND ANALYSIS IN APPLIED CONTROL AND AUTOMATION, Sep 2019, Lisbonne, Portugal. hal-02479106

**HAL Id: hal-02479106**

**<https://hal-amu.archives-ouvertes.fr/hal-02479106>**

Submitted on 14 Feb 2020

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# PATH TRACKING FOR THE CONVOY OF AUTONOMOUS VEHICLES BASED ON A NON-LINEAR PREDICTIVE CONTROL

M. Mahmoud Mohamed Ahmed<sup>1</sup>, Aziz Naamane<sup>1</sup>, Nacer K. M'Sirdi<sup>1</sup>

<sup>1</sup>Aix Marseille University, Université de Toulon, CNRS, LIS UMR 7020, SASV, Marseille, France

<sup>1</sup>[mohamed-mahmoud.mohamed-ahmed@lis-lab.fr](mailto:mohamed-mahmoud.mohamed-ahmed@lis-lab.fr), [aziz.naamane@lis-lab.fr](mailto:aziz.naamane@lis-lab.fr), [nacer.msirdi@lis-lab.fr](mailto:nacer.msirdi@lis-lab.fr)

## ABSTRACT

In this paper, a nonlinear predictive control of a platoon of several vehicles is proposed by using non-linear robotic form model of the vehicles. The model used represents the longitudinal, lateral and yaw movement for each vehicle in the fleet. This control approach allows controlling the fleet, uses the available information, ensures a safe distance between vehicles to avoid collisions and follows the path of the leader. The robustness of the control will be studied in order to assess the different errors occurring in the estimated parameters values.

## 1. INTRODUCTION

The explosion in the number of vehicles put into circulation each year in the world poses problems for road infrastructures today. There is also air pollution and the safety of people. Today, peri-urban networks are affected by recurrent congestion phenomena, due to the increasing number of urban-urban journeys. How to increase the capacity of the infrastructures, while improving the safety and the comfort of the motorists? Solutions can then be considered: better use of available space by automating vehicles at low speeds or streamline all travel. The first strategy led to the behavioral study of inter-vehicular distances [Ali et al. \[2015\]](#), [Nouveliere et al. \[2002\]](#). In the field of road transport, the constraints related to the safety and the capacity of the traffic lanes make the knowledge of inter-vehicle distances and possibly their control necessary.

The vehicle fleet is a very efficient means of transportation for passengers, merchandise and increased traffic capacity. For example, a convoy of trucks carries goods, with a single driver [Ali \[2015\]](#). Other benefits such as reducing fuel consumption and minimizing manpower. The convoy is composed of a vehicle in the head and other cars are followers. The leader vehicle can be autonomous or driven by a driver, the other vehicles follow the leader with a safety distance to avoid collisions between vehicles. Two spacing approaches for the safety distance between vehicles have been proposed in the literature; an established distance and a distance proportionally with the speed [Swaroop \[1994\]](#) . [Nouveliere et al. \[2002\]](#). For a longitudinal displacement, the dis-

tances are constant. For overall control of longitudinal and lateral movements, the distance between vehicles can be proportional to speed and depends on the reference path for lateral deviation.

Several control approaches have been proposed in the literature for vehicle fleets, in [Ali et al. \[2015\]](#) a linear dual integrator dynamic model is used after an exact linearization for a vehicle convoy. The longitudinal movement is controlled with a linear control to ensure a safe distance between the vehicles. The lateral movement is controlled by acting on the vehicle orientation angle with respect to the desired trajectory. Longitudinal and lateral control are independent. Another control approach has been proposed in [Xiang and Bräunl \[2010\]](#) which represents a distribution algorithm based on the relative error of the previous vehicle, position, the vehicle model used for this approach is the kinematic one.

In order to obtain precise data, sensors are placed on board of each vehicle of the fleet, for this mission. The local strategy is based on data or information that are shared between close neighbors. In the literature, most Leader-Follower control approaches belong to this category [Avanzini et al. \[2010\]](#), [Avanzini \[2010\]](#). The Leader vehicle can move autonomously to follow a desired path. It serves as a target or reference for the vehicle following it. Each vehicle in the convoy group plays the role of Leader for the vehicle following it (the follower). The driven vehicle is dependent of the data of its predecessor, the control architecture here is unidirectional. The global architecture uses the information of all the vehicles of the convoy as the state of the leader and neighboring vehicles, for example, the control referenced on the previous vehicle and the leader. This control approach is divided into two categories, using either a centralized or decentralized architectures. For the centralized architecture, the control law applied to each vehicle in the fleet is based on the data of all vehicles in the convoy [Yazbeck \[2014\]](#). On the other hand, the decentralized architecture is based on the data of a part of the convoy, to minimize the numbers of the sensors used. A review on modelling and control strategies has been presented in [M'Sirdi \[2018\]](#).

Several convoy project are realized in the literature that

are based on these approaches of control, we can mention the AutoNet2030 project for a self-driving vehicle cooperation system and a manual drive based on the decentralized approach to make their decision. The control laws are based on the information of the neighbors. And the SARTE project funded by the European Union in 2012 with the aim of driving a convoy of vehicles with high speed on a motorway without modifying the infrastructure. The control law applied on each vehicle of the convoy is based on the decentralized global approach, such that the leader information and the neighbors used to build this control Avanzini [2010]. Another important project is the one called Chauffeur, which deals with the conveying of trucks. The leading vehicle was controlled manually by a driver and the other vehicles (trucks) automatically follow the truck ahead.

In this work, we propose a coupled longitudinal and lateral control of a fleet of autonomous vehicles using non-linear predictive control. The model used for this control approach represents the non-linear robotic form model of a vehicle. The model represents the longitudinal and lateral movement of the fleet and the movement of the yaw. The longitudinal movement of the fleet is controlled by the driving/braking wheels torque and the lateral movement is controlled by the steering angle. In this control approach, the model of the fleet is not linearized. The kinematic model will be considered for moving the fleet in the reference frame. The lateral control of the fleet is coupled with the longitudinal movement according to the speed, as the lateral movement is controlled by imposing a lateral acceleration and this desired acceleration is calculated according to the longitudinal speed and the reference trajectory for each vehicle of the convoy. The overall control of the convoy makes it possible to follow a desired trajectory for the convoy and to ensure a safe distance between the vehicles of the fleet to avoid collisions.

## 2. MODELING

The dynamic model is considered in this part to control the fleet by the efforts that are applied for each movement of the convoy.

### 2.1. Dynamic model

Several methods of modeling can be found in the literature to determine the model of a vehicle. These different methods lead to sets of equations that represent the dynamic motion of the vehicle DeSantis [1995] Jaballah [2011] Rabhi [2005]. The dynamic model used of a vehicle was determined using the robotic formalism Chebly [2017]. The vehicle is represented in the figure 1 with the following variables in  $(G, x, y)$  the vehicle reference frame. G is the gravity center.

$L_f$  is distance from the front wheel to G .

$L_r$ : is the distance from the rear wheel center to G.

$m, I_z$ : the mass and Inertia Moment of the vehicles.

$m_w, I_w$  :the mass and the rotational inertia of the wheel.

$\dot{x}, v_x$  : longitudinal vehicle velocity along  $x$  axis.

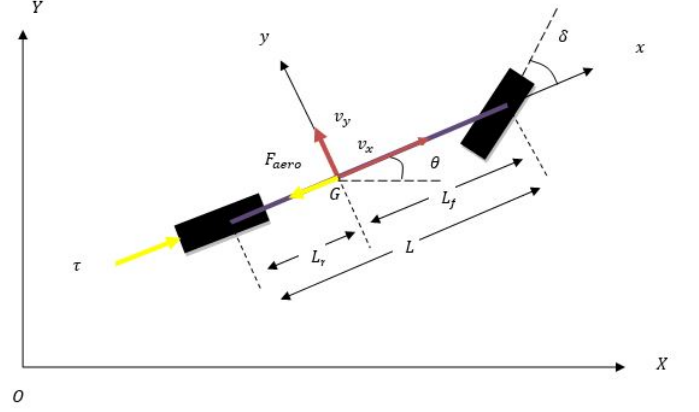


Figure 1: The Vehicle Description

$\dot{y}, v_y$  : lateral velocity (axis  $y$ ).

$\theta$ : yaw angle and  $\dot{\theta}$  : yaw rate.

$a_x = \ddot{x} - \dot{y}\dot{\theta}$  : longitudinal acceleration.

$a_y = \ddot{y} + \dot{x}\dot{\theta}$  : lateral acceleration.

$C_{\alpha f}, C_{\alpha r}$  : are respectively the cornering stiffness of the front and the rear wheels.

$\tau$ : driving/braking wheels torque.

$\delta$  : steering wheel angle.

$F_{aero} = \frac{1}{2}\rho c s \dot{x}^2$ : aerodynamic force, where  $\rho, s$  and  $c$  : are the air density, the vehicle frontal surface and the aerodynamic constant.

$R_t$  :Radius of the tire and  $E$ : Vehicle's track.

We define  $m_e, L_3$  and  $I_3$  as follows :

$$m_e = m + 4 \frac{I_w}{R_t^2}, L_3 = 2m_w(L_r - L_f) \text{ and } I_3 = I_z + m_w E^2.$$

The generalized coordinates  $q \in R^3$  are defined as :  $q_i = [x_i, y_i, \theta_i]^T$ . The dynamic model of a vehicle is presented as follows:

$$M_i(q_i) \cdot \ddot{q}_i + H_i(\dot{q}_i, q_i) = U_i \quad (1)$$

Where the inertia Matrix  $M_i(q_i)$  is:

$$M_i = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_i & -L_{3i} \\ 0 & -L_{3i} & I_{3i} \end{pmatrix}$$

And the vector  $H_i(\dot{q}_i, q_i)$  is equal to

$$H_i(\dot{q}_i, q_i) = \begin{pmatrix} -m_i \dot{q}_2 \dot{q}_3 + L_{3i} \dot{q}_3^2 + \delta_i (2C_{\alpha f i} \delta_i - 2C_{\alpha f i} \frac{\dot{q}_{1i}(\dot{q}_{2i} + L_{fi} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i / 2)^2}) + F_{aero i} \\ m_i \dot{q}_{1i} \dot{q}_3 + 2C_{\alpha f i} \frac{\dot{q}_{1i}(\dot{q}_{2i} + L_{fi} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i / 2)^2} + 2C_{\alpha r i} \frac{\dot{q}_{1i}(\dot{q}_{2i} - L_{ri} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i / 2)^2} \\ 2L_{fi} C_{\alpha f i} \frac{\dot{q}_{1i}(\dot{q}_{2i} + L_{fi} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i / 2)^2} - 2L_{ri} C_{\alpha r i} \frac{\dot{q}_{1i}(\dot{q}_{2i} - L_{ri} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i / 2)^2} - L_{3i} \dot{q}_{1i} \dot{q}_3 \end{pmatrix}$$

And the input vector  $U_i = (u_{1i}, u_{2i}, u_{3i})^T$  :

$$U_i = \begin{pmatrix} \frac{\tau_i}{R_{ti}} \\ (2C_{\alpha f i} - 2 \frac{I_{wi}}{R_{ti}^2} \dot{q}_{1i}) \delta_i \\ L_{fi} u_{2i} - (\frac{E_i}{2} C_{\alpha f i} \frac{E_i \dot{q}_{3i}(\dot{q}_{2i} + L_{fi} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i / 2)^2}) \delta_i \end{pmatrix}$$

The inputs of the system are the control of the torque and the steering wheel angle.

### 2.2. Kinematic Equations

The transformation matrix of the velocity, from the absolute vehicle frame  $(G, x, y)$  to the velocity in the reference

frame  $R(0,X,Y)$  is defined by:

$$\begin{bmatrix} \dot{X}_i \\ \dot{Y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{pmatrix} \cos \theta_i & -\sin \theta_i & 0 \\ \sin \theta_i & \cos \theta_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} \quad (2)$$

Such as we get the kinematics of the  $i^{th}$  vehicle:

$$\begin{cases} \dot{X}_i = \dot{x}_i \cos \theta_i - \dot{y}_i \sin \theta_i \\ \dot{Y}_i = \dot{x}_i \sin \theta_i + \dot{y}_i \cos \theta_i \end{cases} \quad (3)$$

### 2.3. Convoy Motion

The movement of the fleet in a path of reference is presented in the Fig. 2, the convoy moves in this trajectory with a distance that separates every two vehicles. The curvilinear inter-distance error is calculated as a function of the travel distance for each two neighboring vehicles in the curvature of the reference path. Let  $M$  the center of gravity of the vehicle ( $i$ ), the curvilinear error between the vehicle ( $i$ ) and the vehicle ( $i-1$ ) is defined as follows:

$$e_{s_i} = S_{i-1} - S_i - l_{d_i} \quad (4)$$

$S_i$  : represents the curvilinear abscissa of the vehicle ( $i$ ) at the center of gravity, is calculated as follows:

$$\dot{S}_i = \sqrt{\dot{x}_i^2 + \dot{y}_i^2} \quad (5)$$

By replacing equation (5) in equation (4), the curvilinear error will be defined as follows:

$$e_{s_i} = \int_0^t (\dot{x}_{i-1}^2 + \dot{y}_{i-1}^2)^{\frac{1}{2}} dt - \int_0^t (\dot{x}_i^2 + \dot{y}_i^2)^{\frac{1}{2}} dt - l_{d_i} \quad (6)$$

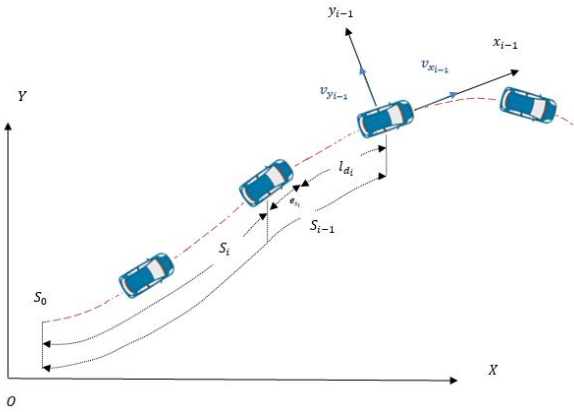


Figure 2: Geometric description of the convoy motion

## 3. CONTROL

### 3.1. Vehicle State Space Model

We have as a state vector, the position and speed of each vehicle:

$$z_i = \begin{pmatrix} z_{1i} \\ z_{2i} \end{pmatrix} \quad (7)$$

$$\dot{z}_{2i} = M^{-1}(z_{1i})(-H(z_{1i}, z_{2i}) + U_i) \quad (8)$$

with positions:  $z_{1i} = [x_i, y_i, \theta_i]^T$

and velocities:  $z_{2i} = [\dot{x}_i, \dot{y}_i, \dot{\theta}_i]^T$

The dynamic model of a vehicle  $i$  of the convoy is represented in canonical forms :

$$\begin{cases} \dot{z}_{1i} = z_{2i} \\ \dot{z}_{2i} = f(z_{1i}, z_{2i}) + g(z_{1i})U_i \end{cases} \quad (9)$$

For our model we have

$$f(z_{1i}, z_{2i}) = -M^{-1}(z_{1i})H(z_{1i}, z_{2i})$$

and  $g(z_{1i}) = M^{-1}(z_{1i})$

### 3.2. The objectives

The aims of the control are to :

- Control the vehicles to follow the trajectories of the leader by ensuring a safe distance between the vehicles to avoid collisions,

- Use the available information to calculate the law of the control and ensures local stability for each vehicle and global one for the fleet,

- Ensure robustness of control over errors in model parameter estimates with the presence of a non-linear model

### 3.3. Longitudinal and lateral control

Tracking accuracy can be improved by using non-linear predictive control based on knowledge of the reference trajectory Hedjar et al. [2005] Merabet and Gu [2008]. This control approach is based on the optimization of the cost function with the objective of controlling the fleet to follow the trajectory of the leader with a safety distance between the vehicles to avoid the collision Song et al. [2017].

$$J_i = \frac{1}{2} \int_0^h e_i(t+T)^T Q_i e_i(t+T) dT + \frac{1}{2} U_i^T R_i U_i \quad (10)$$

With:  $h$  represents the horizon of the prediction,  $T$  is the time of the prediction.  $e_i(t+T)$  : the tracking errors at the next step. For the model defined in the equation (9), we have that  $e_i = (e_{1i}, e_{2i})$  with  $e_i$  represents the position errors,  $e_2$  speed errors and  $Q = \begin{pmatrix} Q_1 & 0 \\ 0 & T^2 Q_2 \end{pmatrix}$  With  $Q_1, Q_2, R$  are weighting matrices.

The aim of the longitudinal control of the fleet is to impose a longitudinal speed on the fleet and to ensure a safety distance between each two neighboring vehicles Fig. 3.

To simplify the writing we define the error of the fleet as defined in the dynamic model as  $e_i = (e_{1i}, e_{2i}, e_{3i})$  with  $e_1$  the longitudinal error of the position,  $e_2$  the error of the lateral position and  $e_3$  the error of the position of the yaw.

We define the curvilinear spacing error between the vehicles of the convoy:

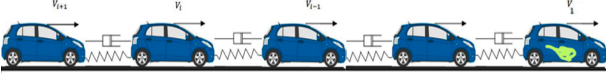


Figure 3: The longitudinal movement of the convoy

$$e_{1i}(t+T) = S_i(t+T) - S_{i-1}(t+T) + l_d(t+T)$$

With  $l_d$  : the safety distance.  $S_i(t+T)$  : represents the curvilinear abscissa of the vehicle (i) at the next step. The error of the longitudinal velocity is defined as follows:

$$\dot{e}_{1i}(t+T) = \dot{x}_i(t+T) - \dot{x}_{i-1}(t+T)$$

The lateral movement of the fleet is controlled by the steering angle of vehicles. The error of the lateral acceleration is defined as follows:

$$\ddot{e}_{2i}(t+T) = a_{yi}(t+T) - a_{y_{di}}(t+T) \quad (11)$$

The reference lateral acceleration is calculated as follows:  $a_{y_{di}} = \dot{x}_i^2/r_i$ . Such as  $r_i$  represents the radius of the leader's trajectory. We have that  $a_{yi} = \ddot{y}_i + \dot{x}_i \dot{\theta}_i$ . replacing the two previous expressions in the equation (11), We can write the lateral error in the following form :

$$\ddot{e}_{2i}(t+T) = \ddot{y}_i(t+T) - \ddot{y}_{di}(t+T)$$

With :  $\ddot{y}_{di} = \dot{x}_i^2/r_i - \dot{x}_i \dot{\theta}_i$ .

The prediction of the tracking error (longitudinal and lateral) can be made using the Taylor approximation and based on the model defined in (9), such as  $e_i$  represents the position error:

$$e_i(t+T) = e_i(t) + T\dot{e}_i + \frac{T^2}{2!}(f(z_i) - \dot{z}_{id}) + \frac{T^2}{2!}g(z_i)U_i$$

$$\dot{e}_i(t+T) = \dot{e}_i + T(f(z_i) - \dot{z}_{id}) + Tg(z_i)U_i$$

The minimization of the cost function is obtained such that:  $\partial J_i / \partial U_i = 0$

$$\begin{aligned} U_i = & -g(z_i)^{-1} \left( \frac{h^5}{20}(Q_{1i} + 4Q_{2i}) \right. \\ & + g(z_{1i})^{-T} R g(z_{1i})^{-1} \left. \right)^{-1} \left( \frac{h^3}{6} Q_{1i} e_i + \dots \right. \\ & \left. + \frac{h^4}{8} (Q_{1i} + 2Q_{2i}) \dot{e}_i + \frac{h^5}{20} (Q_{1i} + 4Q_{2i}) (f(z_{1i}, z_{2i}) - \dot{z}_{i-1}) \right) \end{aligned} \quad (12)$$

$U_i$  controls the longitudinal movement of the fleet by the torque ( $u_{1i}$ ) of each vehicle and the lateral movement by the steering angle ( $u_{2i}$ ). In our case, the longitudinal and lateral control are coupled by the longitudinal velocity. The steering angle is used to calculate the third control ( $u_{3i}$ ) (yaw movement) to calculate the yaw rate and present the movement of the fleet in the reference frame (0,X,Y) by the transformation matrix.

### 3.3.1. Convergence Analysis

The stability study for each vehicle in the convoy is based on the vehicle error and the lateral error with respect to the leader's trajectory. We define the parameters:  $K_{1i} = \frac{h^3}{6} Q_{1i}$ ,  $K_{2i} = \frac{h^4}{8} (Q_{1i} + 2Q_{2i})$  and  $K_{3i} = \frac{h^5}{20} (Q_{1i} + 4Q_{2i})$ . For the stability study according to the errors (longitudinal and lateral), we neglect the weighting on the control. Let the candidate Lyapunov function :

$$V_i = \frac{1}{2} \dot{e}_i^T \dot{e}_i + \frac{1}{2} e_i^T \frac{K_1}{K_3} e_i \quad (13)$$

Deriving this function we find:

$$\dot{V}_i = \dot{e}_i^T \ddot{e}_i + \dot{e}_i^T \frac{K_1}{K_3} e_i \quad (14)$$

Replacing  $\ddot{e}$  (the acceleration error) with its expression ( $\ddot{e}_i = \ddot{z}_i - \ddot{z}_{i-1}$ ) :

$$\dot{V}_i = \dot{e}_i^T (f(z_{1i}, z_{2i}) + g(z_{1i})U_i - \ddot{z}_{i-1}) + \dot{e}_i^T \frac{K_1}{K_3} e_i \quad (15)$$

$$\begin{aligned} = & \dot{e}_i^T (f(z_{1i}, z_{2i}) - \ddot{z}_{i-1} - \frac{K_1}{k_3} e + \dots \\ & - \frac{K_2}{K_3} \dot{e}_i - f(z_{1i}, z_{2i}) + \ddot{z}_{i-1}) + \dot{e}_i^T \frac{K_1}{K_3} e_i \end{aligned} \quad (16)$$

It is clear that the stability condition is verified when the gains of the weighting matrices are positive such that :  $\dot{V}_i = -\dot{e}_i^T \frac{K_2}{K_3} \dot{e}_i < 0$ . The choice of  $K_1, K_2$  and  $K_3$  depends on the weighting matrices and the horizon of the prediction that is around ms. By increasing the gains of the matrices  $Q_1$  and  $Q_2$ , the stability is still checked and ensured.

## 4. SIMULATIONS

To validate this result we used the parameters of a vehicle of the Scanner Studio. 10 vehicles are simulated in Matlab Simulink using both dynamic and kinematic models. The simulation achieved to validate the control law in both directions of longitudinal and lateral motions and to check the stability and accuracy of trajectory tracking. The leader has been controlled using a chosen reference speed and a desired trajectory. The other vehicles using the predecessor's information to calculate their control and follow the path of the leader and ensure distances between each neighboring pair.

The longitudinal velocity is limited to  $v_x < 50 \text{ km/h}$ , and the imposed lateral acceleration has been bounded by two values ;  $a_{ymin} < a_y < a_{ymax}$  and as a function of the longitudinal velocity and the radius of the reference trajectory of the leader and the convoy. The inter-vehicle distance is limited between  $l_{dmin} < l_d < l_{dmax}$ . The displacement of the fleet in the fixed reference is presented using the following kinematic model:

$$X_i = \int_0^t (\dot{x}_i \cos \theta_i - \dot{y}_i \sin \theta_i) dt \quad (17)$$

Parameter	Value	Parameter	Value
$m$	1500 kg	$m_w$	23.2kg
$I_z$	1652.7kg.m <sup>2</sup>	$I_w$	2kg.m <sup>2</sup>
$C_{\alpha_f}$	67689N/rad	$C_{\alpha_r}$	69253N/rad
$L_r$	1.441m	$L_f$	1.099m
$s$	2m <sup>2</sup>	$E$	1.5m
$c$	0.3	$\rho$	1.3

Table 1: Vehicle parameters values SCANer-Studio

$$Y_i = \int_0^t (\dot{x}_i \sin \theta_i + \dot{y}_i \cos \theta_i) dt \quad (18)$$

The two previous equations are used to calculate the positions of the fleet in the reference frame R(O,X,Y). The Fig. 4 shows the movement of the fleet in the leader's trajectory. The convoy follows the path of the leader, the lateral error is almost negligible; that is to say, no angular deviation between the fleet and the desired trajectory. We can see that even with this control approach that uses the information from its predecessor, the fleet is still on the same path and the accumulation of tracking error is almost negligible too. The path tracking accuracy of the convoy Fig. 4 shows the robustness of the non-linear predictive control for tracking the trajectory of a fleet of 10 vehicles that takes into account the non-linear dynamics of each vehicle in the convoy. This control approach makes it possible to control the movements of the fleet, based on the available data, longitudinal and lateral movements.

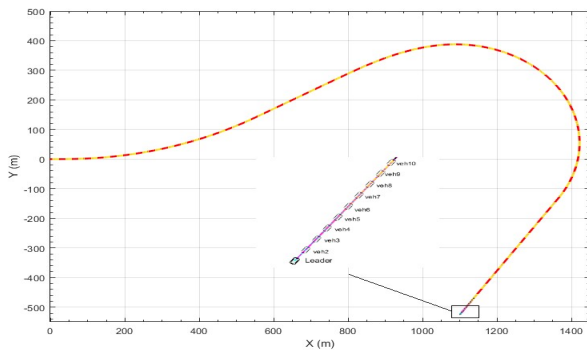


Figure 4: Trajectory of the convoy

The safety distance between the fleet is shown in the Fig 5. We can see a deviation of this distance between 2 and 4.5 m then it stabilizes for a value of 3.5 m. This safety distance was chosen for speed around 43 km/h. Generally, the convoy moves at low speed. The safety distance is almost the same for convoy vehicles. For a convoy that moves with a high speed, the distance must be higher because the risk of collision increases with a high speed and a small distance

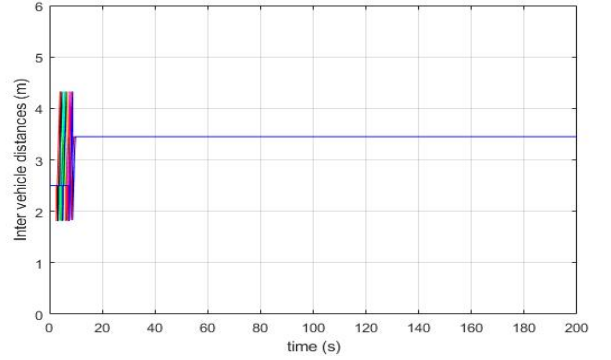


Figure 5: Inter vehicle distance

The Fig. 6 shows the different steering angles for the fleet vehicles. These angles are calculated using the second term of the global control ( $u_{2i}$ ) such as:

$$\delta_i = u_{2i} / (2C_{\alpha_{fi}} - 2 \frac{I_{wi}}{R_{fi}^2} \dot{x}_i) \quad (19)$$

This control approach makes it possible to control the longitudinal and lateral movements of the fleet. The lateral movement of the fleet is based on the trajectory of the leader as shown in the Fig. 4 and the longitudinal velocity. That is, both controls are coupled by lateral acceleration. We can clearly see a lateral movement or a lateral deviation by carrying the x-axis of the longitudinal movement from the  $t = 5$  s. The steering angle is almost constant between the interval  $t \in [10, 55$  s] with a value of 0.03 rad. This value is always dependent on the speed of the fleet and the desired trajectory.

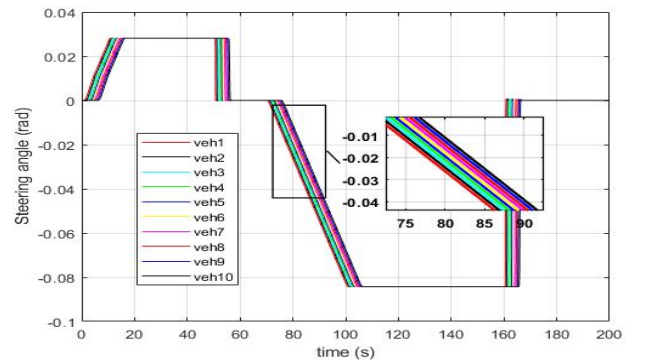


Figure 6: Steering angle

The Longitudinal speed of the fleet is presented in the Fig. 7. This speed has been imposed for the leader. By the law of the control and with the predictive control it is clear that the vehicle speeds of the convoy converge quickly to the speed of the leader. The speed of the convoy compared to the speed of the leader which is propagated in the convoy is almost negligible, which shows

the best precision and performance given by the predictive control for a convoy of 10 vehicles.

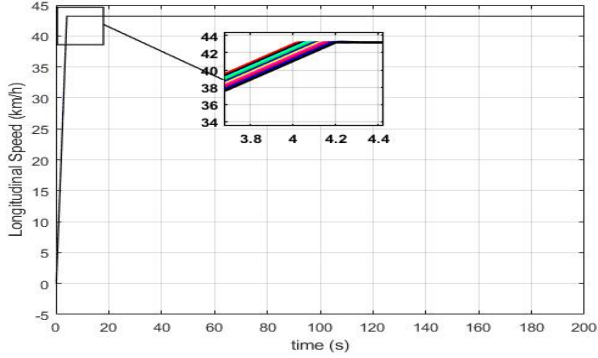


Figure 7: velocity  $v_x$  of the convoy

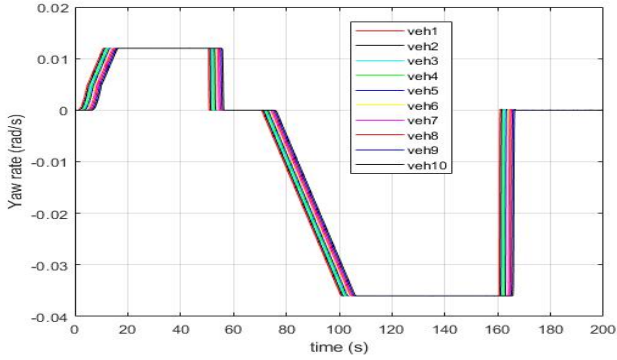


Figure 8: Yaw rate of the convoy

The lateral acceleration of the fleet is presented in Fig. 9. In our case, we took into account the speed of the yaw as  $a_{yi} = \ddot{y}_i + \dot{\theta}_i \dot{x}_i$ . This acceleration is proportional to the longitudinal velocity and the radius of the leader's trajectory. We see clearly at  $t = 5$  s a presence of the lateral movement to wait for a value  $0.15 \text{ m/s}^2$ . This acceleration is positive for  $t \in [5, 55 \text{ s}]$  and allows vehicles to be oriented for a positive lateral deviation along the y-axis. For  $t \in [55, 75 \text{ s}]$  the lateral acceleration is zero, that proves, that the fleet remains in the same direction (longitudinal direction), then between  $[75, 170 \text{ s}]$ , we can see a deceleration. Fig. 10 and Fig. 8 represents the lateral velocities of the fleet and the yaw rate which are proportional to the lateral acceleration.

To test the robustness of the control on the parameters of the model; we assumed that the parameters are not well estimated, that is, 20% errors of  $f \Rightarrow \Delta f = f - \hat{f} = 20\%f$  and 20 of  $g \Rightarrow \Delta g = g - \hat{g} = 20\%g$ . The results Fig. 11 and 12 show that the fleet is still following the leader's trajectory and the safety distance remains the same. The lateral deviation from the reference trajectory is still negligible, which proves the robustness of the control compared to the estimation errors on the model parameters.

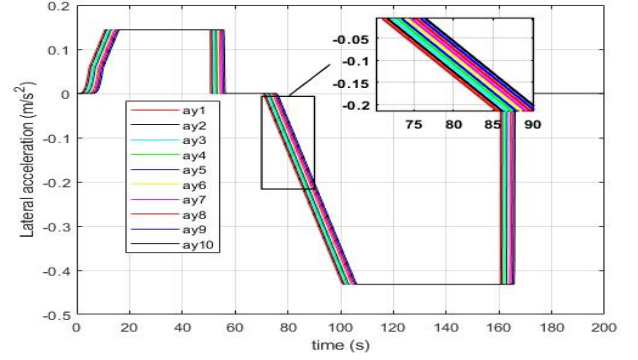


Figure 9: Lateral acceleration  $a_y$  of the convoy

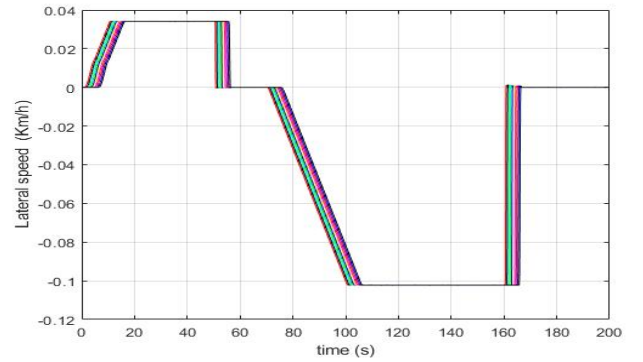


Figure 10: velocity  $v_y$  of the convoy

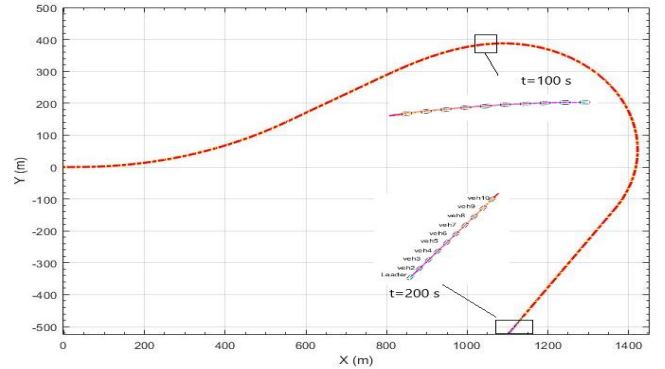


Figure 11: Trajectory of the convoy +20% of estimation errors

## 5. CONCLUSIONS

In this paper, we proposed a coupled longitudinal and lateral control for a convoy of autonomous vehicles. This approach uses nonlinear predictive control for tracking trajectory. The proposed approach allows to control the fleet by the available information and to follow the reference trajectory of the leader. Dynamic and kinematic modeling was presented to control and represent the movement of the fleet in the reference frame. This

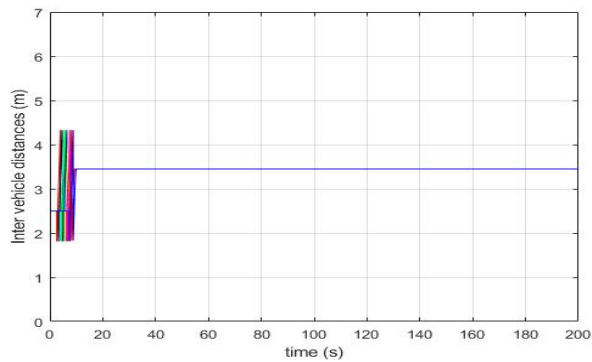


Figure 12: Inter vehicle distance +20% of estimation errors

nonlinear control approach has shown a precision performance with respect to the trajectory tracking for the lateral movement of the fleet and robustness when the parameters are not well estimated. The control law makes it possible to ensure a safe distance between the vehicles to avoid collisions by the longitudinal control, such that the fleet moves with the same speed of the leader. Accumulation of fleet tracking error is negligible when using this control approach.

## REFERENCES

- A. Ali. *Modélisation et commande d'un convoi de véhicules urbains*. PhD thesis, Université Nantes Angers Le Mans, 2015.
- Alan Ali, Gaetan Garcia, and Philippe Martinet. Urban platooning using a flatbed tow truck model. In *Intelligent Vehicles Symposium (IV), 2015 IEEE*, pages 374–379. IEEE, 2015.
- Pierre Avanzini. *Modélisation et commande d'un convoi de véhicules urbains par vision*. PhD thesis, Université Blaise Pascal-Clermont-Ferrand II, 2010.
- Pierre Avanzini, Benoit Thuilot, and Philippe Martinet. Accurate platoon control of urban vehicles, based solely on monocular vision. In *Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on*, pages 6077–6082. IEEE, 2010.
- Alia Chebly. *Trajectory planning and tracking for autonomous vehicles navigation*. PhD thesis, Université de Technologie de Compiègne, 2017.
- RM DeSantis. Path-tracking for car-like robots with single and double steering. *IEEE Transactions on vehicular technology*, 44(2):366–377, 1995.
- Ramdane Hedjar, Redouane Toumi, Patrick Boucher, and Didier Dumur. Finite horizon nonlinear predictive control by the taylor approximation: application to robot tracking trajectory. *International Journal of Applied Mathematics and Computer Science*, 15:527–540, 2005.
- Belgacem Jaballah. *Observateurs robustes pour le diagnostic et la dynamique des véhicules*. PhD thesis, Université Paul Cézanne-Aix-Marseille III, 2011.
- A Merabet and J Gu. Robust nonlinear predictive control based on state estimation for robot manipulator. *International Journal of Applied Mathematics and Mechanics*, 5(1):48–64, 2008.
- Nacer K. M'Sirdi. Vehicle platooning: an overview on modelling and control approaches. In *International Conference on Applied Smart Systems (ICASS'18)*. Medea University, 2018.
- L Nouveliere, J Sainte Marie, and S. Mammarran N K M'Sirdi. Contrôle longitudinal de véhicules par commande sous optimale. In *CIFA 2002*, pages 906–911. Nantes Juillet, 2002.
- Abdelhamid Rabhi. *Estimation de la dynamique du véhicule en interaction avec son environnement*. PhD thesis, Versailles-St Quentin en Yvelines, 2005.
- Linhan Song, Hongyan Guo, Fei Wang, Jun Liu, and Hong Chen. Model predictive control oriented shared steering control for intelligent vehicles. In *Control And Decision Conference (CCDC), 2017 29th Chinese*, pages 7568–7573. IEEE, 2017.
- D. Swaroop. *String Stability of Interconnected Systems : An application to platooning in AHS*. PhD thesis, University of California at Berkeley, 1994.
- Ji Xiang and Thomas Bräunl. String formations of multiple vehicles via pursuit strategy. *IET control theory & applications*, 4(6):1027–1038, 2010.
- Jano Yazbeck. *Accrochage immatériel sûr et précis de véhicules automatiques*. PhD thesis, Université de Lorraine, 2014.