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Abstract

As illustrated by some French departments, how can we explain the existence of equilibria with different fertility and growth rates in economies with the same fundamentals, preferences, technologies and initial conditions? To answer this question we develop an endogenous growth model with altruism and love for children. We show that independently from the type of altruism, a multiplicity of equilibria might emerge if the degree of love for children is high enough. We refer to this condition as the *love for children hypothesis*. Then, the fertility rate is determined by expectations on the future growth rate and the dynamics are not path-dependent. Our model is able to reproduce different fertility behaviours in a context of completed demographic transition independently from fundamentals, preferences, technologies and initial conditions.

JEL Classification: J13, O41, D11

Key Words: Fertility, Love for Children, Expectations, Endogenous Growth, Balanced Growth Path.

1 Introduction

Many countries have almost surely completed their demographic transition. For instance, in France, the total fertility rate of all women has reached its *plateau*. Data from Insee[1] show that, from 2006 to nowadays, the total fertility rate of French women is on average stable at the replacement level of two children per woman.[2] Even though nowadays the total fertility rate of French women is on

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[2] The total fertility rate is expressed in per 1000 women (considering women in reproductive ages, i.e. 15-49 years-old.). We only consider the total fertility rate for Metropolitan France. See Appendix B for data sources.

average stable at replacement level, if we focus on small geographical areas, such as regions or departments, very different fertility patterns emerge. The demographic literature has already highlighted the existence of these geographical differences in France dating back to the previous centuries.[3]

Figure 1 maps the total fertility rate of French women in 2014 at department level. It clearly show that differentials in fertility rates still persist nowadays. More precisely, if we concentrate on fertility rates in French departments belonging to the same regional entity, we can clearly observe very different fertility behaviours. The coefficients of spatial autocorrelation of Moran and Geary at department level confirm the existence of a strong fertility heterogeneity across departments.[4]

[Figure 1 here]

This is, in our opinion, a very interesting phenomenon that deserves to be analysed in details. Indeed, in a context of completed fertility transition without heterogeneity of individuals' preferences, abilities, norms or initial conditions, such as in departments belonging to the same regional entity, we would expect a convergence of fertility rates across departments. However, this expectation is not confirmed by the empirical evidence. The case of Alsace is emblematic.[5] Even though this region is composed by two departments, Bas-Rhin and Haut-Rhin, with similar, if not the same, fundamentals and initial conditions, we can observe the existence of an important fertility differential. We cannot ascribe this phenomenon to heterogeneity of preferences of individuals populating two neighbourhood departments, since their cultural fundamentals, religious values, historical heritages and social norms are the same. Of course, this fact is not specific to Alsace and can be observed in other adjacent French departments. At the same time, there also exists some French departments, adjacent and belonging to the same region, that do not exhibit any differentials but a convergence of fertility rates.

To understand if these different current fertility behaviours have persisted over time, we use time-series data for French departments. We observe that time-persistence might arise in different geographical areas of France. Figure 2 clearly shows this statement.[6] In particular, it depicts the dynamic evolution of the total fertility rate of all women over the period 1975-2014 for four couples

[3] For instance, in the second half of the XIX century, the total fertility rate of all women was much more higher in the North, North-West and East compared to the South-West, center and South of France. See Desplanques (2011) and Vivier (2014).

[4] While the first coefficient is a measure of global spatial autocorrelation, the second measures local spatial autocorrelation. Negative values indicate negative spatial autocorrelation while positive values indicate positive one. Both coefficients measure the correlation among values of a single variable strictly attributable to their relatively close locational positions on a two-dimensional surface (Griffith (1987)).

[5] Take in mind that we define regions as they were before the law of December 12, 2014, that reduces the number of French metropolitan regions from 22 to 13 from January 1, 2016.

[6] For this figure we consider two adjacent departments in the following regions: Alsace (Bas-Rhin and Haut-Rhin), Lorraine (Meurthe-et-Moselle and Meuse), Bourgogne (Côte-d'Or and Yonne), PACA (Bouches-du-Rhône and Vaucluse)

of two adjacent departments belonging to a particular region for which this persistence of fertility behaviours is pronounced.[7]

[Figure 2 here]

Looking at other adjacent departments in Figure 1 we can clearly observe that differences in fertility rates at department level is not the only possible empirical outcome observable in French data. We also find patterns characterized by convergence of fertility rates across adjacent departments and over time. To support this claim, we present in Figure 3 four cases of adjacent departments belonging to the same regional entity in which we do not report any persistence of fertility differentials over time.[8] The fertility behaviour of women in each department tends to be much more homogeneous with respect to the case of persistent fertility gap. Indeed, in all four cases in Figure 3, fertility rates at department level fluctuate around very similar values and cross each other several times.

[Figure 3 here]

These important empirical facts we observe in French departments ask to answer to the following theoretical questions: how can we explain, on the one hand, the persistence and, on the other hand, the absence of fertility differentials in a context of completed fertility transition in which households' preferences, such as altruism, norms, religious beliefs, technologies and initial conditions are homogeneous? Can an economic model of fertility explain this diversity when is not assumed the heterogeneity of fundamentals?

We claim that a standard fertility model with homogeneous preferences, fundamentals, technologies and initial conditions cannot explain this diversity of configurations because it would necessarily predict the convergence of fertility rates. Therefore, the main motivation of this paper is to fill the gap between economic theory on fertility and the empirical evidence we have highlighted in this section. To this end we propose a new theoretical model based on the role of expectations that might help to understand the variability of fertility behaviours across French departments. Put differently, the main objective of this paper is to develop a new model of fertility that is able to reproduce theoretically the evidence we observe in the French data at department level in a context of completed fertility transition. In the next section, we will present an economic model with endogenous growth, fertility, love for children and either paternalistic or dynastic altruism that is able to provide an explanation, based on the multiplicity of equilibria, of the different stylized facts presented in this section.

[7]We choose adjacent departments to guarantee the convergence in terms of cultural characteristics and initial conditions. Moreover, we choose departments in different regions to cover all the territory of France. For conciseness, we only consider 4 metropolitan regions of France but this phenomenon can be found in other geographical areas.

[8]For Figure 3, we consider two adjacent departments in the following regions: Rhône-Alpes (Ain and Rhône), Languedoc-Roussillon (Aude and Pyrénées-Orientales), Limousin (Corrèze et Creuse), Poitou-Charentes (Charente and Charente-Maritime).

In the literature, several papers have explained the existence of fertility differentials, because of heterogeneous endowments in human capital and/or skills to work (see for instance Dahan and Tsiddon (1998), de la Croix and Doepke (2003, 2004), Kremer and Chen (1999, 2002)). However, differences in fertility collapse in the absence of heterogeneities across households, as it seems to be the case among the French departments we focus on in Section 2. Other contributions interesting for our research questions develop models with endogenous fertility and traps due to a multiplicity of steady states. This multiplicity may be explained by the net return of capital which is no more always decreasing with respect to capital (Palivos (1995), Cai (2002)) or the difference between the returns of investing in capital and having children (Becker *et al.* (1990), Galor and Weil (1996)). In these approaches, one converges to a steady state with high or low fertility depending on the initial condition on capital. The dynamics are typically path-dependent. This means that with the same initial conditions, as in the French departments we consider, one converges to the same equilibrium and fertility rate. Therefore, these different types of analysis cannot explain the diversity of configurations we highlight among French departments, such as in the region of Alsace. In addition, both the existence and the absence of fertility gaps across adjacent departments cannot be explained by such models in which individuals are homogeneous in terms of preferences, as we believe it is the case for individuals living in neighbourhood departments within the same regional entity.

For these reasons, we look at another explanation based on the multiplicity of equilibria. Depending on whether agents living in two adjacent departments coordinate their expectations on the same or on different equilibria, the model might reproduce the absence or the persistence of disparities in fertility rates. In other words, we develop a model which is able to replicate both empirical evidences without considering heterogeneity of preferences or different initial socio-economic conditions. We obtain these conclusions considering an endogenous growth model with fertility, love for children and either paternalistic or dynastic altruism. We recall that in the first case, utility depends on the amount of bequest for each child, while in the second one, utility of parents depends on utility of children. To have a model with growth as tractable as possible, we consider an Ak technology. We show that whatever the type of altruism, there exist two balanced growth paths under similar conditions. Therefore, our results do not depend on the type of altruism considered, which highlights the robustness of the findings.

Above all, our analysis emphasizes the crucial role of love for children, formalized by a utility that depends on the number of children *per se*. As we show, the multiplicity of equilibria emerges if love for children is significant, referred as the *love for children hypothesis*, and the marginal utility of having children does not strongly depend on the number of children, meaning that households are quite indifferent between two (stationary) fertility rates. In this case, the fertility rate is determined by expectations on the future growth rate. Indeed, depending on their expectations, agents coordinate on a high fertility rate or rather a lower one.[9] This result can be related to the Matsuyama (1990) model with real money balances in the utility in which multiplicities can occur. Indeed, in our framework, the number of children which is an endogenous argument of the utility function is also the price of capital. This may explain the diversity of configurations we observe among adjacent French departments belonging to the same region. If we associate our economy to a French department, households of two departments sharing the same fundamentals can coordinate their expectations on the same or on different equilibria. Finally, on the theoretical ground, our result is new for two reasons. First, it does not depend on a form of heterogeneity among agents (preferences, skills, initial conditions). Second, our economy can jump on one equilibrium or the other one because, in contrast to what we usually find in growth models with fertility, dynamics are not path-dependent. Because of love for children, the fertility rate is determined by expectations on the future growth rate.

Our analysis is also related to the *value of children* theory formulated in the sociological literature by the pioneering work of Hoffmann and Hoffmann (1973). These authors develop a sociological approach based on the idea that the combination of socio-economic and normative factors is able to influence fertility decisions. The main assumption of their sociological model is that the value that parents give to their children determines the intra-family relationships and the fertility behaviours. Empirical studies have tested if the value of children hypothesis is able to explain fertility differentials within and between countries. The results indicate that this hypothesis is quite predictive of fertility intentions (Mares and Mozny (2005), Nauck (2006), Nauck and Klaus (2007)). To our knowledge, no economic contributions have tried to give an explicit theoretical formulation of this theory. We show that this can be done extending basic growth model of rational fertility choice to the idea that the marginal utility of having children does not strongly decrease with the number of children. In this case, expectations on the value of children could explain the appearance of fertility differentials between households and geographical areas characterized by the same fundamentals. However, in an economic setting with altruism and love for children, it seems to us more appropriate to refer to this theoretical approach as *love for children hypothesis*, that is, a welfare that strongly weights utility for the number of children.

This paper is organized as follows. In the next section, we develop our model with endogenous growth, fertility, love for children and either paternalistic or dynastic altruism that exhibits a multiplicity of equilibria. We discuss and interpret our results in Section 3. We conclude in Section 4, while technical proofs are relegated to an Appendix.

[9] A similar result is found by Chabé-Ferret and Melindi-Ghidi (2012). They show that ethnic minorities are often found to exhibit in some circumstances higher fertility levels and lower educational investments than the majority group because parents' fertility decisions are affected by the uncertainty concerning the future economic status of their offspring. However, this concept is related to the concept of minority status and the size of each groups with respect to the majority. In this work we do not concentrate on the minority status hypothesis but on the concept of value of children and expectations on future growth rates.

2 The models with love for children and altruism

We present two models with love for children, i.e. parents derive utility for the number of children they have, and altruism, respectively paternalistic one and dynastic one. We show that whatever the model chosen, a multiplicity of equilibria occur under similar conditions: the degree of love for children should be high enough, i.e. what we call the love for children hypothesis is satisfied, and the marginal utility of having children should not strongly depend on the number of children. These models may explain the diversity of experiences met by adjacent French departments. Depending on whether agents of two such departments coordinate their expectations on the same equilibrium or not, one explains non persistent or persistent fertility differentials even in presence of homogeneity of fundamentals and initial conditions.

Time is discrete ($t = 0, 1, ...$) and there are two types of agents: firms and altruistic consumers. We start by presenting the production sector.

2.1 Production

A continuum of unit size of identical firms produces the final good using capital K_t and labor L_t. In order to introduce in a simple way endogenous growth, a learning by doing process results in knowledge accumulation. Congestion effects are taken into account assuming that the externality of knowledge depends on the capital intensity (Bosi and Seegmuller (2012), Frankel (1962), Ljungqvist and Sargent (2004), Chapter 14). Therefore, the quantity of final good produced by each firm is given by $Y_t = F(K_t, \bar{k}_t L_t)$, where $\bar{k}_t \equiv \bar{K}_t/\bar{L}_t$ denotes the average capital-labor ratio. The technology $F(K_t, \bar{k}_t L_t)$ has the usual neoclassical properties, i.e. is a strictly increasing and concave production function satisfying the Inada conditions, and is homogeneous of degree one with respect to its two arguments.

Each firm considers knowledge spillovers as given and maximizes its profits with respect to capital and labor under perfect competition. Since we note r_t the real interest rate and w_t the real wage, profit maximization gives:

$$r_t = F_1(K_t, \bar{k}_t L_t) \quad (1)$$

$$w_t = F_2(K_t, \bar{k}_t L_t)\bar{k}_t \quad (2)$$

We now present the model with paternalistic altruism.

2.2 Paternalistic altruism

2.2.1 Households

The economy is populated by individuals in overlapping generations whose finite lifespan is divided up into two periods: youth (inactive period) and adult age (working period). Despite consumption, households have preferences for fertility with a weight $\epsilon > 0$, and altruism, with a weight $\gamma > 0$. Paternalistic altruism means that households take care about the amount of bequest, capital in our framework, they leave to their children (see Michel *et al.* (2006)). The size of the generation of adults born in $t-1$ is N_t, growing at an endogenous factor $n_t \in (0; +\infty)$. Therefore, the population size of successive generations evolves according to $N_{t+1} = n_t N_t$.

When adult, an agent born at time $t-1$ derives utility from consumption, c_t, having children n_t and bequest per child through capital holding κ_{t+1}:

$$\ln c_t + \epsilon \frac{n_t^{1-\mu}}{1-\mu} + \gamma \ln \kappa_{t+1} \tag{3}$$

with $\mu > 0$ and $\mu \neq 1$. In the limit case where $\mu = 1$, the utility becomes log-linear, i.e. $\ln c_t + \epsilon \ln n_t + \gamma \ln k_{t+1}$, as in a lot of existing contributions (Galor and Weil (1996), de la Croix and Doepke (2003)). We generalize this specification introducing a specification of the utility that allows us to discuss the results according to the concavity degree with respect to the fertility rate. Note that a quite similar specification where considered in the seminal contribution by Razin and Ben-Zion (1975). Parameter μ measures the elasticity of marginal utility with respect to the number of children. In a related setting, Cordoba and Ripoll (2016) show that the relationship between family income and fertility is in accordance with the empirical evidence when consumption and fertility are intertemporal substitutes. It is therefore relevant to consider μ different and even smaller than one. The utility function (3) also implies operative bequests at each period of time, because of an Inada condition with respect to κ_{t+1}.

Each young individual induces a rearing time cost $b > 0$ to her parents. Each household supplies $1 - bn_t$ units of labour to firms, earning the competitive wage rate w_t. Moreover, she receives income from capital bequests κ_t coming from her parents remunerated by firms at the rate r_t. This income is shared between family consumption and capital bequests to children. Considering solutions with n_t strictly positive, the budget constraint of an adult born in $t-1$ writes as follows:

$$c_t + n_t(\kappa_{t+1} + bw_t) = R_t \kappa_t + w_t \tag{4}$$

with $R_t \equiv 1 - \delta + r_t$ the gross return of capital and $\delta \in (0,1)$ the depreciation rate of capital.

An adult household determines her optimal choice maximizing the utility (3) taking into account the budget constraint (4). One get:

$$n_t = \left[\frac{\kappa_{t+1} \epsilon}{\gamma(\kappa_{t+1} + bw_t)} \right]^{\frac{1}{\mu-1}} \tag{5}$$

$$c_t = \frac{\kappa_t R_t + w_t(1 - bn_t)}{1+\gamma} \tag{6}$$

Lemma 1 *There exists $\epsilon_1 > 0$ such that the second order conditions are satisfied for the utility maximisation (3) under the constraint (4) if $\epsilon > \epsilon_1$.*

Proof. See Appendix A.1. ■

Lemma 1 states that if the weight associated to love for children, i.e. ϵ is sufficiently high, household's utility is maximised whatever the value of μ. The second order conditions are in particular satisfied for $\mu < 1$, a configuration in which we will show that a multiplicity of equilibria exists. We refer to this analytical condition as the *love for children hypothesis*.[10]

2.2.2 Equilibrium and balanced growth path (BGP): uniqueness versus multiplicity

Because of a continuum of unit size of identical firms, individual and average capital labor ratios coincide at the equilibrium, that is: $\bar{k}_t = k_t = K_t/L_t$. Defining $s \equiv F_1(1,1)/F(1,1) \in (0,1)$ the capital share in total production and $A \equiv F(1,1) > 0$ the total factor productivity, (1) and (2) rewrite:

$$\begin{aligned} r_t &= sA \quad (7)\\ w_t &= (1-s)Ak_t \quad (8) \end{aligned}$$

Equilibrium on the labor market is satisfied if $L_t = N_t(1-bn_t)$. Equilibrium on the capital market requires $k_t = K_t/L_t = \kappa_t N_t/[(1-bn_t)N_t] = \kappa_t/(1-bn_t)$.

Define $g_t \equiv k_t/k_{t-1}$ the growth factor of the capital-labor ratio. We can rewrite (5) and (6) as follows:

$$\begin{aligned} n_t &= \left[\frac{\gamma}{\epsilon}\left(1+\frac{(1-s)Ab}{(1-bn_{t+1})g_{t+1}}\right)\right]^{\frac{1}{1-\mu}} \quad (9)\\ c_t &= \frac{k_t[(1-\delta+sA)(1-bn_t)+(1-s)A(1-bn_t)]}{1+\gamma} \quad (10) \end{aligned}$$

with $0 < n_t < 1/b$. Substituting (7), (8) and (10) into (4), we obtain:

$$n_t\left(\frac{1+\gamma}{\gamma}g_{t+1}(1-bn_{t+1})+(1-s)Ab\right) = (1-\delta+sA)(1-bn_t)+(1-s)A \quad (11)$$

Equation (9) rewrites:

$$g_{t+1} = \frac{(1-s)Ab}{(1-bn_{t+1})(n_t^{1-\mu}\epsilon/\gamma-1)} \quad (12)$$

Substituting this equation in (11), we get:

$$G(n_t) = H(n_t) \quad (13)$$

with:

$$\begin{aligned} G(n) &\equiv (1-s)Abn(\epsilon n^{1-\mu}+1) \quad (14)\\ H(n) &\equiv [(1-\delta+sA)(1-bn)+(1-s)A](\epsilon n^{1-\mu}-\gamma) \quad (15) \end{aligned}$$

[10] Note that the denomination "love" for children can already be met in the literature. See the survey by Nerlove and Raut (1997).

Equation (13) defines the solutions $n_t \in (0, 1/b)$. Note that this equation becomes a static one. Given (n_t), we deduce the value of (g_{t+1}) using (12). We show that:

Proposition 1 *Let*

$$\underline{\epsilon} \quad \equiv \quad (2b)^{1-\mu}[1 - s + \gamma(2 - s)] \tag{16}$$

Assuming $\epsilon > \epsilon_1$, the following holds:

(i) For $\mu = 1$, there is a unique equilibrium $0 < n_0 < 1/b$ if and only if $\epsilon > \gamma$;

(ii) For $\mu > 1$, there is a unique equilibrium $0 < n_0 < 1/b$;

(iii) For $0 < \mu < 1$, there exist two equilibria n_1 and n_2, such that $(\gamma/\epsilon)^{\frac{1}{1-\mu}} < n_1 < 1/(2b) < n_2 < 1/b$, if $\epsilon \geqslant \underline{\epsilon}$.

Proof. See Appendix A.2. ■

A BGP is an equilibrium with $g_t = g_{t+1} = g$, defined by:

$$g = \frac{(1 - s)Ab}{(1 - bn)(n^{1-\mu}\epsilon/\gamma - 1)} \tag{17}$$

with $n = n_0$ for $\mu \geqslant 1$ and $n = n_1$ or $n = n_2$ for $\mu \in (0, 1)$. This last case is of special interest because it entails a form of global indeterminacy. The agents coordinate their expectations either on the equilibrium n_1, or on n_2. This means that, with the same fundamentals, such as preferences and socio-economic initial conditions, an economy can either be on the equilibrium n_1, or on n_2. Different fertility rates may be obtained depending on agents' expectations. If our economy corresponds to a French department, this explains the possible existence and persistence of fertility differentials between two adjacent departments with the same cultural and fundamental characteristics.

Two main ingredients are important to obtain this multiplicity. First, the love for children hypothesis should be satisfied so that ϵ should be large enough to satisfy the second order conditions whatever the value of μ. Second, μ should be low enough, which means that the marginal utility of having children $(\epsilon n^{-\mu})$ weakly depends on the number of children. Then, households are quite indifferent between two equilibria with different fertility rates. They can coordinate on different values of the fertility rate.

We show now that this result of multiplicity is not specific to the assumption of paternalistic altruism.

2.3 Dynastic altruism

To show the robustness of our previous results, we introduce dynastic altruism. Otherwise, the model is similar to the previous one. We show that multiplicities occur under similar conditions. Therefore, the same problem of coordination of expectations will be able to explain fertility differentials under the same fundamentals.

2.3.1 Households

The population size of the generation born in $t-1$ is N_t. In $t-1$, the young agents of this generation are inactive, while in t, they become active. As in the previous model, agents derive utility from consumption, the number of children and a form of altruism. The utility for consumption and the number of children is similar than in the case of paternalistic altruism, but the altruism is now dynastic. As in Razin and Ben-Zion (1975), agents care about the utility of their children. Considering that the degree of altruism is now measured by $\beta \in (0,1)$, the utility of an adult consumer at period t is given by:

$$U_t = \ln c_t + \epsilon \frac{n_t^{1-\mu}}{1-\mu} + \beta U_{t+1}$$

We take into account that bequests are operative. At equilibrium, this will occur, because otherwise, income and therefore consumption become equal to zero. Accordingly, the utility of a dynasty writes:

$$\sum_{t=0}^{+\infty} \beta^t \left(\ln c_t + \epsilon \frac{n_t^{1-\mu}}{1-\mu} \right) \tag{18}$$

with $\epsilon > 0$, $\mu > 0$ and $\mu \neq 1$. In the limit case where $\mu = 1$, the utility writes $\sum_{t=0}^{+\infty} \beta^t \left(\ln c_t + \epsilon \ln n_t\right)$. As it is explained in the previous model, our specification of the utility allows us to discuss the results according to the concavity degree with respect to the fertility rate. The budget constraints faced at each period are similar to the previous model, i.e. are given by equation (4).

We determine the optimal behaviour of households maximizing the utility (18) under the constraints (4), focusing again on n_t strictly positive. We get the following conditions:

$$c_t = n_t^{\mu} \frac{bw_t + \kappa_{t+1}}{\epsilon} \tag{19}$$

$$\frac{n_t}{c_t} = \beta \frac{R_{t+1}}{c_{t+1}} \tag{20}$$

$$\lim_{t \to +\infty} \beta^t \frac{n_t}{c_t} \kappa_{t+1} = 0 \tag{21}$$

Lemma 2 *There exists $\epsilon_2 > 0$ such that the second order conditions are satisfied for the utility maximisation (18) under the constraint (4) if $\epsilon > \epsilon_2$.*

Proof. See Appendix A.3. ■

Lemma 2 indicates that household's maximisation holds whatever the value of μ under a similar condition than in the model with paternalistic altruism (see Lemma 1), i.e. a high enough weight of love for children ϵ. This means that again, the so-called *love for children hypothesis* must be satisfied.

2.3.2 Equilibrium and BGP: uniqueness versus multiplicity

Substituting (19) in (4) and (20), we get:

$$\frac{n_t^{1-\mu}}{bw_t + \kappa_{t+1}} = \beta \frac{R_{t+1}}{(bw_{t+1} + \kappa_{t+2}) n_{t+1}^{\mu}} \quad (22)$$

$$n_t^{\mu} \frac{w_t b + \kappa_{t+1}}{\epsilon} + n_t(\kappa_{t+1} + bw_t) = R_t \kappa_t + w_t \quad (23)$$

Using $k_t = K_t/L_t = \kappa_t/(1 - \beta n_t)$, (7) and (8), equations (22) and (23) become:

$$\frac{n_t^{1-\mu} g_{t+1}}{b(1-s)A + (1 - bn_{t+1})g_{t+1}} = \beta \frac{1 - \delta + sA}{[b(1-s)A + (1 - bn_{t+2})g_{t+2}] n_{t+1}^{\mu}} \quad (24)$$

$$g_{t+1}(1 - bn_{t+1}) + b(1-s)A = \epsilon \frac{(1 - \delta + sA)(1 - bn_t) + (1-s)A}{n_t^{\mu} + \epsilon n_t} \quad (25)$$

A BGP is a steady state of the system (24)-(25), i.e. a solution (n, g) solving:

$$g = \frac{\beta(1 - \delta + sA)}{n} \quad (26)$$

$$\left(\frac{n^{\mu}}{\epsilon} + n\right)[g(1 - bn) + b(1-s)A] = (1 - \delta + sA)(1 - bn) + (1-s)A \quad (27)$$

Substituting (26) into (27), a BGP is a value of n satisfying $I(n) = J(n)$, with:

$$I(n) \equiv (1 + \epsilon n^{1-\mu})[(1 - bn)\beta(1 - \delta + sA) + nb(1-s)A] \quad (28)$$

$$J(n) \equiv [(1 - \delta + sA)(1 - bn) + (1-s)A]\epsilon n^{1-\mu} \quad (29)$$

On a BGP, $\kappa_{t+1}/\kappa_t = k_{t+1}/k_t = g$ because n is constant. Using the budget constraint, we also have $c_t/\kappa_t + ng = 1 - \delta + A$. This implies that $c_{t+1}/c_t = \kappa_{t+1}/\kappa_t = g$. Therefore, the transversality condition (21) is always satisfied on a BGP.

We examine now the existence and the number of BGP:

Proposition 2 *Let*

$$\widehat{\epsilon} \equiv \frac{\beta(1 - \delta + sA)}{1 - \delta + A - \beta(1 - \delta + sA)}$$

$$\widetilde{\epsilon} \equiv \frac{(2b)^{1-\mu}[(1 - \delta + sA)\beta + (1-s)A]}{(1-\delta)(1-\beta) + A(1 - \beta s)}$$

Assuming $\epsilon > \epsilon_2$, the following holds.

(i) For $\mu = 1$, there is a unique BGP (n_0, g_0), with $0 < n_0 < 1/b$, if and only if $\epsilon > \widehat{\epsilon}$;

(ii) For $\mu > 1$, there is a unique BGP (n_0, g_0), with $0 < n_0 < 1/b$;

(*iii*) *For* $0 < \mu < 1$*, there exist two BGP,* (n_1, g_1) *and* (n_2, g_2)*, such that* $0 < n_1 < 1/(2b) < n_2 < 1/b$ *and* $g_1 > g_2$*, if* $\epsilon > \widetilde{\epsilon}$*.*

Proof. See Appendix A.4. ■

Before discussing this result on the uniqueness versus the multiplicity of BGP, we analyze the dynamics of the model. To analyze the stability of the BGP in the different configurations highlighted in Proposition 2, let us rewrite the dynamic system (24)-(25) as follows. Using (25), we substitute g_{t+1} in equation (24) to get:

$$\frac{\widetilde{I}(n_{t+1})}{(1-\delta+sA)(1-bn_{t+1})+(1-s)A} = \frac{\widetilde{J}(n_t)}{(1-\delta+sA)(1-bn_t)+(1-s)A} \quad (30)$$

with:

$$\widetilde{I}(n_{t+1}) \equiv \beta(1-\delta+sA)(1-bn_{t+1})(\epsilon n_{t+1}^{1-\mu}+1) \quad (31)$$

$$\widetilde{J}(n_t) \equiv \epsilon n_t^{1-\mu}(1-\delta+A)(1-bn_t)-(1-s)Abn_t \quad (32)$$

This means that the dynamics are driven by a one-dimensional dynamic equation that gives the sequence of the non-predetermined variable n_t. Of course, given (n_t), we are able to deduce the sequence of growth factor (g_t) using equation (25). Equation (30) allows us to show:

Proposition 3 *Assuming* $\epsilon > \epsilon_2$*, the following holds.*

(*i*) *For* $\mu = 1$ *and* $\epsilon > \widehat{\epsilon}$*, or* $\mu > 1$*, the unique BGP* (n_0, g_0) *is globally determinate;*

(*ii*) *For* $0 < \mu < 1$ *and* $\epsilon > \widetilde{\epsilon}$*, the two BGP* (n_1, g_1) *and* (n_2, g_2) *are locally determinate.*

Proof. See Appendix A.5. ■

Propositions 2 and 3 show that when $\mu \geqslant 1$, there is a unique BGP which is globally determinate (see also Figure 4). This is the only equilibrium and one immediately jumps on this BGP. Taking the fundamentals of the economy as given, rational expectations imply that $n_t = n_0$. Therefore, this configuration is not able to explain the diversity of situations among French departments, i.e. both the existence and the absence of a fertility differential when we compare some of them that are adjacent and belong to the same region.

[Figure 4 here]

When $0 < \mu < 1$, there are two BGP and each one is locally determinate (see also Figure 5). The coexistence of the two equilibria n_1 and n_2 means that there is a form of global indeterminacy associated to the multiplicity of BGP.

The agents may coordinate their expectations on one of the two BGP.[11] This is interesting for our aim: as in the case of paternalistic altruism, this model can explain that economies with the same fundamentals may be characterized by persistent heterogeneous fertility rates. This may explain the diversity of situations experienced by some adjacent French departments that can either be characterized by the same fertility rates or by persistent fertility differentials.

[Figure 5 here]

As in the model with paternalistic altruism, two key conditions are required to get this result: the love for children hypothesis (ϵ large enough) and a marginal utility of having children that weakly depends on the fertility rate (μ smaller than one).[12] Therefore, the utility to have children $\epsilon n_t^{1-\mu}/(1-\mu)$ plays the key role to have the multiplicity equilibria and fertility rates. This conclusion is reinforced if we refer to the paper by Bosi and Seegmuller (2012). Except that they also introduce heterogeneous agents and exogenous mortality rates, the main difference with our model with dynastic altruism lies in the fact that households are of the Barro and Becker (1989) type and there is no love for children as in our framework. In contrast to us, they always get a unique BGP.

3 Discussion of the theoretical results

We have shown that whatever the type of altruism, paternalistic or dynastic, there is a multiplicity of BGP and of associated fertility rates. Therefore, our result seems to be quite robust. In both cases, it requires the same conditions. First, the love for children hypothesis must be satisfied, i.e. ϵ should be sufficiently high. Empirical evidence in different disciplines strongly supports this hypothesis. For instance, love for children can be proxied by the willingness to pay for a child. In the medical care research, Neumann and Johannesson (1994) estimate that the ex-ante willingness to pay for a child of infertile adults was $1,8 million. In psychology, infertility stress is also seen as a crucial factor in explaining life quality of infertile patients (Moura-Ramos *et al.* (2012)). Second, the elasticity of the marginal utility of having children with respect to the number of children measured in absolute value, μ, needs to be low enough. This means that the marginal utility for children does not strongly depend on the number of children and households are quite indifferent between two (stationary) fertility rates. This condition is also supported by empirical evidence. In

[11] Even if we are not especially concerned with dynamic paths with oscillations and fluctuations, we can note that the model may feature complex dynamics. By direct inspection of Figure 5, we see that there is a threshold value $\widehat{n}$ that separates the dynamics in two regions. For either $0 < n_t < \widehat{n}$ or $\widehat{n} < n_t < 1/b$, we observe that there is a form of global indeterminacy, because there exist values of n_t for which it corresponds two values of n_{t+1}. In addition, it is a priori possible to switch from the region with $0 < n_t < \widehat{n}$ to the one with $\widehat{n} < n_t < 1/b$, and vice-versa.

[12] Note that, despite the fact that we consider endogenous growth, our model with dynastic altruism is close to the one developed in the seminal contribution by Razin and Ben-Zion (1975). They have no multiplicities because they restrict their attention to the configuration where $\mu > 1$.

a related paper, Cordoba and Ripoll (2016) find that the relationship between income and fertility is empirically relevant when consumption and fertility are sufficient substitutes over time, which corresponds to $\mu < 1$. Along these lines, Jones *et al.* (2010) show that models where consumption and children are high substitutes perform better replicating the empirical evidence of the relationship between income and fertility.

The question now is to understand why our models with altruism generate a multiplicity of stationary solutions, i.e. a form of global indeterminacy. As it is well known, a large number of contributions have analyzed the conditions for the occurrence of local indeterminacy. Early references in this literature are Kehoe *et al.* (1992) and Spear (1991). They show that sufficiently strong externalities lead to local indeterminacy. Despite the fact that we have a multiplicity of BGP and no local indeterminacy, we can ask whether a similar externality characterizes our models with paternalistic and dynastic altruism. Considering the case of paternalistic altruism, we notice that when an adult chooses the (positive) level of bequest, she does not take into account all the effects of capital per capita on the following periods. Hence, this choice generates an external effect. However, in the case of dynastic altruism, an adult decides her choices of consumption, number of children and bequest taking into account the effects on all the following generations. Therefore, there is no external effect due to bequest, as the one described above. Since the multiplicity of BGP emerges under similar conditions in both models with paternalistic and dynastic altruisms, we argue that a common mechanism to both models should explain the multiplicity of equilibria.

Note that usually, optimal growth models (i.e. without inefficiencies) have no multiplicities of equilibria. Notable exceptions are however economies where the price of an asset enters the utility function. One can refer, for instance, to models with real money balances in the utility (Matsuyama (1990)) or with spirit of capitalism (Clain-Chamosset-Yvrard (2016), Kamihigashi (2008)). In our framework, we argue that we have a closely related feature. Indeed, n_t, which is an argument of the utility function and is endogenous, is also the price of capital. In line with Matsuyama (1990), multiplicity occurs in our two models, because it is possible to decrease the price of the asset, i.e. the number of children, to increase the level of capital, and get a different level of utility.[13]

We now investigate more precisely how such a mechanism is compatible with the equilibrium conditions of the two models. Let us rewrite the main arbitrage conditions. In the case of paternalistic altruism (PA), we have (see (B.1) and (B.2) in Appendix A.1):

$$\frac{n_t}{c_t} = \frac{\gamma}{\kappa_{t+1}} \tag{33}$$

$$\epsilon n_t^{1-\mu} = \gamma \frac{\kappa_{t+1} + b w_t}{\kappa_{t+1}} \tag{34}$$

[13]Note that we do not analyze global dynamics. Figure 5 suggests that global indeterminacy, bifurcations and cycles may occur. Such a study is however out of the scope of this paper and could be considered for future research.

In the case of dynastic altruism (DA), the arbitrage conditions are (see (B.6) and (B.7) in Appendix A.3):

$$\frac{n_t}{c_t} = \beta \frac{R}{c_{t+1}} \tag{35}$$

$$\epsilon n_t^{1-\mu} = \frac{\beta R(\kappa_{t+1} + bw_t)}{c_{t+1}} \tag{36}$$

where $R = 1 - \delta + sA$.

In both models, the first equation, (33) and (35) respectively, determines the choice between current consumption and the bequest transmitted to the next generation. The cost of bequest in terms of the consumption good, n_t, is equal to the marginal utility of bequest over the marginal utility of current consumption.

We recall that on a BGP, $k_t = \kappa_t/(1 - bn)$, which means that $\kappa_t/\kappa_{t-1} = k_t/k_{t-1} = g$. Using the budget constraint faced by the household, we also have $c_t + n\kappa_{t+1} = (1 - \delta + A)\kappa_t$, which is equivalent to $c_t/\kappa_t = 1 - \delta + A - ng$. This implies that $c_t/c_{t-1} = \kappa_t/\kappa_{t-1} = g$. Hence, in a neighborhood of a BGP, equation (35) also writes:

$$n_t = \beta \frac{R}{g_{t+1}} \tag{37}$$

and equation (33) gives an expression for n_t proportional to the right-hand side of (37). Because of the decreasing marginal utilities, the endogenous cost of the capital bequeathed, n_t, is therefore inversely related to the expected growth factor.

Let us focus on the other arbitrage equation, (34) in the model with PA and (36) in the model with DA. The left-hand side of both these equations is similar and corresponds to the cost of capital n_t times the marginal utility of having children $\epsilon n_t^{-\mu}$. The right-hand side is therefore equal to the price of having an adding child $\kappa_{t+1} + bw_t$ times the marginal utility associated to an increase of bequest in terms of capital, which is equal to γ/κ_{t+1} in the model with PA and $\beta R/c_{t+1}$ in the model with DA.

On the one hand, we see by direct inspection of the right-hand side of (34) and (36) that, following an increase in the expected growth factor, the increase of the price of a child does not cross out the decrease of the marginal utility associated to altruism. On the other hand, if μ is lower than one, we observe that, following a decrease of n_t, the effect through the price of capital dominates the opposite one that goes through the marginal utility of having children.

Hence, this arbitrage between the number of children and the amount of capital bequeathed to each child shows us that if households expect an increase of the future growth rate, they choose to have a lower number of children. This is compatible with the other arbitrage between current consumption and bequest (see equation (37)). Indeed, in both models with PA or DA, a lower fertility, which also corresponds to a lower cost of investment, leads to a larger capital investment, which boosts growth.

Of course, if μ is larger or equal to one, this cannot happen. Any increase of the expected growth factor implies either a raise or no effect on the number of children (see (34) and (36)). It contradicts the arbitrage between current consumption and bequest, which describes a negative relationship between growth and fertility rates (see equation (37)). This implies the uniqueness of BGP.

Finally, when $\mu < 1$, equations (33) and (34) in the model with PA and equations (35) and (36) in the model with DA describe two negative relationships between the fertility rate n and the growth rate g. This means that the two existing BGP, (n_1, g_1) and (n_2, g_2), satisfy $n_1 < n_2$ and $g_1 > g_2$. The BGP with a lower fertility rate experiences a larger growth. This is in line with the theoretical and empirical literature highlighting a negative relationship between fertility and income variables when the demographic transition is completed (Galor (2005)).

4 Conclusion

In this paper we provided an economic interpretation of the *value of children* hypothesis developed by the literature in sociology during the 70's. Following this approach, we claimed that expectations could explain the appearance of fertility differentials within geographical areas characterized by the same fundamentals. Since we employed an economic setting with altruism and love for children, we referred to this new theoretical approach as the *love for children hypothesis*, that is, a household's welfare that strongly weights the utility for the number of children.

From a theoretical perspective, we developed two growth models with love for children and altruism. We showed that independently from the type of altruism chosen, i.e. paternalistic or dynastic, a multiplicity of equilibria might occur if the degree of love for children is high enough. With respect to the previous economic literature, our model is the first able to explain simultaneously the possibility of different configurations of fertility patterns without assuming any form of heterogeneity in preferences and/or technology. More precisely, our theoretical conclusions do not depend on heterogeneity in utilities or initial conditions. Because of love for children and endogenous growth, the fertility rate is determined by expectations on the future growth rate and the dynamics are not path-dependent.

This theoretical finding explains important empirical facts. Indeed, despite the fact that France has completed its demographic transition, fertility differentials seem to persist over time in some areas. If one can claim that adjacent departments within the same regional entity have the same fundamentals, technologies, social norms, religious beliefs, to our knowledge, our model is able to predict simultaneously the possibility of persistence and non-persistence of fertility differentials over time without assuming some forms of heterogeneity in preferences and/or technology. Even though we recognize the importance of socio-economic factors in explaining fertility behaviours, we also highlight that expectations could complete the overall picture. We believe that this is an important theoretical result that nicely contributes to the economic literature in fertility and growth theory.

A Proofs

A.1 Proof of Lemma 1

In the model with paternalistic altruism, the objective function maximized by the household can be written:

$$L_{1t} \equiv \ln\left[R_t\kappa_t + w_t - n_t(\kappa_{t+1} + bw_t)\right] + \gamma \ln \kappa_{t+1} + \epsilon \frac{n_t^{1-\mu}}{1-\mu}$$

The first order conditions are given by:

$$\frac{\partial L_{1t}}{\partial n_t} = -\frac{\kappa_{t+1} + bw_t}{c_t} + \epsilon n_t^{-\mu} = 0 \tag{B.1}$$

$$\frac{\partial L_{1t}}{\kappa_{t+1}} = -\frac{n_t}{c_t} + \frac{\gamma}{\kappa_{t+1}} = 0 \tag{B.2}$$

with $c_t = R_t\kappa_t + w_t - n_t(\kappa_{t+1} + bw_t)$. We easily deduce the following derivatives:

$$\frac{\partial^2 L_{1t}}{\partial n_t^2} = -\frac{(\kappa_{t+1} + bw_t)^2}{c_t^2} - \mu\epsilon n_t^{-\mu-1} < 0 \tag{B.3}$$

$$\frac{\partial^2 L_{1t}}{\kappa_{t+1}^2} = -\frac{n_t^2}{c_t^2} - \frac{\gamma}{\kappa_{t+1}^2} < 0 \tag{B.4}$$

$$\frac{\partial^2 L_{1t}}{\partial\kappa_{t+1}\partial n_t} = -\frac{1}{c_t} - n_t\frac{\kappa_{t+1} + bw_t}{c_t^2} \tag{B.5}$$

The second order conditions are fulfilled if the Hessian $H_{1t} \equiv \frac{\partial^2 L_{1t}}{\partial n_t^2}\frac{\partial^2 L_{1t}}{\kappa_{t+1}^2} - (\frac{\partial^2 L_{1t}}{\partial\kappa_{t+1}\partial n_t})^2 > 0$. Using (B.3)-(B.5), we get:

$$H_{1t} = \left[\frac{(\kappa_{t+1} + bw_t)^2}{c_t^2} + \mu\epsilon n_t^{-\mu-1}\right]\left[\frac{n_t^2}{c_t^2} + \frac{\gamma}{\kappa_{t+1}^2}\right] - \left[\frac{1}{c_t} + n_t\frac{\kappa_{t+1} + bw_t}{c_t^2}\right]^2$$

Using (B.1) and (B.2), this equation becomes:

$$H_{1t} = \frac{n_t(\kappa_{t+1} + bw_t)[\mu c_t + n_t(\kappa_{t+1} + bw_t)]}{c_t^4}\frac{1+\gamma}{\gamma} - \left[\frac{c_t + n_t(\kappa_{t+1} + bw_t)}{c_t^2}\right]^2$$

For $\mu \geqslant 1$, $H_{1t} > 0$ if:

$$\frac{1+\gamma}{\gamma}n_t(\kappa_{t+1} + bw_t) > c_t + n_t(\kappa_{t+1} + bw_t)$$

which is equivalent to:

$$n_t(\kappa_{t+1} + bw_t) > \gamma c_t$$

Using (B.2), we have $n_t\kappa_{t+1} = \gamma c_t$, which means that the second order conditions are satisfied for all $\mu \geqslant 1$.

For $\mu < 1$, we have $H_{1t} > 0$ if:

$$\frac{1+\gamma}{\gamma} > \left[\frac{c_t + n_t(\kappa_{t+1} + bw_t)}{n_t(\kappa_{t+1} + bw_t)}\right]^2$$

which is equivalent to:

$$\frac{n_t(\kappa_{t+1} + bw_t)}{c_t} > \frac{1}{\sqrt{1+1/\gamma} - 1}$$

Because the utility function satisfies standard Inada conditions, n_t cannot be zero, but is always strictly positive. This means that there exists $\widetilde{n} > 0$ such that $n_t > \widetilde{n}$. Using (B.1), we deduce that:

$$n_t \frac{\kappa_{t+1} + bw_t}{c_t} = \epsilon n_t^{1-\mu} > \epsilon \widetilde{n}^{1-\mu}$$

Therefore, for ϵ sufficiently large, such that:

$$\epsilon > \widetilde{n}^{\mu-1} \frac{1}{\sqrt{1+1/\gamma} - 1} \equiv \epsilon_1,$$

$H_{1t} > 0$ and the second order conditions are satisfied for $\mu < 1$.

A.2 Proof of Proposition 1

Let us start with the limit case where $\mu = 1$. Equations (14) and (15) become:

$$\begin{aligned} G(n) &\equiv (1-s)Abn(\epsilon+1) \\ H(n) &\equiv [(1-\delta+sA)(1-bn) + (1-s)A](\epsilon - \gamma) \end{aligned}$$

If $\epsilon > \gamma$, there is a unique equilibrium given by:

$$n = \frac{(\epsilon-\gamma)(1-\delta+A)}{(\epsilon-\gamma)b(1-\delta+sA) + (\epsilon+1)(1-s)Ab} \in (0, 1/b)$$

Otherwise, no equilibrium exists.

Let us focus on the case where $\mu \neq 1$. Using (14) and (15), we show that:

$$\begin{aligned} G''(n) &= (1-s)Ab(2-\mu)(1-\mu)\epsilon n^{-\mu} \\ H''(n) &= -\mu n^{-\mu-1}(1-\mu)\epsilon(1-\delta+A) - (1-\mu)n^{-\mu}\epsilon b(1-\delta+sA)(2-\mu) \\ H(1/b) &= (1-s)A(\epsilon b^{\mu-1} - \gamma) < G(1/b) = (1-s)A(\epsilon b^{\mu-1} + 1) \end{aligned}$$

We especially deduce that:

$$G''(n) - H''(n) = (1-\mu)(1-\delta+A)\epsilon n^{-\mu-1}[bn(2-\mu) + \mu]$$

This implies that $G''(n) - H''(n) < 0$ if $\mu > 1$ and $G''(n) - H''(n) > 0$ if $\mu < 1$.

If $\mu > 1$, we have $G(0) < H(0)$. Since $G(1/b) > H(1/b)$ and $G(n) - H(n)$ is strictly concave, there is one solution to $G(n) - H(n) = 0$, which proves (ii) of the proposition.

If $\mu \in (0,1)$, $G(n) - H(n)$ is strictly convex and $H(0) = -\gamma(1-\delta+A) < 0 = G(0)$. Since $H(n) \geqslant 0$ for all $n \geqslant (\gamma/\epsilon)^{\frac{1}{1-\mu}}$, there exist two solutions n_1 and n_2 that belong to $((\gamma/\epsilon)^{\frac{1}{1-\mu}}, 1/b)$ to equation (13) if there is a value of $n \in ((\gamma/\epsilon)^{\frac{1}{1-\mu}}, 1/b)$ such that $H(n) > G(n)$. Take $n = 1/(2b)$ that belongs to this interval under $\epsilon > \gamma(2b)^{1-\mu}$. $G(1/(2b)) < H(1/(2b))$ is equivalent to:

$$A[(2b)^{1-\mu}(1-s+\gamma(2-s)) - \epsilon] < (1-\delta)[\epsilon - \gamma(2b)^{1-\mu}]$$

This inequality is satisfied if $\epsilon \geqslant \underline{\epsilon}$. In this case, there exist two solutions n_1 and n_2 to equation (13) such that $(\gamma/\epsilon)^{\frac{1}{1-\mu}} < n_1 < 1/(2b) < n_2 < 1/b$, which proves (iii) of the proposition.

A.3 Proof of Lemma 2

In the model with dynastic altruism, the objective function maximized by the household can be written:

$$L_{2t} \equiv \sum_{t=0}^{+\infty} \beta^t \left[\ln\left(R_t\kappa_t + w_t - n_t(\kappa_{t+1} + bw_t)\right) + \epsilon \frac{n_t^{1-\mu}}{1-\mu} \right]$$

The first order conditions are given by:

$$\frac{\partial L_{2t}}{\partial n_t} = -\frac{\kappa_{t+1} + bw_t}{c_t} + \epsilon n_t^{-\mu} = 0 \qquad \text{(B.6)}$$

$$\frac{\partial L_{2t}}{\kappa_{t+1}} = -\frac{n_t}{c_t} + \beta \frac{R_{t+1}}{c_{t+1}} = 0 \qquad \text{(B.7)}$$

with $c_t = R_t\kappa_t + w_t - n_t(\kappa_{t+1} + bw_t)$. We easily deduce the following derivatives:

$$\frac{\partial^2 L_{2t}}{\partial n_t^2} = -\frac{(\kappa_{t+1} + bw_t)^2}{c_t^2} - \mu\epsilon n_t^{-\mu-1} < 0 \qquad \text{(B.8)}$$

$$\frac{\partial^2 L_{2t}}{\kappa_{t+1}^2} = -\frac{n_t^2}{c_t^2} - \beta \frac{R_{t+1}^2}{c_{t+1}^2} < 0 \qquad \text{(B.9)}$$

$$\frac{\partial^2 L_{2t}}{\partial \kappa_{t+1} \partial n_t} = -\frac{1}{c_t} - n_t \frac{\kappa_{t+1} + bw_t}{c_t^2} \qquad \text{(B.10)}$$

The second order conditions are fulfilled if the Hessian $H_{2t} \equiv \frac{\partial^2 L_{2t}}{\partial n_t^2} \frac{\partial^2 L_{2t}}{\kappa_{t+1}^2} - (\frac{\partial^2 L_{2t}}{\partial \kappa_{t+1} \partial n_t})^2 > 0$. Using (B.8)-(B.10), we get:

$$H_{2t} = \left[\frac{(\kappa_{t+1} + bw_t)^2}{c_t^2} + \mu\epsilon n_t^{-\mu-1} \right] \left[\frac{n_t^2}{c_t^2} + \beta \frac{R_{t+1}^2}{c_{t+1}^2} \right] - \left[\frac{1}{c_t} + n_t \frac{\kappa_{t+1} + bw_t}{c_t^2} \right]^2$$

Using (B.6) and (B.7), this equation becomes:

$$H_{2t} = \frac{n_t(\kappa_{t+1} + bw_t)[\mu c_t + n_t(\kappa_{t+1} + bw_t)]}{c_t^4} \frac{1+\beta}{\beta} - \left[\frac{c_t + n_t(\kappa_{t+1} + bw_t)}{c_t^2}\right]^2$$

For $\mu \geqslant 1$, $H_{2t} > 0$ if:

$$\frac{1+\beta}{\beta} n_t(\kappa_{t+1} + bw_t) > c_t + n_t(\kappa_{t+1} + bw_t)$$

Using (B.6), this is equivalent to $\epsilon > \beta n_t^{\mu-1}$. Since $n_t < 1/b$, this inequality is ensured by $\epsilon > \beta b^{1-\mu} \equiv \widehat{\epsilon}_2$.

For $\mu < 1$, we have $H_{2t} > 0$ if:

$$\frac{1+\beta}{\beta} > \left[\frac{c_t + n_t(\kappa_{t+1} + bw_t)}{n_t(\kappa_{t+1} + bw_t)}\right]^2$$

which is equivalent to:

$$\frac{n_t(\kappa_{t+1} + bw_t)}{c_t} > \frac{1}{\sqrt{1 + 1/\beta} - 1}$$

Because the utility function satisfies standard Inada conditions, n_t cannot be zero, but is always strictly positive. This means that there exists $\underline{n} > 0$ such that $n_t > \underline{n}$. Using (B.6), we deduce that:

$$n_t \frac{\kappa_{t+1} + bw_t}{c_t} = \epsilon n_t^{1-\mu} > \epsilon \underline{n}^{1-\mu}$$

Therefore, for ϵ sufficiently large, such that:

$$\epsilon > \underline{n}^{\mu-1} \frac{1}{\sqrt{1+1/\beta} - 1} \equiv \widetilde{\epsilon}_2,$$

$H_{2t} > 0$ and the second order conditions are satisfied for $\mu < 1$.

Let us define $\epsilon_2 \equiv \max\{\widehat{\epsilon}_2, \widetilde{\epsilon}_2\}$. The second order conditions are satisfied for all $\mu > 0$ if $\epsilon > \epsilon_2$.

A.4 Proof of Proposition 2

Let us start with the limit case where $\mu = 1$. Equations (28) and (29) become:

$$\begin{aligned} I(n) &\equiv (1+\epsilon)[(1-bn)\beta(1-\delta+sA) + nb(1-s)A] \\ J(n) &= [(1-\delta+sA)(1-bn) + (1-s)A]\epsilon \end{aligned}$$

There is a unique equilibrium given by:

$$n = \frac{1}{b} \frac{\epsilon(1-\delta+A) - (1+\epsilon)\beta(1-\delta+sA)}{(1+\epsilon)[(1-s)A - \beta(1-\delta+sA)] + \epsilon(1-\delta+sA)} \in (0, 1/b)$$

if and only if $\epsilon > \widehat{\epsilon}$. Otherwise, no equilibrium exists.

We focus now on the case where $\mu \neq 1$. Using (28) and (29), we have $I(1/b) = (1-s)A(1+\epsilon b^{\mu-1})$ and $J(1/b) = (1-s)A\epsilon b^{\mu-1} < I(1/b)$. Moreover, using (28) and (29), we compute:

$$\begin{aligned} I''(n) &= -\mu(1-\mu)\epsilon n^{-\mu-1}\beta(1-\delta+sA) \\ &\quad +(1-\mu)(2-\mu)\epsilon n^{-\mu}b[(1-s)A-\beta(1-\delta+sA)] \\ J''(n) &= -\mu n^{-\mu-1}(1-\mu)\epsilon(1-\delta+A)-(1-\mu)n^{-\mu}\epsilon b(1-\delta+sA)(2-\mu) \end{aligned}$$

We obtain:

$$I''(n)-J''(n) = (1-\mu)\epsilon n^{-\mu-1}[1-\delta+A-\beta(1-\delta+sA)][\mu+(2-\mu)bn]$$

which shows that $I''(n)-J''(n) < 0$ if $\mu > 1$ and $I''(n)-J''(n) > 0$ if $\mu < 1$.

If $\mu > 1$, we further have $I(0) < J(0)$. Since $I(1/b) > J(1/b)$ and $I(n)-J(n)$ is strictly concave, there is a unique solution to the equation $I(n)-J(n) = 0$, which proves *(ii)* of the proposition.

If $0 < \mu < 1$, $I(0) = \beta(1-\delta+sA) > 0 = J(0)$ and $I(n)-J(n)$ is strictly convex. Let us consider $n = 1/(2b)$. Using (28) and (29), we deduce that $I(1/(2b)) < J(1/(2b))$ if and only if $\epsilon > \widetilde{\epsilon}$. Using the continuity of $I(n)$ and $J(n)$, we deduce that there are two solutions n_1 and n_2 solving $I(n)-J(n) = 0$, such that $0 < n_1 < 1/(2b) < n_2 < 1/b$. Using equation (26), we determine the associated values g_1 and g_2. This proves *(iii)* of the proposition.

A.5 Proof of Proposition 3

Differentiating equation (30), we get:

$$\begin{aligned} &\frac{\widetilde{I}'(n_{t+1})[(1-\delta+sA)(1-bn_{t+1})+(1-s)A]+\widetilde{I}(n_{t+1})(1-\delta+sA)b}{[(1-\delta+sA)(1-bn_{t+1})+(1-s)A]^2}dn_{t+1} \\ &= \frac{\widetilde{J}'(n_t)[(1-\delta+sA)(1-bn_t)+(1-s)A]+\widetilde{J}(n_t)(1-\delta+sA)b}{[(1-\delta+sA)(1-bn_t)+(1-s)A]^2}dn_t \end{aligned}$$

Using (31) and (32), we obtain $\widehat{I}(n_{t+1})dn_{t+1} = \widehat{J}(n_t)dn_t$, with:

$$\widehat{I}(n) \equiv \frac{\beta(1-\delta+sA)\mathcal{A}(n)}{[(1-\delta+sA)(1-bn)+(1-s)A]^2} \tag{B.11}$$

$$\widehat{J}(n) \equiv \frac{(1-\delta+A)\mathcal{A}(n)}{[(1-\delta+sA)(1-bn)+(1-s)A]^2} \tag{B.12}$$

$$\begin{aligned} \mathcal{A}(n) \equiv\ & (1-\mu)\epsilon n^{-\mu}(1-bn)[(1-\delta+sA)(1-bn)+(1-s)A] \\ & -b(1-s)A(\epsilon n^{1-\mu}+1) \end{aligned} \tag{B.13}$$

We first deduce that at each BGP, we have:

$$\frac{dn_{t+1}}{dn_t} = \frac{1-\delta+A}{\beta(1-\delta+sA)} > 1 \tag{B.14}$$

This means that each BGP is locally determinate.

When $\mu \geqslant 1$, we further have $\widehat{I}(n_{t+1}) < 0$ and $\widehat{J}(n_t) < 0$. This means that n_{t+1} is a strictly increasing function of n_t for all $n_t \in (0, 1/b)$. Therefore, since the BGP is unique, it is globally determinate (see also Figure 4).

When $\mu < 1$, the analysis is different. By direct inspection of equations (B.11)-(B.13), we have $\widehat{I}(0) = \widehat{J}(0) = +\infty$ and $\widehat{I}(1/b) = \widehat{J}(1/b) < 0$. Since $\mathcal{A}(n)$ is strictly decreasing, there is a unique $\widehat{n} \in (0, 1/b)$ such that $\widehat{I}(\widehat{n}) = \widehat{J}(\widehat{n}) = 0$. In addition, we have $\widehat{J}(n_t) > 0$ ($\widehat{I}(n_{t+1}) > 0$) for $n_t < \widehat{n}$ ($n_{t+1} < \widehat{n}$) and $\widehat{J}(n_t) < 0$ ($\widehat{I}(n_{t+1}) < 0$) for $n_t > \widehat{n}$ ($n_{t+1} > \widehat{n}$). We deduce that $dn_{t+1}/dn_t > 0$ for all $(n_t, n_{t+1}) \in (0, \widehat{n})^2$ and $(n_t, n_{t+1}) \in (\widehat{n}, 1/b)^2$ and $dn_{t+1}/dn_t < 0$ for all $(n_t, n_{t+1}) \in (0, \widehat{n}) \times (\widehat{n}, 1/b)$ and $(n_t, n_{t+1}) \in (\widehat{n}, 1/b) \times (0, \widehat{n})$. Because of these results and inequality (B.14) holds at each BGP, the two BGP are such that $n_1 < \widehat{n} < n_2$. Each BGP is locally determinate. Using the different ingredients of this proof, we can draw Figure 5. The proposition immediately follows.

B The Insee dataset and Philcarto

The Insee provides demographic, economic and social data for France. Data are available at www.insee.fr. Data are provided at national, regional, departmental and municipal levels. In our analysis, we concentrate on data at departmental level. In particular, we use time series for the total fertility rate of women of all ages (15-49 years-old) between 1975 and 2014. Following the Insee's definition: *total period fertility measures the number of children a woman would have in the course of her life if the fertility rates observed at each age in the year considered remain unchanged*. We have merged data on total fertility rate with shape files provided by Philcarto to produce the geographical map to illustrate total fertility rate differentials at departmental level in 2014. The program and the shape files are publicly available at http://philcarto.free.fr. To describe geographical borders at regional level, we use shape files defining regional borders before the law of December 12, 2014, that reduces the number of France metropolitan regions from 22 to 13 from January 1, 2016. We do not consider TOM, i.e. overseas territories of France.

Compliance with Ethical Standards:

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The authors declare that they have no conflict of interest.

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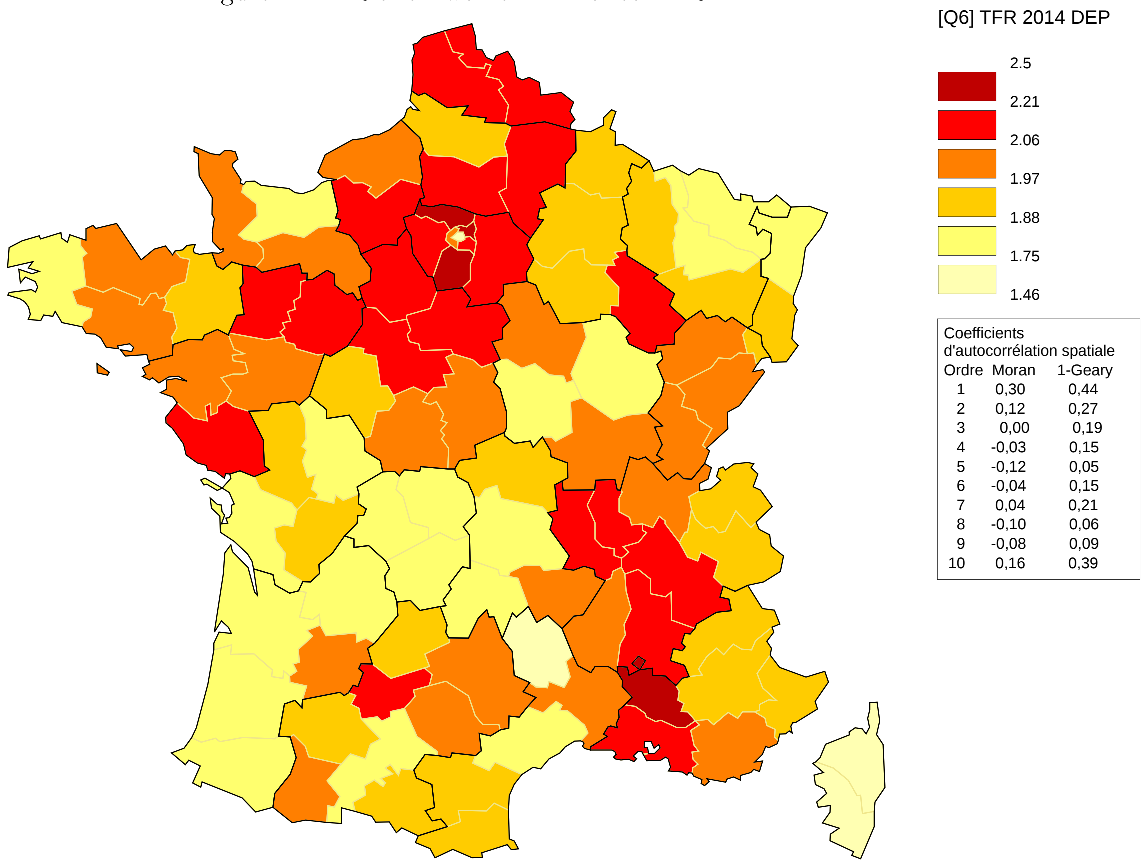
Figure 1: TFR of all women in France in 2014



Ordre	Moran	1-Geary
1	0,30	0,44
2	0,12	0,27
3	0,00	0,19
4	-0,03	0,15
5	-0,12	0,05
6	-0,04	0,15
7	0,04	0,21
8	-0,10	0,06
9	-0,08	0,09
10	0,16	0,39

Figure 2: Persistent Fertility Differentials at department level in 4 Regions of France 1975 - 2014

Figure 3: Non-persistent Fertility Differentials at department level in 4 Regions of France 1975 - 2014

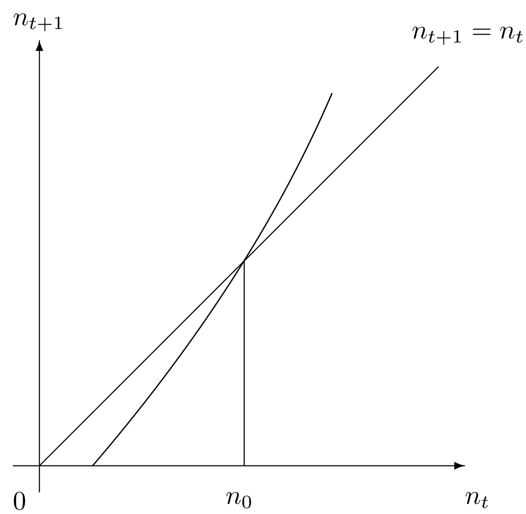


Figure 4: Dynastic altruism with $\mu \geqslant 1$

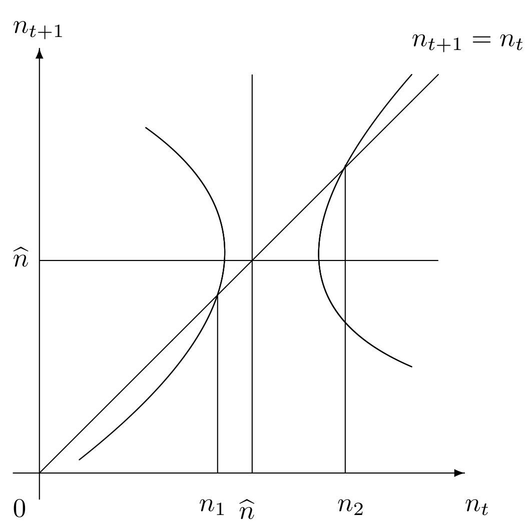


Figure 5: Dynastic altruism with $0 < \mu < 1$