Can harmful events be another source of environmental traps?

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Abstract

This paper aims to present a new explanation for environmental traps through the presence of endogenous hazard rate. We show that adaptation and mitigation policies affect the occurrence of environmental traps differently. The former could cause environmental traps, whereas the latter could help society avoid such traps by decreasing the probability of a harmful event occurring. As a result, we present a new trade-off between adaptation and mitigation policies different than the usual dynamic trade-off that is highlighted in many studies and is crucial to developing countries. Contrary to the literature, when an economy is in a trap, an economy with a high environmental quality equilibrium tends to be more conservative in terms of resource exploitation than an economy with a low environmental quality equilibrium, which implies a heterogeneous reaction against the endogenous hazard rate.

Keywords : Environmental damage, Harmful event, Occurrence hazard, Tipping points, Multiple equilibria, Environmental traps, Adaptation, Mitigation.

JEL Classification : O13, D81,Q2, Q54

1 Introduction

A social planner should consider methods for avoiding damage as a result of hazardous events. A direct response requires action to reduce the probability of a harmful event taking place. In many cases, mitigation activities reduce the risk of a harmful event by improving environmental quality.[1] Still, mitigation activities cannot eliminate risk completely. When risk is unavoidable, a possible action could be to alleviate the negative consequences of damage caused by a harmful event. The measures taken to reduce losses due to the harmful event can be categorized as adaptation activities. The management of adaptation and mitigation activities raises an interesting dynamic trade-off that can be described as an "adaptation and mitigation dilemma" in the field of environmental economics literature (Zemel (2015), Tsur and Zemel (2015), Crépin et al. (2012)).

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[1] Since we focus on environmental quality in the paper, mitigation activities are intended to increase environmental quality. A possible definition of mitigation activity can be reforestation, which enhances carbon sinks. (IPCC (2007))

To elaborate on these concepts, we consider concrete examples. Improvements in energy efficiency through activities such as carbon capture and storage and reforestation address the root causes by decreasing greenhouse gas emissions and reducing the risk of a harmful event occurring. Therefore, such activities can be referred to as mitigation activities[2]. Differently, installing flood defenses and developing irrigation systems aim to reduce the damage inflicted by a harmful event rather than stop the event from occurring. Hence, such activities can be classified as adaptation activities. In this context, adaptation plays a *proactive* role, which means that it has no noticeable effect prior to the harmful event's occurrence (Smit et al. (2000) ; Shalizi and Lecocq (2009)). In this example, the problem is to decide on an optimal combination of risk-reducing and damage-reducing measures within a given budget.

In this paper, we study the optimal management of natural resources and its link with adaptation and mitigation policies in a simple growth model under endogenous hazard rate depending on the environmental quality level[3]. For example, the rates of harmful events such as droughts, crop failures, and floods[4] are linked to the exploitation of natural capital.

The contribution of this paper is threefold. The first contribution of the paper is to analyze the implications of the endogenous hazard rate on the occurrence of the multiplicity of equilibria (i.e., the environmental trap). We show that one of the reasons for environmental traps is the endogenous hazard rate.

When an economy faces an endogenous hazard rate, a second trade-off arises between consumption and the endogenous hazard rate other than the usual intertemporal trade-off between present and future consumption. An economy with serious environmental quality problems is expected to be impatient due to the high endogenous hazard rate. Therefore, agents tend to increase their consumption at earlier dates, which again stresses the environmental quality over time. This trade-off between consumption and harmful events results in a vicious cycle of a "low level of environmental quality and consumption" in the long run and leads to an environmental trap. Since the multiplicity of equilibria can trap an economy to a lower welfare level, it is legitimate to try to avoid it. In that sense, differently from the existing literature, our paper aims to present qualitative implications of the endogenous hazard regarding the long term dynamics of an economy.

The possibility of multiple stationary equilibria in growth models with endogenous hazard[5] has been mentioned by Tsur and Zemel (2016) but not examined in greater depth. In this paper, not only do we offer an economic explanation of environmental traps, but we also prove the existence of the multiplicity of equilibria by giving explicit mathematical conditions (see Appendix A*).

Our study relates also to the substantial literature on resource exploitation under uncertainty. The consideration of uncertain events for optimal management began with the work of Cropper (1976), who finds that the depletion of resources is either faster or slower than expected if the available resource stock is uncertain. Clarke and Reed (1994) find that the endogenous hazard rate either increases or decreases the pollution stock when there is a single occurrence event that indefinitely reduces utility to a constant level.

Our paper differs substantially from these studies, since we treat an economy facing recurrent harmful events. Having recurrent events in the model allows us to offer a more realistic setup.

[2] Mitigation activities can also be seen as tools used to avert a harmful event (Martin and Pindyck (2015)).

[3] We use the terms environmental quality and natural resource stock interchangeably.

[4] The depletion of forests in a region increases the probability of floods because the soil loses its ability to absorb rainfall.

[5] There is also a large amount of literature on growth models with endogenous discounting which points out the possibility of multiple equilibria (Das (2003), Drugeon (1996), Schumacher (2009)). However, these papers include neither uncertainty components nor environmental aspects.

The existing literature attempts to determine the reaction of an economy to uncertain events. A common argument is that uncertainty pushes the economy to become more precautious by conserving natural resources (Polasky et al. (2011), de Zeeuw and Zemel (2012), Ren and Polasky (2014)).

The second contribution of this paper is to show that there can be a heterogeneous reaction to uncertain events. We show that an economy with a high environmental quality level is more precautious than one with a low environmental quality level. In other words, an economy with a low environmental quality equilibrium adopts an "aggressive" exploitation policy relative to an economy with a high environmental quality equilibrium.

Apart from the literature on resource exploitation under uncertainty, recent contributions have been made regarding the implications of harmful events on the long-term behavior of an economy. A paper by van der Ploeg (2014) focuses on how a first-best optimal carbon tax should be adjusted over time when an economy faces harmful event probability. van der Ploeg and de Zeeuw (2017) extend the framework offered by van der Ploeg (2014) and show that a positive saving response can help the economy to dampen the discrete change in consumption at the time of the harmful event. A recent study by Akao and Sakamoto (2018) offers a general framework that can be used to justify the empirical evidence that shows the positive correlation between disasters and long-term economic performance.

Our study differs also from this branch of the literature, since these studies focus on the social cost of carbon stock but say nothing about the necessary policy response in terms of adaptation and mitigation policies in order to deal with harmful events.

The third (and most significant) contribution of this paper is that it presents a new trade-off between adaptation and mitigation different from the dynamic trade-off highlighted by numerous studies (Bréchet et al. (2012), Le Kama and Pommeret (2016), Millner and Dietz (2011)). Zemel (2015) and Tsur and Zemel (2015) investigated the time profile of the optimal mix of adaptation and mitigation in a simple growth model with uncertainty. However, these studies did not consider the multiplicity of equilibria and its implications regarding adaptation and mitigation policies. Our main result is to show that adaptation increases and mitigation decreases the possibility of environmental traps.

What is the mechanism behind the new trade-off between adaptation and mitigation? Adaptation capital is shown to decrease the optimal steady-state level of environmental quality, as agents worry less about the consequences of a harmful event. Then, because the endogenous hazard rate increases, the trade-off between present consumption and the endogenous hazard rate becomes more important, which is likely to raise multiple equilibria. Contrary to this mechanism, mitigation activity improves environmental quality, and the trade-off between present consumption and harmful event probability becomes weaker.

Another contribution is our analysis of how the dynamic trade-off between adaptation and mitigation affects the occurrence of environmental traps. Recent papers by Tsur and Zemel (2015) and Zemel (2015) focus on the dynamic trade-off between adaptation and mitigation but overlook its qualitative implications regarding the multiplicity of equilibria. We show that the unit cost of adaptation changes the optimal mix of adaptation and mitigation policies over time but also affects the occurrence of the multiplicity of equilibria.

The remainder of this paper is organized as follows: Section 2 presents the benchmark model. Section 3 describes the model with adaptation and mitigation policies and explains in depth the effects of adaptation and mitigation on the occurrence of environmental traps. Section 4 provides numerical illustrations. Section 5 concludes the paper.

2 Model

Let $S(t)$ represent the environmental stock available or environmental quality at time t, e.g, the stock of clean water, soil quality, air quality, forests, biomass. We refer to a broad definition of environmental quality which encompasses all environmental amenities and existing natural capital that have an economic value[6].

Obviously, disamenities such as waste and pollution stemming from consumption $c(t)$ decrease environmental quality stock. The stock $S(t)$ evolves in time according to

$$\dot{S}(t) = R(S(t)) - c(t) \tag{1}$$

With a given initial state $S(0)$, an exploitation policy of environmental stock $c(t)$ generates the state process $S(t)$ according to equation (1) and provides the utility $u(c(t))$. Similar to Le Kama and Schubert (2007), we use a framework where consumption comes directly from environmental services and causes environmental damage.

We make use of the following assumptions.

A.1 The regeneration of environmental quality is characterized by $R(.) : \mathbb{R}_+ \to \mathbb{R}_+$, $R(S) > 0$, $R_{SS}(S) < 0$, $R_S(S) \lesseqgtr 0$ and $R(\bar{S}) = 0$ where $\bar{S}$ is the carrying capacity of the natural resource stock.

A.2 The utility function $u(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable with the following properties ; $u_c(c) > 0$, $u_{cc}(c) < 0$, $\forall c$ and $\lim_{c \to 0}$, $u_c(c) = \infty$.

Schumacher (2011) shows that models with endogenous discounting should use a utility function with a positive domain because the utility function appears in the optimal path of consumption. Indeed, since our model focuses on recurrent events such as droughts and floods, the utility function does not appear in the Keynes-Ramsey rule. Therefore, changing the utility function from having a negative domain to a positive domain does not qualitatively change the solutions.

Not only does the environmental stock $S(t)$ represent a source of consumption for the economy, but it also affects the endogenous hazard rate. Forests, for example, considerably influence the environmental conditions in any given area and help to decrease the endogenous hazard rate (see (Dasgupta and Mäler (1997), chapter 1) and Jie-Sheng et al. (2014), Bradshaw et al. (2007))[7].

Let T be the event occurrence time of an harmful event and let $F(t) = Pr\{T \leq t\}$ and $f(t) = F^{'}(t)$ denote the corresponding probability distribution and density functions, respectively. The endogenous hazard rate $h(S)$ is related to $F(t)$ and $f(t)$ with respect to

$$h(S(t))\Delta = Pr\{T \in (t, t+\Delta \mid T > t)\} = \frac{f(t)\Delta}{1 - F(t)} \tag{2}$$

where Δ is an infinitesimal time interval. We have $h(S(t)) = -\frac{dln(1-F(t))}{dt}$. The term $h(S(t))\Delta$ specifies the conditional probability that a harmful event will occur between $[t, t+\Delta]$, given that the event has not occurred by time t. A formal specification for probability distribution and density functions gives

$$F(t) = 1 - exp\left(-\int_0^t h(S(\tau))\, d\tau\right) \; and \; f(t) = h(S(t))\,[1 - F(t)] \tag{3}$$

[6]We exclude mining and oil industries from our definition of natural capital.

[7]An interesting real-world example is the reforestation project in Samboja Lestari conducted by the Borneo Orangutan Survival Foundation. The project helped to increase rainfall by 25% and to avoid droughts by lowering the air temperature by 3 degrees Celcius to 5 degrees Celcius . (Boer (2010), Normile (2009))

We assume that the hazard function converges to a constant $\bar{h}$ when the natural resource stock tends to infinity.

A.3 The endogenous hazard rate $h(S) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable with the following properties ; $h(S) > 0$, $h_S(S) < 0$.

Since $S(t)$ is a beneficial state, the endogenous hazard rate h is a non-increasing function (a higher environmental quality stock entails a lower occurrence probability). We do not make any assumptions on for the second derivative of the endogenous hazard rate.

Recurrent events are repeated events such as droughts and floods that occur frequently in our current society and entail an immediate damage $\bar{\psi}$. In addition, the economy is always under the risk of more events occurring later, with value function $V(S)$. The post-value function describing the economy with recurrent is defined as

$$\varphi(S) = V(S) - \bar{\psi} \tag{4}$$

Given the uncertain arrival time T, the exploitation policy $c(t)$ yields the following payoff

$$\int_0^T u(c(t))\, e^{-\rho t} dt + \varphi(S(T))\, e^{-\rho T} \tag{5}$$

where ρ is the social discount rate. Taking expectations of the expression (5) according to the distribution of T and considering (3) gives the expected payoff

$$V(S) = \max_{c(t)} \int_0^\infty \left[u(c(t)) + h(S(t)) \left[V(S(t)) - \bar{\psi}\right]\right] \exp\left(-\int_0^t \left[\rho + h(S(\tau))\right] d\tau\right) dt \tag{6}$$

The maximization problem (6) at hand is deterministic, yielding a value that corresponds to the maximum expected value of the uncertainty problem (see Appendix A.1). The uncertain time T of the harmful event arrival and assuming a constant value of harmful damage $\bar{\psi}$ introduce a modelling asymmetry. However, we can interpret the term $\bar{\psi}h(S)$ as the expected harmful damage.

The solution of maximizing (6) with respect to the evolution of environmental stock (1) leads to the Keynes-Ramsey rule (see Appendix (A) for details),

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)}\right] \tag{7}$$

When harmful events are recurrent, the economy adopts precautionary behavior due to the presence of the precaution term $\frac{\bar{\psi}h_S(S)}{u_c(c)}$ in equation (7). [8]

By using equations (1) and (7), the steady state of the economy can be described by the function $G(S)$.

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(R(S))} \tag{8}$$

Note that the roots of the function $G(S)$ are steady-states of the economy.

Proposition 1. *(i) The necessary condition for multiple equilibria in an economy exposed to an endogenous hazard rate is the following: If $A = \{S : G(S) = 0\}$, then $card(A) > 1$. If the hazard rate is exogenous, the multiplicity of equilibria is not a possible outcome.*

[8] This result is similar to that presented by de Zeeuw and Zemel (2012), by which the authors show that recurrent events induce a precautionary behavior.

(ii) The sufficient conditions to have an environmental trap (i.e, multiple equilibria) for recurrent events are $G\left(\widetilde{S}\right) > 0$ and $\exists S < \widetilde{S}$, $G(S) < 0$ where $\widetilde{S}$ is a value between S_{mid} and S_{high} and also

$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi} h_{SS}(S)}{u_c(R(S))} + \frac{\bar{\psi} h_S(S)\, u_{cc}(R(S))\, R_S(S)}{\left(u_c(R(S))\right)^2} > 0 \tag{9}$$

Proof. *See Appendix (B)*

The cardinality (with notation $card(A)$) of a set is a measure for the number of elements of the set A [9]. The first part of the sufficient conditions holds for the existence of low steady state. The second part ensures the existence of a high steady state (see Appendix (B) for the existence of three different steady states).

Without the precaution term $\frac{\bar{\psi} h_S(S)}{u_c(c)}$, the function $G_S(S)$ becomes a negative term due to A.1 and the economy always admits a unique equilibrium. The explanation for the mathematical expressions in (9) are as follows: The first term relates to the convexity of regeneration function, which is a negative term. The second term represents the convexity of the endogenous hazard rate[10]. The more sensitive the endogenous hazard rate is to marginal changes of environmental quality, the greater the chance that $G_S(S) > 0$. The third term relates to the concavity of the utility function. A more concave utility function (i.e., higher risk aversion) results in decreases in $G_S(S)$ if $R_S(S) > 0$.

Overall, the multiple equilibria condition depends on the close relationship between the endogenous hazard rate and consumption. The precaution term $\frac{\bar{\psi} h_S(S)}{u_c(c)}$ in equation (7) is the expression representing the second trade-off between present consumption and harmful event risk which is the source of environmental traps. If the constant penalty rate is equal to zero, the trade-off disappears[11]. Therefore, the economy admits a unique equilibrium.

Lemma 1. *The low and high steady-states S_{low} and S_{high} are saddle path stable. However, S_{mid} could have complex eigenvalues*[12].

Proof. *See Appendix (D)*

This means that the economy could converge to either a high or low environmental quality equilibrium. Once the economy reaches either a high or low equilibrium, it remains in this state permanently. When an economy reaches a low-quality equilibrium, it is said to be "trapped" in a state where the consumption and environmental quality are low relative to an economy with a high-quality equilibrium.

2.1 Phase Diagram Analysis

Considering equation (7), we can observe that the steady state curve $\dot{c} = 0$ is non-linear on a phase plane (S, c) due to the endogenous hazard rate . Therefore, multiplicity of equilibria is a possible outcome[13].

We present two different phase diagram configurations[14] on a plane (S, λ) where λ is the shadow price for environmental quality. The first case is for multiple equilibria with a unique path. This means that, given

[9] For example, when we note $card(A) = 3$, it means that the set contains three elements.

[10] Assume that the endogenous hazard rate has a convex form.

[11] We have the same dynamics as in a classical Ramsey-Cass-Koopmans model, where the economy admits a unique equilibrium.

[12] This lemma is related to the analysis of the multiple equilibria with a unique growth path. This concerns the local stability of steady-states.

[13] If the steady state curve $\dot{c} = 0$ is not non linear but vertical, there is always a unique equilibrium

[14] Of course, there can be different kinds of phase diagrams. However, the analyses for other phase diagram configurations are not different than what we present in this section.

an initial value of S, there is only one reachable steady state and it is optimal to pursue this steady-state. The second case is where, given an initial value of S, multiple growth paths exist. We conduct the phase diagram analysis on a plane (S, λ) and not (S, c) because differentiating Hamiltonian with respect to a co-state variable (here λ) directly gives the isocline $\dot{S} = 0$ (See Appendix (A) for details). Then, it is easier to find the value of different optimal trajectories and compare them to find the global optimum[15].

Of course, our analysis in following sections concentrates on the case of multiple equilibria with a unique growth path, since all steady-states are optimal and relevant. However, it is worthwhile to present the phase diagram with multiple growth paths.

First, we present the dynamics of the co-state and state variables (see Appendix (A))

$$\dot{\lambda} = -\lambda \left(R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{\lambda} \right) \tag{10}$$

$$\dot{S} = R(S) - c(\lambda) \tag{11}$$

2.1.1 Multiple equilibria: Unique growth path

Figure 1 shows that S_{mid} acts as a threshold. An economy starting with an initial stock of natural resources $S_0 < S_{\text{mid}}$ always converges to S_{low}, while an economy which starts with $S_0 > S_{\text{mid}}$ converges to S_{high}. There are two convergence groups and the choice between them is made depending on history (initial condition).

The phase diagram with a unique path is as follows

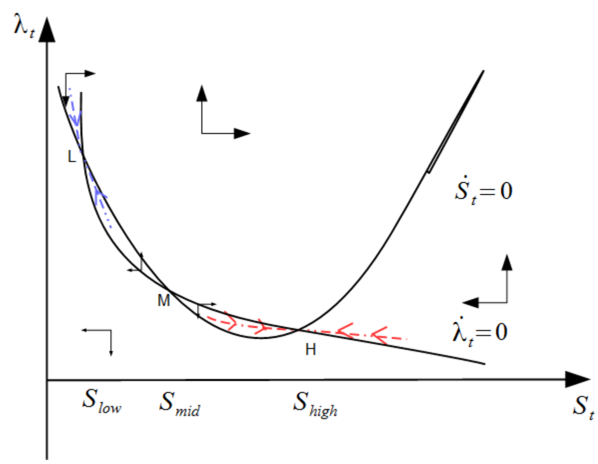


Figure 1: Phase diagram on the plane (S, λ): unique growth path

The economic explanation in Figure 1 is as follows[16]: Postponing consumption is too costly for survival $(u_c(0) = \infty)$ for very poor agents, and preferences are directed toward the present. In this case, the weight of the precaution term moves toward zero, and we have the dynamics of a standard Ramsey growth model with a unique equilibrium. Conversely, if the agents rich enough, they tend to be very precautious regarding the environment, and the economy would assume a unique equilibrium. However, when agents are neither very poor nor very rich, multiplicity of equilibria occurs.

[15]This is particularly important for the second phase diagram that we present in this study.

[16]Note that Figure 1 does not stem from a numerical exercise and stands for representative purposes in order to show the optimal dynamics on a single graphic.

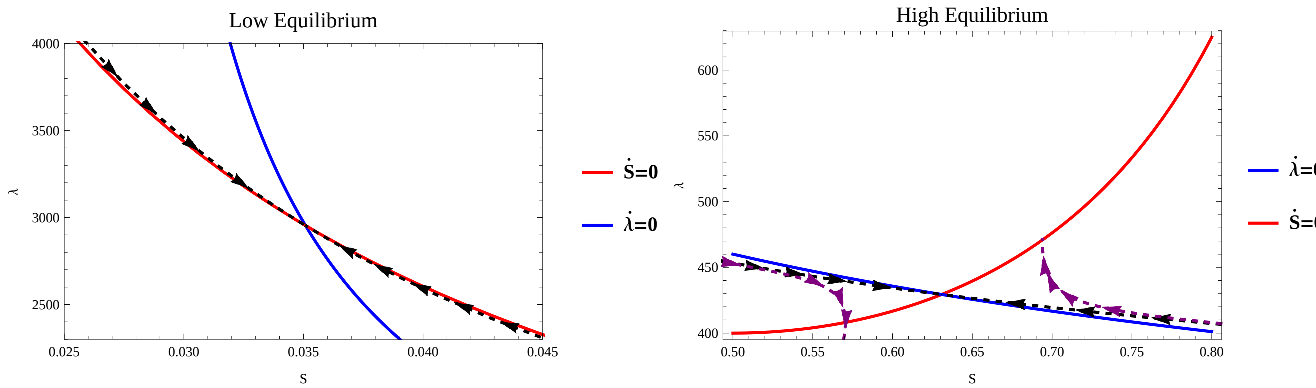


Figure 2: Phase diagram with converging trajectories for low and high equilibrium

For the sake of clarity, we present trajectories for both equilibria separately and show that different optimal converging paths exist to both low and high steady states. Figure 2[17] shows that depending on the initial conditions, either the economy converges to a steady state or diverges from a steady state. The phase diagram analysis clearly shows that the existence of different convergence groups rules out the possibility of global stability in the model.

When there is a multiplicity of equilibria, an important issue concerns the stability of the equilibrium and the corresponding trajectories converging to different equilibria. Wagener (2003) analyses the shallow lake dynamics and shows that there could be heteroclinic connections between different saddle points. In the existence of heteroclinic bifurcations, only one of the saddle points may be relevant (see Figure 3 in Wagener (2003)). In this study, Lemma 1 rules out the possibility of heteroclinic bifurcations.

2.1.2 Multiple equilibria: Multiple growth paths

When the Hamiltonian is not concave in S, the uniqueness of Pontryagin paths is no longer ensured. Therefore, starting with an initial value of natural resource stock, several paths may exist, and these may lead to different saddle points (i.e., there may be multiple reachable equilibria) (see Palivos (1995)). In this case, the values of different trajectories, starting from a given initial condition of S, should be compared to determine the global optimum (Skiba (1978), Davidson and Harris (1981)).

As illustrated in Figure 3 there are three steady states B, D, H.

[17]The parameters used for the simulation is as follows; $\sigma = 2.1$, $\rho = 0.0095$, $g = 0.01$, $\bar{S} = 1$, $\eta = 1.75$ $\bar{h} = 0.03$, $\bar{\psi} = 140$.

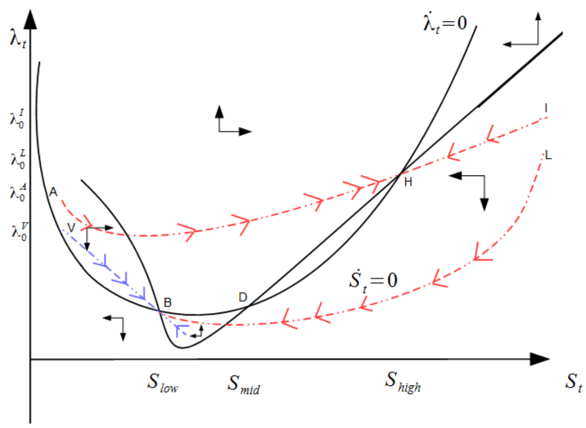


Figure 3: Phase diagram on the plane (S, λ): multiple growth paths

Figure 3 [18] shows that an economy can benefit from a high consumption level and low natural resource stock or vice versa (see equilibrium B and H). Consider an economy with an initial value of $S_0 < S_{\text{low}}$ (see figure 3). Between two feasible paths AH and VB, which one yields the global maximum? To answer this question, we calculate the value of paths V and compare them. The value of the infinite horizon problem, denoted $V(S_0)$ is given by

$$V(S_0) = \frac{\mathcal{H}(S_0, \lambda_0)}{\rho} \tag{12}$$

where $\mathcal{H}$ is the Hamiltonian associated with the infinite horizon program presented in (6) (See Appendix (A) for details). From (12), the choice between AH and VB is immediate. Given that $\mathcal{H}_\lambda = \dot{S} = R(S) - c(\lambda) > 0$ for all λ above $\dot{S} = 0$ locus and that $\lambda_0^A < \lambda_0^V$ (i.e, $c_0^A > c_0^V$), the optimal solution is to take path AH. Lemma 1 implies that it is impossible to be optimal for one country to start at $S_0 < S_{\text{low}}$ and to move toward H while another country starts at $S_0 > S_{\text{high}}$ and moves toward B.

For the case with multiple reachable steady states, one would like to know whether there is a threshold at which the economy is indifferent toward going to either of these steady states. This leads us to perform an analysis to find a Skiba point.

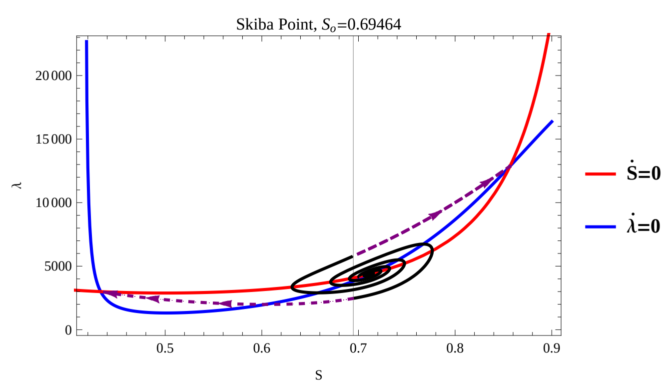


Figure 4: Skiba point with trajectories converging to saddle-point equilibria

[18] Figure 3 does not stem from a numerical exercice. It only aims to show clearly the optimal trajectories and to understanding which trajectory is a global maximum.

Figure 4[19] is line with the work of Wagener (2003) in which it is shown that a Skiba point may not be associated with an unstable equilibrium. Departing from the initial condition $S_0 = 0.69464$, the value of the optimal path on the left and on the right of Figure 4 (see thick dashed lines) is the same[20]. The existence of an indifference point is understandable; Either the economy prefers having higher consumption with a lower natural resource stock or vice versa.

Proposition 2. *$S^*_{high} - S^*_{NR} > S^*_{low} - S^*_{NR}$ where S^*_{NR}, S^*_{high} and S^*_{low} are the optimal steady-state value of natural resources in an economy without risk and the optimal steady-state for high and low environmental quality in an economy with risk, respectively. This means that a high environmental quality equilibrium economy adopts a more conservative exploitation policy than low environmental quality equilibrium economy.*

Proof. *See Appendix (E)*

According to the precaution term in equation (7), when there are recurrent events, the endogenous hazard rate always makes an economy more precautious in terms of resource exploitation (see de Zeeuw and Zemel (2012), Tsur and Zemel (2016), Polasky et al. (2011), Clarke and Reed (1994)).

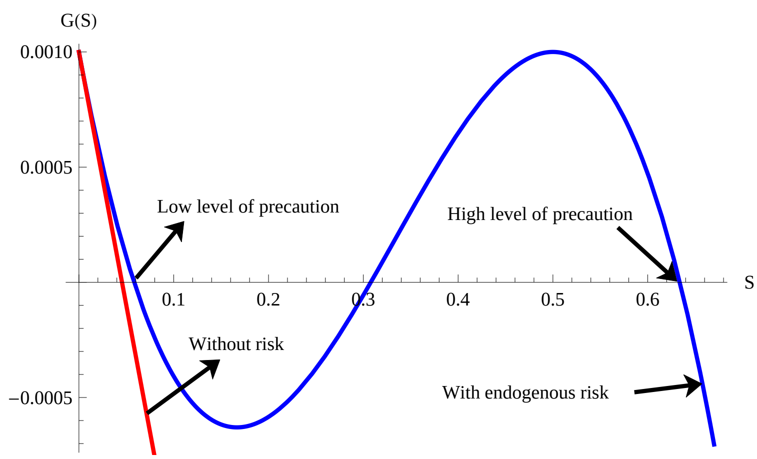


Figure 5: Different reaction against the endogenous hazard rate

As seen in Figure 5, the novelty of this proposition with respect to the literature is that the endogenous hazard rate leads to different levels of precaution (different steady states) against harmful events when there is a multiplicity of equilibria.

3 Can environmental policy cause/prevent an environmental trap?

The focus of our study is to determine the effects of adaptation and mitigation policies on environmental traps. To facilitate an understanding of their mechanisms, we prefer to analyze separately the effects of adaptation and mitigation policies on environmental traps.

[19]The parameters used for the simulation is as follows; $\sigma = 2.1$, $\rho = 0.015$, $g = 0.09$, $\bar{S} = 1$, $\eta = 12$ $\bar{h} = 0.5$, $\bar{\psi} = 715$. The functional forms for the simulation of the phase diagram are given in section 4.

[20]The Skiba point is the point where both value functions of optimal trajectories leading to low and to high steady-state are equal. Then, the economy is indifferent toward converging to either of these steady states (see Haunschmied et al. (2003)). The numerical calculation with the calibration given in footnote 19 yields the Skiba point $S_0 = 0.69464$.

3.1 An economy with an adaptation-only policy

A social planner could reduce the damage ψ via adaptation capital K_A. This way of modeling adaptation is consistent with Zemel (2015) and Tsur and Zemel (2016). However, this differs from the work of Bréchet et al. (2012) and Le Kama and Pommeret (2016), where the adaptation capital directly affects the damage function for all time t. In our model, adaptation plays a *proactive* role. Hence, the tangible benefits of adaptation can be gained only if a harmful event occurs. However, this is not to say that investing in the adaptation decision does not make a difference. Its contribution is accounted for by the objective function of the social planner. Investing at rate A contributes to adaptation capital K_A which is reduced by constant depreciation $\delta > 0$. The evolution of the adaptation capital is as follows

$$\dot{K}_A(t) = A(t) - \delta K_A(t) \tag{13}$$

with $K_A(0)$ given. Damage function $\psi(K_A)$ decreases when adaptation capital K_A increases. We assume that

A.4 The damage function is characterized by $\psi(.)$: $\mathbb{R}_+ \to \mathbb{R}_+$, $\psi(0) = \bar{\psi}$, $\psi(\infty) = \underline{\psi}$, $\psi(K_A) > 0$, $\psi_{K_A}(K_A) < 0$ and $\psi_{K_A K_A}(K_A) > 0$

When there is no adaptation capital, the inflicted damage will be a constant term. Moreover, it is realistic to assume that damage reduction has a limit, as we cannot eliminate the negative effects of a harmful event by accumulating adaptation capital. For that reason, we assume that the damage function is constrained within an upper $\bar{\psi}$ and a lower bound $\underline{\psi}$.

Investing in adaptation capital has the cost $Q^1(A) = \phi_A \frac{A^2}{2}$ with the unit cost of adaptation ϕ_A which has a convex form. The cost function for adaptation enters in the utility function as a social cost. This is in line with Zemel (2015) and Tsur and Zemel (2016). Consumption $c(t)$ provides utility $u(c(t))$. The expected payoff is

$$\begin{aligned} V(S, K_A) = \max_{\{c(t), A(t)\}} \int_0^\infty & \left[u(c(t)) - Q^1(A(t))\right. \\ & \left. + h(S(t))\left[V(S(t), K_A(t)) - \psi(K_A(t))\right]\right] \exp\left(-\int_0^t \left[\rho + h(S(\tau))\right] d\tau\right) dt \end{aligned} \tag{14}$$

$V(S, K_A)$ stands for the value of the maximization problem. The optimal policy is to maximize (14) subject to (1) and (13) (see Appendix F). Optimal dynamics for consumption and adaptation are as follows

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{u_c(c)}\right] \tag{15}$$

$$\dot{A} = \frac{Q^1_A(A)}{Q^1_{AA}(A)}\left(\rho + \delta + \frac{h(S)\psi_{K_A}(K_A)}{Q^1_A(A)}\right) \tag{16}$$

When an economy invests in an adaptation policy, we have an additional equation (16), which shows the optimal dynamics of adaptation investment.

In order to assess the effect of adaptation capital on environmental traps, we calculate the necessary conditions for multiple equilibria (see Appendix (F)).

$$G_S(S) = R_{SS}(S) - \frac{h_{SS}(S)\,\psi(K_A)}{u_c(R(S))} + \frac{h_S(S)\,\psi(K_A)\,u_{cc}(R(S))\,R_S(S)}{(u_c(R(S)))^2} - \underbrace{\left[\frac{\psi_{K_A}(K_A)\,h_S(S)}{u_c(R(S))}\right]\frac{dK_A}{dS}}_{=Z_1} > 0 \tag{17}$$

where the term Z_1 stands for the effect of adaptation capital on the necessary condition for the environmental trap.

To assess the effect of this term, we must understand how environmental quality changes the adaptation capital level at steady state. To do this, we write equation (16) at the steady-state

$$Q_A^1(\delta K_A)(\rho+\delta) = -\psi_{K_A}(K_A)\,h(S) \tag{18}$$

which defines the function $K_A(S)$. The right-hand side $-\psi_{K_A}(K_A)\,h(S)$ can be interpreted as the marginal benefit of adaptation capital, and the left-hand side $Q_A^1(\delta K_A)(\rho+\delta)$ is the marginal cost of adaptation capital. Optimal steady states are located on this curve. We can find the slope of the steady state curve by taking the total derivative of equation (18)

$$\frac{dK_A}{dS} = \frac{h_S(S)\,\psi_{K_A}(K_A)}{(\rho+\delta)\,Q_A^1(\delta K_A) + \psi_{K_A K_A}(K_A)\,h(S)} < 0 \tag{19}$$

Proposition 3. *A higher adaptation capital $K_A(S)$ makes the necessary condition $G_S(S) > 0$ easier to hold, meaning that the possibility of multiple equilibria increases with adaptation capital.*

Proof. See Appendix (F)

Because we have $\frac{dK_A}{dS} < 0$, the term Z_1 is negative; this means that the possibility of multiple equilibria increases with an increase in adaptation capital.

According to equation (19), a higher level of environmental quality requires a lower adaptation capital level at a steady state. The numerator in (19) can be considered the variation of the marginal benefit of adaptation with respect to environmental quality, which is a negative term. This means that the marginal benefit of adaptation decreases as the environmental quality level increases. The first and second terms in the denominator represent the marginal cost of adaptation capital and concavity of penalty function with respect to adaptation capital.

Another interpretation of this result can be as follows: Because agents expect to face less damage with an adaptation policy and can more easily bear the negative consequences of a harmful event, they tend to care less about environmental quality.

How can we explain the multiplicity of equilibria when the economy implements an adaptation policy? When there is investment in adaptation capital, environmental quality decreases as shown in (19), and the endogenous hazard rate increases. As a result, due to a higher impatience level, an economy becomes less precautionary in exploiting natural resources. It follows that the endogenous hazard rate amplifies again, and the need for adaptation capital increases again[21]. This mechanism, causing multiple equilibria, explains how an adaptation-only policy may trap a country into a low welfare state.

Indeed, the trade-off between present consumption and a harmful event becomes more significant when an adaptation policy is in place because the endogenous hazard rate increases.

[21]Because the endogenous hazard rate increases, the marginal value of adaptation capital increases.

3.2 An economy with a mitigation-only policy

We now show that an economy implementing only a mitigation policy could escape an environmental trap. Because consumption comes from environmental assets, improving environmental quality (mitigation) increases the consumption level in the long run.

Investing at rate M for mitigation improves environmental quality by $\Gamma(M)$. Then, in the presence of mitigation activity, environmental quality evolves according to

$$\dot{S}(t) = R(S(t)) + \Gamma(M(t)) - c(t) \tag{20}$$

where $\Gamma(M)$ represents the effects of mitigation activities such as reforestation, desalination of water stock, enhancing carbon sinks, etc. Mitigation is defined as a "human intervention to reduce the sources or enhance the sinks of greenhouse gases." (IPCC (2014), p.4) In this sense, reforestation can be considered as a means to enhance carbon sinks because forests allow for carbon sequestration.

The specification of the mitigation variable is similar to that proposed in Chimeli and Braden (2005). Alternatively, function $\Gamma(M)$ can be considered an "environmental protection function". The expenditures for environmental protection may be directed not only toward pollution mitigation but also toward the protection of forests and the recovery of degraded areas. As such, mitigation activity can be seen as a means of improving environmental quality.

To keep the model as simple as possible, we choose to consider mitigation as a flow variable. We use the following assumption

A.4 The mitigation function given by $\Gamma(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, with the following properties ; $\Gamma_M(M) > 0$, $\Gamma_{MM}(M) < 0$.

The mitigation function is assumed to be an increasing and concave function. Mitigation activity can be considered a complement to the regeneration of the environment. Investing in mitigation activity has the cost $Q^2(M) = P_M M$ with the unit price for mitigation P_M that enters in the utility function as a social cost. In an economy with mitigation activity, the expected payoff is

$$V(S) = \max_{\{c(t),M(t)\}} \int_0^\infty \left[u(c(t)) - Q^2(M) \ + h(S(t))\left[V(S(t)) - \bar{\psi}\right]\right] \exp\left(-\int_0^t \left[\rho + h(S(\tau))\right] d\tau\right) dt \tag{21}$$

Assuming a concave function for mitigation activity $\Gamma(M)$ gives optimal interior solutions and simplifies calculations.

The optimal policy is to maximize (21) subject to (20) (see details in Appendix (H)). Optimal dynamics for consumption and mitigation are as follows

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)}\right] \tag{22}$$

$$\dot{M} = -\frac{u_{cc}(c)}{u_c(c)}\frac{\mathit{\Gamma}_M(M)}{\mathit{\Gamma}_{MM}(M)}\left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)}\right] \tag{23}$$

In fact, mitigation activity renders the trade-off between present consumption and the endogenous hazard rate less binding. It follows that there is a lower risk of an economy being trapped in a multiplicity of steady states.

Due to the concavity of the mitigation function, we notice that the signs of $\dot{c}$ and $\dot{M}$ are different. It follows that the optimal level of mitigation activity comes at the cost of lower consumption. This also explains that the economy causes less harm to the environment when mitigation activity is invested in. Consequently, the presence of mitigation activity weakens the trade-off between present consumption and the endogenous hazard rate.

We provide the necessary condition for multiple equilibria by using equations (22) and (23) (Appendix (G) for details)

$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)} + \frac{\bar{\psi}h_S(S)\,u_{cc}R_S(S)}{(u_c(c))^2} + \underbrace{\frac{\bar{\psi}h_S(S)\,u_{cc}R_S(S)}{(u_c(c))^2}\frac{dM}{dS}}_{=Z_2} > 0 \tag{24}$$

In the above equation, Z_2 represents the effect of mitigation on the necessary condition for multiple equilibria.

We look at how environmental quality levels change the steady state level of mitigation investment. To do that, we write the first-order condition for mitigation activity (see equations (G.3) and (G.4) in Appendix (G))

$$u_c(R(S) + \Gamma(M))\,\Gamma_M(M) = Q_M^2(M) = P_M \tag{25}$$

and take the total derivative of (25) to find

$$\frac{dM}{dS} = -\frac{u_{cc}(c)\,R_S(S)\,\Gamma_M(M)}{u_{cc}(c)\,(\Gamma_M(M))^2 + u_c(c)\,\Gamma_{MM}(M)} < 0 \tag{26}$$

The economic reasoning behind equation (26) is as follows: When environmental quality is high, the social planner decreases mitigation. In other words, the economy needs less mitigation as environmental quality improves.

Proposition 4. *Mitigation activity M decreases the possibility of multiple equilibria, since the necessary condition for multiple equilibria $G_S(S) > 0$ is less likely to hold.*

Proof. See Appendix (G)

Because we have $\frac{dM}{dS} < 0$, the term Z_2 is negative and decreases $G_S(S)$. This implies that it is more difficult to ensure the multiple-equilibria condition (24).

To better understand how mitigation can prevent an environmental trap, suppose that mitigation activity could reduce the endogenous hazard rate to zero. It follows that the trade-off between present consumption and a harmful event, which causes multiple equilibria, disappears. Based on this, one can understand that mitigation activity weakens the trade-off between present consumption and a harmful event.

Obviously, mitigation activity not only protects the economy against an environmental trap but also increases the steady-state level of environmental quality, as expected. Indeed, since consumption comes from natural resource rents, mitigation policy allows the economy to increase its consumption level. Hence, the impatience level of low-income countries decreases, allowing them to postpone their consumption and to make far-sighted decisions. In summary, a mitigation policy could break the vicious cycle of low consumption and environmental quality that can be triggered by the implementation of an adaptation-only policy. Then, one can conclude that social planners should couple an adaptation policy with a mitigation policy.

4 Numerical analysis

The section aims to illustrate the theoretical findings obtained in previous sections for the effect of adaptation and mitigation on the occurrence of multiple equilibria in the previous sections. For the numerical part, we calibrate the benchmark economy according to New Zealand's economy, which is a rare country in that it publicly announces the likelihood of harmful events such as floods and droughts.

By doing a calibration exercise, we produce a model that matches the EPI index[22] and the endogenous hazard rate that is computed with real data for New Zealand [23]. We use the following functional specifications.

Natural Regeneration Function : $R(S) = gS\left(1 - \frac{S}{\bar{S}}\right)$	Utility function : $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$
Source : Ren and Polasky (2014)	Source : Bommier et al. (2015)
$\bar{S}$ Carrying capacity of environment g Intrinsic growth rate of the resource stock	 σ Degree of relative risk aversion
harmful Damage : $\psi(K_A) = \bar{\psi}\left(\omega + (1-\omega)e^{-\gamma K_A}\right)$	Endogenous Hazard Function : $h(S) = \frac{2\bar{h}}{1+exp\left[\eta\left(s/\bar{S}-1\right)\right]}$
Source : Bréchet et al. (2012)	Source : Ren and Polasky (2014)
$\bar{\psi}$ Damage rate without adaptation policy ω Lower bound of damage when $\psi(\infty)$ γ Elasticity of adaptation w.r.t to damage rate	$\bar{h}$ Upper bound for hazard rate η Endogeneity level of harmful event $\bar{S}$ Carrying capacity of environment
Mitigation function : $\Gamma(M) = M^{\alpha}$	Cost of adaptation investment : $Q_1(A) = \phi_A \frac{A^2}{2}$
Source : Le Kama and Pommeret (2016)	
α Elasticity of mitigation activity	ϕ_A Parameter for the change of marginal cost of adaptation

4.1 An economy with mitigation-only policy

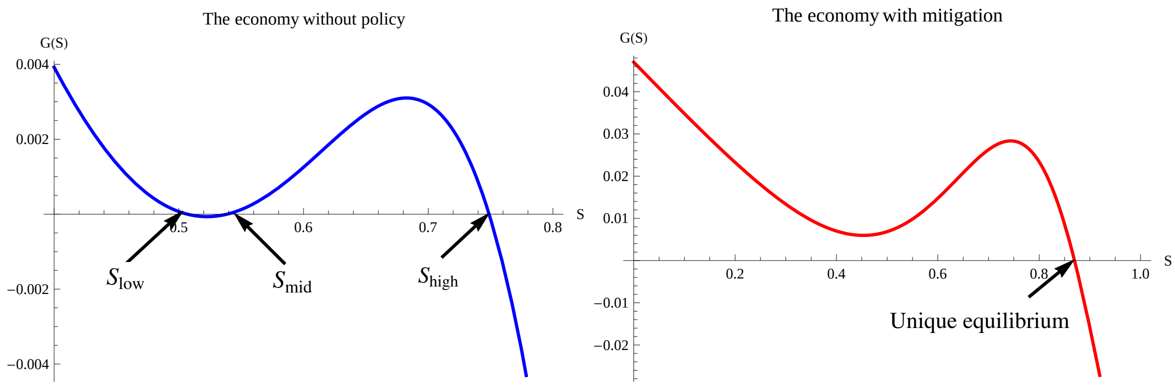
We present a multiple-equilibria economy without a policy and an economy with a mitigation policy. To understand the implications of mitigation on the economy with multiple equilibria, we first calibrate the benchmark model with multiple equilibria to match the features of New Zealand's economy. We calibrate the damage parameter $\bar{\psi}$ and the intrinsic growth rate of the resource stock g such that the high steady-state level of environmental quality S matches the environmental performance index (EPI) score of New Zealand in 2018. We calibrate the important parameters of the endogenous hazard rate (the endogeneity of the hazard η, and the upper bound for hazard rate $\bar{h}$) according to the estimates of New Zealand's government, [24] which show that the probability of a flood occurring in any 30-year period is 45%. For the elasticity of substitution, we choose the conventional value used in the literature, which is 2 and the pure rate of time preference ρ is set to 0.015 as in Nordhaus (2008). We assume that the elasticity of mitigation and the unit price for mitigation activity are equal to 0.5 and to 250, respectively.

[22]The EPI index gives a score for each country between 0 and 100. Because we use a logistic growth function with carrying capacity set to 1 in our calibration, we have a value of S between 0 and 1. Hence, if a country has a score of 60, we take it as 0.60. The EPI score of New Zealand is 0.75 in 2018.

[23]We refer to EMDAT International Disaster Database (2015).

[24]https://www.ecan.govt.nz/your-region/your-environment/natural-hazards/floods/flood-probabilities/

Parameter	Calibrated value
$\bar{S}$	1.01
g	0.062
$\bar{\psi}$	1315
$\bar{h}$	0.25
η	7.8
α	0.5
P_M	250



(a) Multiple equilibria

(b) Unique equilibrium

Figure 6: The benchmark economy vs the economy with mitigation

The most important message portrayed by Figure 6 is that mitigation activity enables an economy to avoid the occurrence of multiple equilibria as shown by Figure 6b. The reason for this result is that mitigation activity reduces the endogenous hazard rate. It follows that the trade-off between present consumption and the endogenous hazard rate which is the source of the multiplicity of equilibria weakens. Hence, the economy avoids the multiplicity of steady states.

Figure 6b also shows that mitigation activity increases environmental quality. It is easy to see on Figure 6b that the unique steady-state level of environmental quality is greater than that of all three steady-state levels of environmental quality on Figure 6a.

Figure 6a shows that, on the one hand, an economy with an initial stock of environmental quality $S_0 < S_{mid}$ converges to S_{low}. On the other hand, the economy converges to S_{high} if the initial stock of environmental quality is $S_0 > S_{mid}$. We can conclude that S_{mid} is unstable and can be considered as a threshold[25] separating two stable steady state S_{low} and S_{high}.

Figure 6b shows that the economy avoids the multiplicity of equilibria by investing in mitigation. The economy admits a unique equilibrium and regardless of the initial stock of environmental quality, there is convergence to the unique steady-state.

4.2 The economy with adaptation-only policy

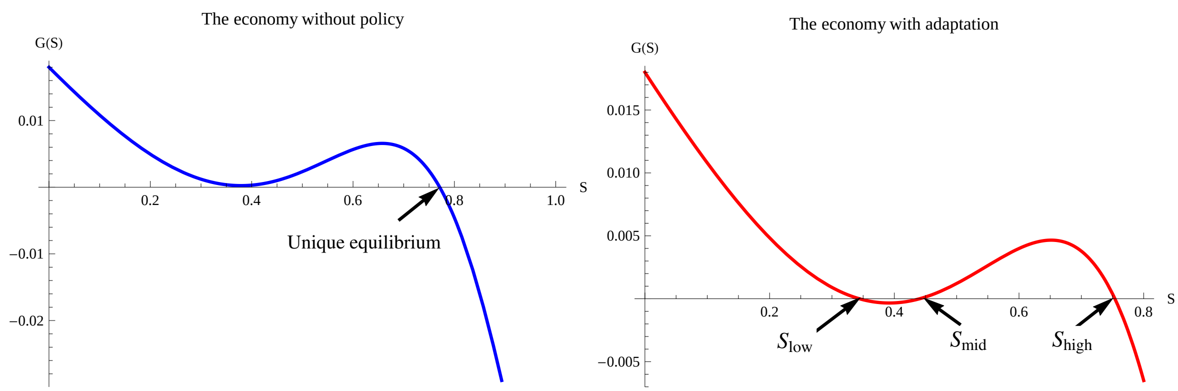
We present the benchmark model without a policy and an economy with an adaptation policy. For this numerical exercise, we calibrated the benchmark economy according to New Zealand's economy. As in the

[25]Note that the numerical example is in line with the phase diagram presented in section 2.1.1.

previous section, we calibrate the damage parameter $\bar{\psi}$ and the intrinsic growth rate of the resource stock g such that the steady-state level of environmental quality S equals the EPI score of New Zealand in 2018.

Parameter	Calibrated value
$\bar{S}$	1
g	0.038
$\bar{\psi}$	30000
$\bar{h}$	0.23
η	8
ω	0.95
γ	0.9
ϕ_A	1

As in the previous numerical exercise for mitigation, we set the elasticity of substitution σ to 2 and the pure rate of time preference ρ to 0.015 as in Nordhaus (2008). Unfortunately, we do not have any available data for the adaptation capital. Therefore, we set parameters ω to 0.95 and γ to 0.9.



(a) Unique equilibrium **(b)** Multiple equilibria

Figure 7: The benchmark economy vs the economy with adaptation

Figure 7a represents the unique equilibrium economy without environmental policy. Figure 7b shows that the economy finds itself in a multiple equilibria by only investing in adaptation capital. If the initial stock of environmental quality is $S_0 < S_{mid}$, the economy converges to S_{low}. When the initial stock of environmental quality is $S_0 > S_{mid}$, there is convergence to S_{high}.

The occurrence of the multiplicity of equilibria relies on the fact that adaptation capital decreases environmental quality (see equation (19)). As a result, the trade-off between present consumption and the endogenous hazard rate becomes more binding and the multiplicity of equilibria occurs. It is worth noting that S_{high} is lower than the environmental quality level in an economy without environmental policy, since adaptation capital tends to decrease environmental quality.

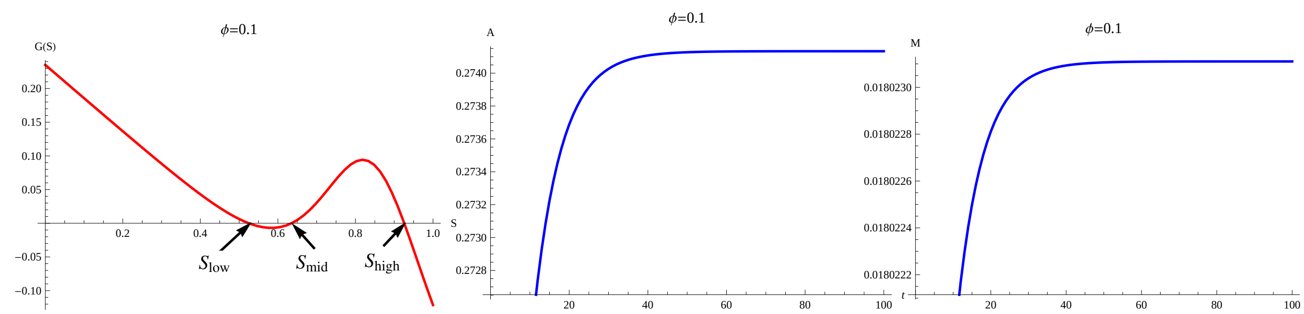
An important policy-based message of this result is that an adaptation policy should be coupled with a mitigation policy in order to avoid environmental traps.

4.3 Adaptation vs mitigation: what about the multiplicity of equilibria?

An important issue regarding adaptation and mitigation policies is the matter of how the trade-off between adaptation and mitigation affects the occurrence of multiple equilibria. The adaptation/mitigation trade-off is intensively discussed in the existing literature. Buob and Stephan (2011) argue that a country's stage of development matters and that high-income countries should invest both in adaptation and mitigation while low-income countries should only invest in adaptation. Bréchet et al. (2012) studied the optimal mix of adaptation and mitigation and conclude that the substitutability between adaptation and mitigation depends on the country's stage of development. A recent study by Tsur and Zemel (2015) focuses on the optimal mix of adaptation and mitigation under harmful event uncertainty, but it remains agnostic about the implications of the trade-off between adaptation and mitigation on the multiplicity of equilibria. We seek to analyze this issue when an economy is subject to an endogenous hazard rate (see Appendix H for calculations).

In this subsection, apart from the calibration, the unit cost of adaptation ϕ is a free parameter. To assess the role of the trade-off between adaptation and mitigation on the occurrence of multiple equilibria, we present two cases with low and high unit cost of adaptation ϕ. Consequently, in the first case, the optimal level of adaptation investment is higher than in the second case.

Figure 8a shows the existence of the multiplicity of equilibria in an economy with adaptation and mitigation policies. Figures 8b and 8c represent the optimal path of adaptation and mitigation corresponding to the economy with a high environmental quality level in the long run. We observe that both adaptation and mitigation increase from the start date and converge to a stable high-environmental-quality equilibrium[26].

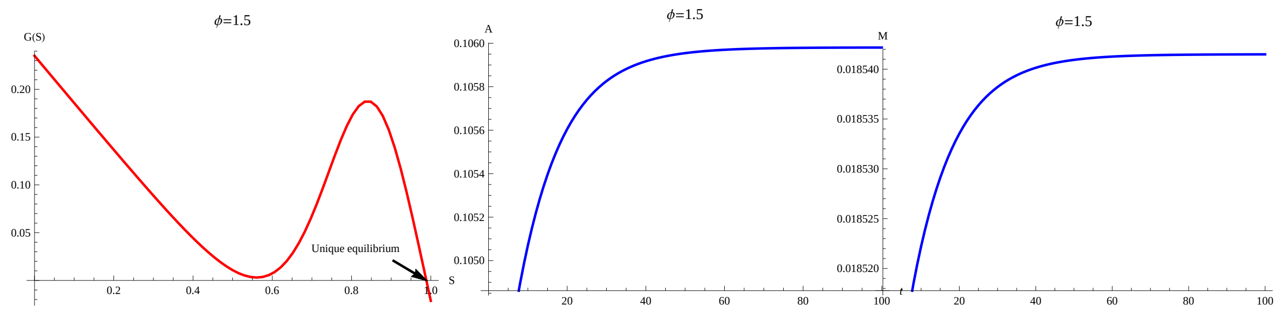


(a) Multiple equilibria **(b)** Adaptation **(c)** Mitigation

Figure 8: Adaptation vs mitigation trade-off and multiple equilibria

Figure 9 shows that when the unit cost of adaptation ϕ is higher, the economy admits a unique equilibrium. When we look at Figures 9b and 9c, both adaptation and mitigation activities increase over time. Compared to the previous case, the economy shifts its resources from adaptation to mitigation due to the higher unit cost of adaptation ϕ. We see that the multiplicity of equilibria disappears when the mitigation activity is relatively higher than adaptation.

[26]Similarly, the convergence to S_{low} can be shown.



(a) Unique equilibrium **(b)** Adaptation **(c)** Mitigation

Figure 9: Adaptation vs mitigation trade-off and unique equilibrium

Note that both adaptation and mitigation aim to decrease the disutility of the presence of the endogenous hazard rate. The former does this by decreasing the level of vulnerability to inflicted damage, the latter by decreasing the endogenous hazard rate. As such, the choice between adaptation and mitigation depends on the relative costs of these policies. When the cost of adaptation increases (i.e, a higher unit cost ϕ), it is evident that the economy starts to give more weight to mitigation than to adaptation in order to cope with the endogenous hazard rate.

As shown, a higher adaptation cost not only affects the dynamic trade-off between adaptation and mitigation but also has qualitative implications, such as those related to the occurrence of environmental traps. When there is more weight given to mitigation activity, the multiplicity of equilibria disappears. The reason for this result is that the trade-off between present consumption and the endogenous hazard rate of harmful events becomes less important.

5 Conclusion

In this paper, we analyzed the effects of adaptation and mitigation policies on environmental traps in an economy subject to an endogenous hazard rate. This study offers a new explanation for environmental traps via endogenous hazard rate and increases the overall understanding of how environmental policy causes or prevents environmental traps. We believe that this new perspective also provides interesting advice to policymakers regarding the opposite effects of adaptation and mitigation policies.

Our main results show that an adaptation policy can lead the economy into an environmental trap, whereas mitigation helps the economy to avoid environmental traps. We show that a new trade-off appears between adaptation and mitigation with respect to their effects on environmental traps. This trade-off is different from the trade-off between adaptation and mitigation over time mentioned in numerous studies (see Zemel (2015), Tsur and Zemel (2015), Bréchet et al. (2012)). The fact that adaptation policy could cause an environmental trap does not mean that social planners should not invest in adaptation activities. On the contrary, since it is impossible to eliminate the endogenous hazard rate, policymakers should invest in adaptation capital but should couple this policy with mitigation activity to avoid the adverse effects of an adaptation-only policy. This is because mitigation activity weakens the trade-off between present consumption and harmful events by improving environmental quality.

Future research could test the model using empirical methods. Currently, this is very challenging, as there are no available data on adaptation investments. However, it is very desirable and is a crucial part of our future research agenda.

Appendix 1

A* Examples with functional forms

This section aims to present examples for the existence of the multiplicity of equilibria which is an issue related to functional forms and to parameter values. The primary purpose of examples is to prove the existence of multiple equilibria rather than giving necessary conditions to ensure its existence.

Example 1: An economy without environmental policy

The regeneration of the environment, the endogenous hazard rate, and the utility function are given as follows:

$$R(S) = g(1-S)S \tag{A*.1}$$

$$h(S) = \left(1 - \bar{h}S^2\right) \tag{A*.2}$$

$$u(c) = log(c) \tag{A*.3}$$

where g is the intrinsic growth rate of the environmental quality and $0 < \bar{h} < 1$ shows the extent to which the endogenous hazard rate depends on the environmental quality level. At the steady state, equation (7) can be written as

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi}h_S(S)}{u_c(c)} = 0 \tag{A*.4}$$

By using the functional specifications, we reformulate equation (A*.4) combined with (1) at the steady state

$$-2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 - 2gS + (g-\rho) = 0 \tag{A*.5}$$

The sufficient condition to have three positive real roots for the third-degree polynomial (A*.5) is as follows[27]:

$$\frac{\left(4g^2 + 36g(g-\rho) - 27(g-\rho)^2\right) - \sqrt{\left(4g^2 + 36g(g-\rho) - 27(g-\rho)^2\right)^2 - 2^9g^3(g-\rho)}}{8(g-\rho)2g\bar{h}}$$
$$< \bar{\psi} < \frac{\left(4g^2 + 36g(g-\rho) - 27(g-\rho)^2\right) + \sqrt{\left(4g^2 + 36g(g-\rho) - 27(g-\rho)^2\right)^2 - 2^9g^3(g-\rho)}}{8(g-\rho)2g\bar{h}} \tag{A*.6}$$

Proof. See Appendix (I)

When the damage rate $\bar{\psi}$ is too high, the economy adopts precautionary behavior because the marginal benefit of protecting the environment is high. On the other hand, when the damage rate is too low, the

[27] We assume that the quantities under the square root are positive. This is also confirmed numerically with plausible parameters.

economy tends to care little about the environment, and natural resources are exploited as a result, leading to a unique equilibrium. However, when the damage rate lies between a range of values that respects the condition (A*.6), the trade-off between harmful events and present consumption becomes relevant because the economy is neither very precautionary nor careless about the environment (see Wirl (2004)[28]).

Example 2: An economy with only an adaptation policy

To prove the existence of multiple equilibria in an economy with an adaptation policy, we start with an example of an economy without any environmental policies that admits a unique equilibrium and uses the following usual functional specifications:

$$R(S) = gS(1-S) \tag{A*.7}$$

$$h(S) = \left(1-\bar{h}S\right) \tag{A*.8}$$

$$u(c) = log(c) \tag{A*.9}$$

$$\psi(K_A) = \bar{\psi}(1-aK_A) \tag{A*.10}$$

$$Q(A) = \phi\frac{A^2}{2} \tag{A*.11}$$

For the sake of analytical tractability, differently from the previous example and from the definition given in A.3[29], we use a linear hazard function and show that such a function ensures a unique equilibrium for an economy without an environmental policy. Then, we prove that an adaptation policy results in a multiple-equilibria economy.

Under the specification of functional forms (A*.7), (A*.8), (A*.9), an economy without an environmental policy admits a unique steady-state. When the economy invests in adaptation activity, multiplicity of equilibria occurs if the following condition on harmful damage $\bar{\psi}$ holds

$$\frac{\left[\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)-\sqrt{\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)^2-2^9g^3(g-\rho)}\right]}{8(g-\rho)g\left(\bar{h}\right)^2} < \bar{\psi} < \frac{\left[\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)+\sqrt{\left(4g^2+36g(g-\rho)-27(g-\rho)^2\right)^2-2^9g^3(g-\rho)}\right]}{8(g-\rho)g\left(\bar{h}\right)^2} \tag{A*.12}$$

Proof. See Appendix (J)

This example clearly shows that an economy which is not initially exposed to multiple equilibria faces the possibility of environmental traps under an adaptation policy if condition ($A*.12$) is ensured.

Indeed, a policy recommendation based on increasing adaptation capital could cause a multiplicity of steady states and, in turn, trap an economy at a low equilibrium.

[28] Wirl (2004) explains that the economy faces multiple equilibria when it becomes difficult to steer between an economic goal (accumulating physical capital) and environmental goal (protecting amenities).

[29] In the numerical analysis, we use a penalty function which is in line with the definition given in A.4.

Example 3: An economy with only mitigation policy

To prove that mitigation can rid the economy of multiple equilibria, we start with an example of a benchmark economy with multiple equilibria and use the following usual functional specifications. In order to obtain analytical results, we use the following linear functional forms:

$$R(S) = gS(1-S) \tag{A*.13}$$

$$h(S) = \left(1 - \bar{h}S^2\right) \tag{A*.14}$$

$$u(c) = log(c) \tag{A*.15}$$

$$\mathit{\Gamma}(M) = M \tag{A*.16}$$

$$Q^2(M) = P_M \frac{M^2}{2} \tag{A*.17}$$

Another simplification is used only for this example in order to ease the calculations. This simplification is that the cost function appears in the dynamic equation of environmental quality $\dot{S}$.

$$\dot{S} = R(S) + \Gamma(M) - Q^2(M) - c \tag{A*.18}$$

In this example, the optimization problem consists of maximizing the following objective function

$$V(S) = \max_{\{c(t),M(t)\}} \int_0^\infty u(c(t)) + h(S(t))\left[V(S(t)) - \bar{\psi}\right] \exp\left(-\int_0^t \left[\rho + h(S(\tau))\right] d\tau\right) dt \tag{A*.19}$$

with respect to (A*.18).

For the sake of analytical tractability, in the example, we use a linear mitigation function and convex cost function for mitigation. We relax these simplifications in the numerical analysis and adopt the specifications in subsection 3.2 to show that our result is robust, with realistic functional forms. With the presence of mitigation activity, the function $G(S)$ is as follows (see Appendix (K))

$$G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 + \left(\frac{\bar{\psi}\bar{h}}{P_M} - 2gS\right) + (g-\rho) = 0 \tag{A*.20}$$

Once we have equation (A*.20), we analyze the region of parameter space which defines the values for a parameter (here the damage parameter $\bar{\psi}$) within which the multiplicity of equilibria occurs. In Appendix K, we show that the region of parameter where the multiplicity of equilibria appears diminishes when the economoy invests in mitigation activity.

Proof. See Appendix (K)

When the economy implements a mitigation policy, there are few parameter combinations, which causes multiple equilibria.

Appendix 2

A Derivation of (7).

To solve the maximization problem, we write the Hamilton-Jacobi-Bellman equation.

$$\rho V^B(S) = \underset{c}{max}\left\{u(c) + V_S^B(S)(R(S) - c) - h(S)\left(V^B(S) - \varphi(S)\right)\right\} \tag{A.1}$$

where $V^B(S)$ is the value of the maximization program before the event. As also stated in the text, the value of the problem after the event is as follows:

$$\varphi(S) = V^B(S) - \bar{\psi} \tag{A.2}$$

The economy is exposed to inflicted damage after the event. The first-order condition is given by

$$u_c(c) = V_S^B(S) \tag{A.3}$$

Computing the derivative of (A.1) with respect to the environmental quality stock S yields

$$\rho V_S^B(S) = -\bar{\psi} h_S(S) + V_{SS}^B(S)(R(S) - c) + V_S^B(S) R_S(S) \tag{A.4}$$

Differentiating equation (A.3) and using equations (A.3) and (A.4) gives

$$\frac{u_{cc}(c)}{u_c(c)}\dot{c} = \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} \tag{A.5}$$

$$\rho = -\frac{\bar{\psi} h_S(S)}{V_S^B(S)} + \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} + R_S(S) \tag{A.6}$$

Arranging equations (A.5) and (A.6) gives the Keynes-Ramsey rule

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)}\right]$$

The associated Hamiltonian of the problem is

$$\mathcal{H} = u(c) + h(S)\left(V(S) - \bar{\psi}\right) + \lambda(R(S) - c) - \mu h(S). \tag{A.7}$$

where λ and μ are the shadow price for natural resources and for the endogenous hazard rate respectively. Because $\mu = V(S)$, it is easy to see that

$$V(S) = \max_c \frac{\mathcal{H}(S, \lambda)}{\rho} \tag{A.8}$$

One can easily understand from optimal control and dynamic programming that we have $\lambda = V_S^B(S) = u_c(c)$. Then, by using equation (A.6), the dynamics of co-state can be derived

$$\dot{\lambda} = -\lambda\left(R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{\lambda}\right) \tag{A.9}$$

A.1 Derivation of the objective function

We can obtain equation (6) by using the Hamilton-Jacobi-Bellman equation. One can reformulate (A.1) as follows

$$(\rho + h(S(t)))\, V^B(S(t)) - V_S^B(S(t))\, \dot{S}(t) = u(c(t)) + h(S(t))\, \varphi(S(t)) \tag{A.10}$$

Multiplying both sides of (A.10) by $e^{-\int_0^t (\rho + h((S(\tau))))d\tau}$, we have

$$\begin{aligned} &\frac{d}{dt}\left[-V^B(S(t))\, e^{-\int_0^t (\rho + h((S(\tau))))d\tau}\right] \\ &= \left[(\rho + h(S(t)))\, V^B(S(t)) - V_S^B(S(t))\, \dot{S}(t)\right] e^{-\int_0^t (\rho + h((S(\tau))))d\tau} = \left[u(c(t)) + h(S(t))\, \varphi(S(t))\right] e^{-\int_0^t (\rho + h((S(\tau))))d\tau} \end{aligned} \tag{A.11}$$

Taking the integral of both sides of (A.11) between 0 and ∞ yields

$$V^B(S_0) = \left[-V^B(S(t))\, e^{-\int_0^t (\rho + h((S(\tau))))d\tau}\right]_{t=0}^{t=\infty} = \int_0^\infty \left[u(c(t)) + h(S(t))\, \varphi(S(t))\right] e^{-\int_0^t (\rho + h((S(\tau))))d\tau} \tag{A.12}$$

B Proof of Proposition 1

The proof starts with an analysis of the limits of a function (let this function be $G(S)$) that describes the steady state of the economy by a single equation in the long run. Then, in the second part of the proof, we focus on the form of $G(S)$ and related necessary conditions for the existence of multiple equilibria.

(a) In a sense, the function $G(S)$ can be considered equation $\dot{c} = 0$ in terms of S at the steady state equilibrium. Writing equations $\dot{c} = 0$ and $\dot{S} = 0$;

$$R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} = 0 \tag{B.1}$$

$$R(S) - c = 0 \tag{B.2}$$

Plugging equation (B.2) in (B.1) yields

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(R(S))} \tag{B.3}$$

Note that we limit our analysis between $S \in \left[0, \bar{S}\right]$. It can be noticed that the function $G(S)$ starts with a positive value and has negative values when S approaches $\bar{S}$. In this framework, $\bar{S}$ is the environmental quality level when the consumption level is equal to zero. With this information, it is easy to verify $\lim_{S\to 0} G(S) = \infty$ and $\lim_{S\to \bar{S}} G(S) = z < 0$.

(b) In this part of the proof, we show the necessary conditions for the existence of multiple equilibria, which also allows us to represent the function $G(S)$;

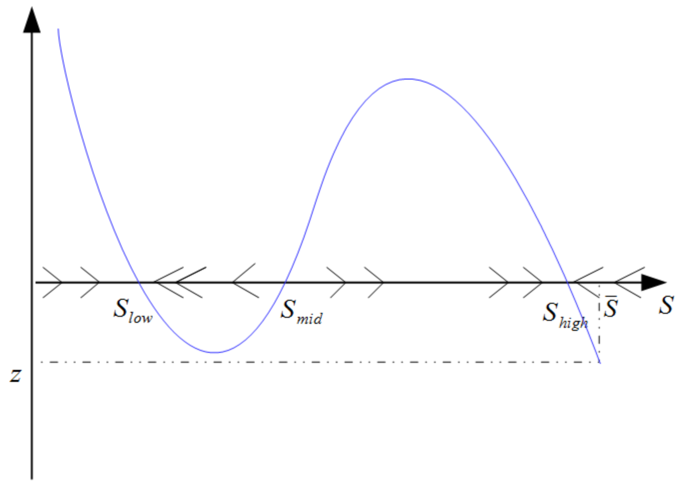


Figure 10: $G(S)$ function with uncertainty

The proof is simple to follow when one looks at the Figure 10. We denote steady states as $0 < S_{\text{low}} < S_{\text{mid}}\left(< \widetilde{S}\right) < S_{\text{high}}\left(< \bar{S}\right)$. A sufficient condition for the existence of S_{low} is that $\exists S < \widetilde{S}$ which ensures (i) $G_S(S) > 0$ and (ii) $G(S) < 0$. Unless the condition (i) $G_S(S) > 0$ is satisfied, $G(S)$ starting from an arbitrary positive value may cross the x-axis just one more time and converge to z, which results in a unique steady state equilibrium. Additionally, the condition (ii) $G(S) < 0$ is also necessary to ensure that $G(S)$ crosses the x-axis by S_{low} at least once.

The sufficient condition for the existence of S_{high} is that $G\left(\widetilde{S}\right) > 0$. If this condition does not hold, $G(S)$ does not cross the x-axis for a second time and converges to z without changing sign. Then, there exists a unique equilibrium. Once the condition $G\left(\widetilde{S}\right) > 0$ is satisfied, we notice that $G(S)$ crosses the x-axis unambiguously by S_{mid} and thereafter converges to z when S approaches $\bar{S}$. Under these conditions, we prove the existence of three steady states, one of which is unstable and two of which are stable.

When there is no endogenous occurrence probability, the necessary condition (9) reduces to $R_{SS}(S) > 0$, which makes multiple equilibria an impossible outcome due to the Assumption 1. Therefore, the model reduces to a standard neoclassical growth model. This completes the proof.

C Slope of the steady-state curve

Using equation (B.3) and the implicit function theorem, we can find the slope of the $\dot{\lambda} = 0$ line.

$$\left.\frac{d\lambda}{dS}\right|_{\dot{\lambda}=0} = \frac{R_{SS}(S) - \frac{\bar{\psi} h_{SS}(S)}{\lambda}}{-\frac{h_S(S)\bar{\psi}}{(\lambda)^2}} \tag{C.1}$$

We can remark that the denominator is always negative. However, the sign of the numerator depends on the convexity of the regeneration function and on the hazard function. This is why we present different phase diagram configurations.

D Proof of Lemma 1

The differential system describing the economy can be written as follows:

$$\begin{bmatrix} \dot{c} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} \frac{d\dot{c}}{dc} & \frac{d\dot{c}}{dS} \\ \frac{d\dot{S}}{dc} & \frac{d\dot{S}}{dS} \end{bmatrix}_{\dot{c}=0,\dot{S}=0} \begin{bmatrix} c - c^{*} \\ S - S^{*} \end{bmatrix}$$

$$\frac{d\dot{c}}{dc} = \rho - R_S(S) \tag{D.1}$$

$$\frac{d\dot{c}}{dS} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)}\right] \tag{D.2}$$

$$\frac{d\dot{S}}{dc} = -1 \tag{D.3}$$

$$\frac{d\dot{S}}{dS} = R_S(S) \tag{D.4}$$

We know that for a saddle-stable path system, it is necessary to have one positive and one negative eigenvalue, denoted as $\mu_{1,2}$. $Tr(J) = \mu_1 + \mu_2$ and $Det(J) = \mu_1\mu_2$. It is sufficient to show that $Tr(J) > 0$ and $Det(J) < 0$. It is easy to see that $Tr(J) = \rho > 0$. By arranging the terms for the determinant, we can see that the determinant reduces to the multiple steady state condition $G(S)$. We conclude that $Det(J)$ is negative if

$$G_S(S) = R_{SS}(S) - \rho - \frac{\bar{\psi}h_{SS}(S)}{u_c(R(S))} + \frac{\bar{\psi}u_{cc}(R(S))}{(u_c(R(S)))^2} < 0 \tag{D.5}$$

$$det(J) = (\rho - R_S(S))R_S(S) - \frac{u_c(c)}{u_{cc}(c)}\left[R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)}\right] \tag{D.6}$$

Complex eigenvalues arise if $(Tr(J))^2 - 4Det(J) < 0$.

$$\rho^2 < 4\left[(\rho - R_S(S))R_S(S) - \frac{u_c(c)}{u_{cc}(c)}\left[R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)}\right]\right] \tag{D.7}$$

As $Det(J)$ is shown to be negative for low and high steady states, this prevents these two steady states from having complex eigenvalues. However, the middle steady state might have complex eigenvalues if condition (D.7) above holds.

E Proof of Proposition 2

The aim of this proof is to show mathematically that all steady-state values of environmental quality S take higher values than the unique steady state of the model without uncertainty. This means that the economy always becomes precautionary when exposed to risk relative to an economy without any harmful events. Suppose that condition (A*.6) holds for the existence of three positive real roots. The function $G(S)$ with risk and the function without risk $G^{WR}(S)$ are as follows:

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} = 0 \tag{E.1}$$

$$G^{WR}(S) = R_S - \rho \tag{E.2}$$

With functional forms in section (A*), we have

$$G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 - 2gS + (g - \rho) = 0 \tag{E.3}$$

$$G^{WR}(S) = -2gS + (g - \rho) = 0 \tag{E.4}$$

Because we use a logistic function for natural resource regeneration, we have $S \in [0;1]$. We know that $G(0) = G^{WR}(0) = (g - \rho)$. This means that two functions start from the same point. It is easy to see that functions $G(S)$ and $G^{WR}(S)$ never intersect, other than at the point where S equals 0, as $G(S) \neq G^{WR}(S)$.

Given this information, we can easily show that all three roots of function $G(S)$ take higher values than the unique root of function $G^{WR}(S)$. The additional term $-2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2$ in $G(S)$ is always a positive term when $S \in [0;1]$. This means that $G(S) > G^{WR}(S)$ for $S \in [0;1]$. This completes the proof.

F Proof of Proposition 3: An economy with adaptation-only policy

The Hamilton-Jacobi-Bellman equation of an economy with only an adaptation policy is as follows

$$\begin{aligned} \rho V^B(S, K_A) = \max_{c,A} \big\{ & u(c) - Q^1(A) + V_S^B(S, K_A)(R(S) - c) \\ & + V_{K_A}^B(S, K_A)(A - \delta K_A) - h(S)\left(V^B(S, K_A) - \varphi(S, K_A)\right) \big\} \end{aligned} \tag{F.1}$$

First order conditions for an internal optimal solution give

$$u_c = V_S^B \tag{F.2}$$

$$V_{K_A}^B = Q_A^1 V_S^B \tag{F.3}$$

Differentiating (F.1) with respect to S and K_A gives

$$\rho V_S^B = V_{SS}^B(R(S) - c) + V_S^B R_S + V_{K_A S}(A - \delta K_A) - h_S(S)\psi(K_A) \tag{F.4}$$

where $V_{K_A S} = \frac{\partial^2 V}{\partial K_A \partial S}$

$$\rho V_{K_A}^B = V_{SK_A}^B(R(S) - c) + V_S^B R_S + V_{K_A K_A}(A - \delta K_A) - h(S)\psi_{K_A}(K_A) - \delta V_{K_A}^B \tag{F.5}$$

The optimal dynamics of consumption and adaptation investment are

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_S(S) - \rho - \frac{\psi(K_A)\, h_S(S)}{u_c(c)}\right] \tag{F.6}$$

$$\dot{A} = \frac{Q_A^1(A)}{Q_{AA}^1(A)}\left[(\rho + \delta) + \frac{h(S)\,\psi_{K_A}(K_A)}{Q_A}\right] \tag{F.7}$$

Using $A = \delta K_A$, at the steady state, we have

$$Q_A^1(\rho + \delta) + h(S)\,\psi_{K_A}(K_A) = 0 \tag{F.8}$$

$$R_S(S) - \rho - \frac{\psi(K_A)\, h_S(S)}{u_c(c)} = 0 \tag{F.9}$$

Using equations (F.6) and $R(S) = c$, we have

$$G(S) = R_S(S) - \rho - \frac{\psi(K_A)\, h_S(S)}{u_c(R(S))} \tag{F.10}$$

Differentiating the equation (F.10) with respect to S yields

$$G_S(S) = R_{SS}(S) - \frac{h_{SS}(S)\,\psi(K_A)}{u_c(R(S))} + \frac{h_S(S)\,\psi(K_A)\,u_{cc}(R(S))\,R_S(S)}{(u_c(R(S)))^2} \underbrace{-\left[\frac{\psi_{K_A}(K_A)\,h_S(S)}{u_c(R(S))}\right]\frac{dK_A}{dS}}_{=Z_1>0} > 0$$

Because the dynamical system is four dimensional, the original condition on $G(S)$ is not sufficient to prove the multiplicity of equilibria. Therefore, we give additional conditions and write the function $G(S) = 0$ given in equation (F.10) in a slighlty different form

$$\underbrace{\frac{(R_S(S) - \rho)\,u_c(R(S))}{h_S(S)}}_{n(S)} = \underbrace{\psi(K_A(S))}_{w(S)} \tag{H.1}$$

By equation (19), we know that $\frac{dK_A}{dS} < 0$. Because the penalty function $\psi(K_A)$ decreases in K_A, it is evident that $w(S)$ increases in S.

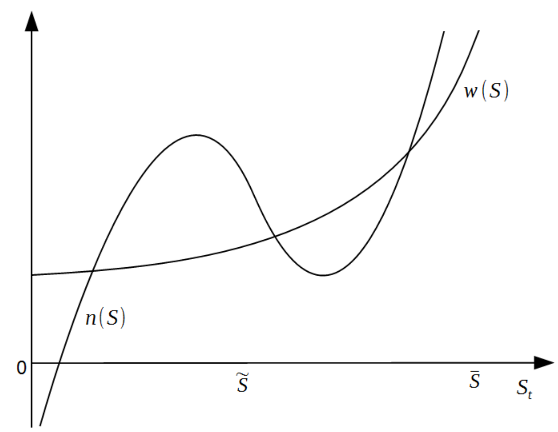


Figure 11: Multiplicity of equilibria with adaptation capital

We remark that $\lim_{S\to 0} n(S) = -\infty$ and $\lim_{S\to \bar{S}} n(S) = +\infty$. A sufficient condition for a low steady state is $\exists S < \tilde{S}$ such that $n_S(S) < 0$ with $n(S) > w(S)$. Note that $\tilde{S}(< \bar{S})$ is between the low and high steady state. A sufficient condition for high steady state is $\exists S > \tilde{S}$ such that $n(S) < w(S)$.

G Proof of Proposition 4: An economy with mitigation-only policy

We write the Hamilton-Jacobi-Bellman equation for the economy with a mitigation activity.

$$\rho V^B(S) = \max_{c,M} \left\{ u(c) - Q^2(M) + V_S^B(S)\left(R(S) + \Gamma(M) - c\right) - h(S)\left(V^B(S) - \varphi(S)\right) \right\} \tag{G.1}$$

where $V^B(S)$ is the value of the maximization program before the event. As also stated in the text, the value of the problem after the event is as follows:

$$\varphi(S) = V^B(S) - \bar{\psi} \tag{G.2}$$

The economy is exposed to inflicted damage after the event. The first-order conditions are given by

$$u_c(c) = V_S^B(S) \tag{G.3}$$

$$Q_M^2 = V_S^B(S)\,\Gamma_M(M) \tag{G.4}$$

Calculating the derivative of (G.1) with respect to the environmental quality stock S yields

$$\rho V_S^B(S) = -\bar{\psi} h_S(S) + V_{SS}^B(S)\left(R(S) + \Gamma(M) - c\right) + V_S^B(S)\,R_S(S) \tag{G.5}$$

Differentiating equation (G.3) and using equations (G.3) and (G.5) gives

$$\frac{u_{cc}(c)}{u_c(c)}\dot{c} = \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} \tag{G.6}$$

$$\rho = -\frac{\bar{\psi} h_S(S)}{V_S^B(S)} + \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} + R_S(S) \tag{G.7}$$

Arranging equations (G.6) and (G.7) gives the Keynes-Ramsey rule

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)}\right]$$

At the steady-state, we have

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c\left(R(S) + \Gamma(M)\right)} \tag{G.8}$$

Note that the optimal steady state level of mitigation M depends on S. Differentiating the equation (G.8) with respect to S yields

$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)} + \frac{\bar{\psi}h_S(S)\,u_{cc}}{(u_c(c))^2}\left(R_S(S) + \Gamma_M(M)\frac{dM}{dS}\right)$$

H The economy with adaptation and mitigation

We solve the model with both adaptation and mitigation policies. The social planner maximizes the following program

$$\begin{aligned} V(S, K_A) = \max_{\{c(t),A(t)\}} \int_0^\infty & \left[u(c(t)) - Q^1(A(t)) - Q^2(M(t))\right. \\ & \left. + h(S(t))\left[V(S(t), K_A(t))\right] - \psi(K_A(t))\right] \exp\left(-\int_0^t \left[\rho + h(S(\tau))\right] d\tau\right) dt \end{aligned} \tag{H.1}$$

subject to

$$\dot{K}_A(t) = A(t) - \delta K_A(t) \tag{H.2}$$

$$\dot{S}(t) = R(S(t)) + \Gamma(M(t)) - c(t) \tag{H.3}$$

The optimal policy is to maximize (H.1) subject to (H.2) and (H.3).

We write the Hamilton-Jacobi-Bellman equation for the economy implementing both adaptation and mitigation policies.

$$\begin{aligned} \rho V^B(S, K_A) = \underset{c,A}{max} & \left\{u(c) - Q^1(A) - Q^2(M) + V_S^B(S, K_A)(R(S) + \Gamma(M) - c)\right. \\ & \left. + V_{K_A}^B(S, K_A)(A - \delta K_A) - h(S)\left(V^B(S) - \varphi(S, K_A)\right)\right\} \end{aligned} \tag{H.4}$$

where $V^B(S)$ is the value of the maximization program before the event. As also stated in the text, the value of the problem after the event is as follows:

$$\varphi(S, K_A) = V^B(S, K_A) - \psi(K_A) \tag{H.5}$$

The economy is exposed to inflicted damage after the event. The first-order conditions are given by

$$u_c(c) = V_S^B(S, K_A) \tag{H.6}$$

$$Q_A^1(A) = V_{K_A}^B(S, K_A) \tag{H.7}$$

$$Q_M^2(M) = V_S^B(S, K_A)\,\Gamma_M(M) \tag{H.8}$$

Computing the derivative of (H.4) with respect to the environmental quality stock S and adaptation capital K_A yields

$$\rho V_S^B(S,K_A) = -h_S(S)\,\psi(K_A) + V_{SS}^B(S,K_A)\,(R(S)+\Gamma(M)-c) + V_{K_AS}^B(S,K_A)\,(A-\delta K_A) + V_S^B(S,K_A)\,R_S(S) \quad \text{(H.9)}$$

$$\rho V_{K_A}^B(S,K_A) = V_{SK_A}^B(S,K_A)\,(R(S)+\Gamma(M)-c) + V_{K_AK_A}^B(S,K_A)\,(A-\delta K_A) - \delta V_{K_A}^B(S,K_A) - h(S)\,\psi_{K_A}(K_A) \quad \text{(H.10)}$$

Differentiating equation (H.6) and using equations (H.6) and (H.9) gives

$$\frac{u_{cc}(c)}{u_c(c)}\dot{c} = \frac{V_{SS}^B(S,K_A)}{V_S^B(S,K_A)}\dot{S} \quad \text{(H.11)}$$

$$\frac{Q_{AA}^1(A)}{Q_A^1(A)}\dot{A} = \frac{V_{K_AK_A}^B(S,K_A)}{V_{K_A}^B(S,K_A)}\dot{K_A} \quad \text{(H.12)}$$

Optimal dynamics for consumption, adaptation and mitigation are as follows

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_S(S) - \rho - \frac{\psi(K_A)\,h_S(S)}{u_c(c)}\right] \quad \text{(H.13)}$$

$$\dot{A} = \frac{Q_A^1(A)}{Q_{AA}^1(A)}\left(\rho + \delta + \frac{h(S)\,\psi_{K_A}(K_A)}{Q_A^1(A)}\right) \quad \text{(H.14)}$$

$$\dot{M} = -\frac{u_{cc}(c)}{u_c(c)}\frac{\Gamma_M(M)}{\Gamma_{MM}(M)}\left[R_S(S) - \rho - \frac{\psi(K_A)\,h_S(S)}{u_c(c)}\right] \quad \text{(H.15)}$$

I Proof of Example 1

The third degree polynomial equation has the following form

$$a_1x^3 + b_1x^2 + c_1x + d_1 = 0 \quad \text{(I.1)}$$

With the functional forms given in the text, we remark that terms $b_1 = -a_1$ with $a_1 = -2g\bar{\psi}\bar{h}$, $c_1 = -2g$ and $d_1 = g - \rho$. This simplifies the proof of the existence for three positive real roots. The discriminant of the cubic equation is as follows:

$$\Delta = 18a_1b_1c_1d_1 - 4b_1^3d + b_1^2c_1^2 - 4_1ac_1^3 - 27a_1^2d_1^2 \quad \text{(I.2)}$$

- $\Delta > 0$, equation (I.1) has three distinct real roots.
- $\Delta = 0$, equation (I.1) has multiple roots and all three roots are real.
- $\Delta < 0$, equation (I.1) has one real root and two non-real, complex roots.

Because $b = -a$, we can reformulate the discriminant (I.2) in the following way

$$\Delta = -b_1 \left[4d_1 b_1^2 - \left(c_1^2 - 18c_1 d_1 - 27d_1^2\right) b_1 - 4c_1^3\right] \tag{I.3}$$

Then, we have a second degree equation to be solved for the value of b. The discriminant of this second degree equation is $\Delta_1 = \left(c^2 - 18cd - 27d^2\right)^2 - 64dc^3$. Equation (I.3) is written as

$$\Delta = \underbrace{-b_1}_{<0} \left[b_1 - \left(\frac{\left(c_1^2 - 18c_1 d_1 - 27d_1^2\right) + \sqrt{\left(c_1^2 - 18c_1 d_1 - 27d_1^2\right)^2 + 64d_1 c_1^3}}{8d_1}\right)\right] \left[b_1 - \left(\frac{\left(c_1^2 - 18c_1 d_1 - 27d_1^2\right) - \sqrt{\left(c_1^2 - 18c_1 d_1 - 27d_1^2\right)^2 + 64d_1 c_1^3}}{8d_1}\right)\right] \tag{I.4}$$

Then, by assuming $\Delta_1 > 0$, the discriminant Δ has a positive value if

$$\underbrace{\left(\frac{\left(c_1^2 - 18c_1 d_1 - 27d_1^2\right) - \sqrt{\left(c_1^2 - 18c_1 d_1 - 27d_1^2\right)^2 + 64d_1 c_1^3}}{8d_1}\right)}_{L_E} < b_1 < \underbrace{\left(\frac{\left(c_1^2 - 18c_1 d_1 - 27d_1^2\right) + \sqrt{\left(c_1^2 - 18c_1 d_1 - 27d_1^2\right)^2 + 64d_1 c_1^3}}{8d_1}\right)}_{U_E} \tag{I.5}$$

where L_E and U_E are lower and upper extremes of the inequality (I.5). By replacing the terms a_1, b_1, c_1 and d_1 with their corresponding values, condition (I.5) becomes

$$\frac{\left(4g^2 + 36g\left(g-\rho\right) - 27\left(g-\rho\right)^2\right) - \sqrt{\left(4g^2 + 36g\left(g-\rho\right) - 27\left(g-\rho\right)^2\right)^2 - 2^9\left(g-\rho\right)}}{8\left(g-\rho\right) g\bar{h}} < \bar{\psi} < \frac{\left(4g^2 + 36g\left(g-\rho\right) - 27\left(g-\rho\right)^2\right) + \sqrt{\left(4g^2 + 36g\left(g-\rho\right) - 27\left(g-\rho\right)^2\right)^2 - 2^9\left(g-\rho\right)}}{8\left(g-\rho\right) g\bar{h}} \tag{I.6}$$

However, the positive discriminant Δ proves only the existence of real roots and does not indicate whether these roots are positive or not. For this purpose, we use the Descartes rule. To see if the equation $G\left(S\right) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 - 2gS + \left(g-\rho\right) = 0$ has three positive real roots, we write $G\left(-S\right)$

$$G\left(-S\right) = -2g\bar{\psi}\bar{h}\left(-S\right)^3 + 2g\bar{\psi}\bar{h}\left(-S\right)^2 - 2g\left(-S\right) + \left(g-\rho\right) = 0 \tag{I.7}$$

This yields

$$G\left(-S\right) = 2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 + 2gS + \left(g-\rho\right) = 0 \tag{I.8}$$

By assuming $g-\rho>0$, we observe that there is no sign change. This proves all roots are positive for $G(S)$. This completes the proof.

J Proof of Example 2 (only-adaptation)

Equation (7) combined with (1) at the steady state can be reformulated as

$$G(S) = -\bar{\psi}\bar{h}gS^2 + \left(\bar{\psi}\bar{h}g - 2g\right)S + (g-\rho) = 0 \tag{J.1}$$

It is easy to show that this equation has one positive and one negative real root when $g-\rho>0$:

$$S_{1,2} = \frac{\left(\bar{\psi}\bar{h}g - 2g\right) \pm \sqrt{\left(\bar{\psi}\bar{h}g - 2g\right)^2 + 4\bar{\psi}\bar{h}g\,(g-\rho)}}{2\bar{\psi}\bar{h}g} \tag{J.2}$$

We exclude the negative root because it has no economic meaning. Consequently, the economy has a unique equilibrium without any possibility of multiple equilibria. What happens if this economy starts to invest in adaptation capital? For the sake of analytical tractability, we use a linear hazard and adaptation function. In the numerical analysis, we relax the linearity assumption on functional forms and use more general functional forms to show the robustness of our results.

$$\psi(K_A) = \bar{\psi}f(K_A) = \bar{\psi}(1-aK_A) \tag{J.3}$$

$$Q(A) = \phi\frac{A^2}{2} \tag{J.4}$$

where a shows the extent to which the adaptation capital is able to decrease the vulnerability against the inflicted damage $\bar{\psi}$ and ϕ stands for a scale parameter for the adaptation cost function. Using equations (F.6), (F.7) and functional forms presented in section (A*), the economy with an adaptation policy ends up with the following $G(S)$ (see Appendix F)

$$G(S) = -\bar{h}gzS^3 + \left(\left(g+\bar{h}g\right)z - \bar{\psi}\bar{h}g\right)S^2 + \left(\bar{\psi}\bar{h}g - 2g - zg\right)S + (g-\rho) = 0 \tag{J.5}$$

where $z = \frac{\bar{h}\left(a\bar{\psi}\right)^2}{\phi\delta(\rho+\delta)}$. For analytical tractability, we suppose that $\bar{\psi} = \frac{z}{\bar{h}}$. Then, equation (J.5) can be reformulated in a simplified form

$$G(S) = -\bar{h}gzS^3 + \bar{h}gzS^2 - 2gS + (g-\rho) = 0$$

This equation is similar to (A*.5) which is shown to have three positive real roots. Following the same method as that used in proof of Proposition 3., the condition of having multiple equilibria in the benchmark economy augmented by the adaptation investments is as follows[30]

[30]The proof is available upon request.

$$\frac{\left[\left(4g^2+36g\left(g-\rho\right)-27\left(g-\rho\right)^2\right)-\sqrt{\left(4g^2+36g\left(g-\rho\right)-27\left(g-\rho\right)^2\right)^2-2^9g^3\left(g-\rho\right)}\right]}{8\left(g-\rho\right)g\left(\bar{h}\right)^2}$$
$$<\bar{\psi}<\frac{\left[\left(4g^2+36g\left(g-\rho\right)-27\left(g-\rho\right)^2\right)+\sqrt{\left(4g^2+36g\left(g-\rho\right)-27\left(g-\rho\right)^2\right)^2-2^9g^3\left(g-\rho\right)}\right]}{8\left(g-\rho\right)g\left(\bar{h}\right)^2} \tag{J.6}$$

K Proof of Example 3 (only-mitigation)

The optimization problem holds only for the Example 3 and not for the subsection 3.2, since we put the cost function of the mitigation activity in the resource constraint in the example for the sake of analytical tractability. We write the Hamilton-Jacobi-Bellman equation

$$\rho V^B\left(S\right)=\max_{c,M}\left\{u\left(c\right)+V^B_S\left(S\right)\left(R\left(S\right)+\Gamma\left(M\right)-Q_2\left(M\right)-c\right)-h\left(S\right)\left(V^B\left(S\right)-\varphi\left(S\right)\right)\right\} \tag{K.1}$$

where $V^B\left(S\right)$ is the value of the maximization program before the event. The economy is exposed to constant inflicted damage after the occurrence of the event. As also stated in the text, the value of the problem after the event is as follows

$$\varphi\left(S\right)=V^B\left(S\right)-\bar{\psi} \tag{K.2}$$

The first-order conditions are given by

$$u_c\left(c\right)=V^B_S\left(S\right) \tag{K.3}$$

$$\Gamma_M\left(M\right)=Q^2_M\left(M\right)=P_M \tag{K.4}$$

Using the envelop theorem, we have

$$\rho V^B_S\left(S\right)=-\bar{\psi}h_S\left(S\right)+V^B_{SS}\left(S\right)\left(R\left(S\right)+\Gamma\left(M\right)-Q_2\left(M\right)-c\right)+V^B_S\left(S\right)R_S\left(S\right) \tag{K.5}$$

Differentiating equation (K.3) and using equations (K.3) and (K.5) gives

$$\frac{u_{cc}\left(c\right)}{u_c\left(c\right)}\dot{c}=\frac{V^B_{SS}\left(S\right)}{V^B_S\left(S\right)}\dot{S} \tag{K.6}$$

$$\rho=-\frac{\bar{\psi}h_S\left(S\right)}{V^B_S\left(S\right)}+\frac{V^B_{SS}\left(S\right)}{V^B_S\left(S\right)}\dot{S}+R_S\left(S\right) \tag{K.7}$$

Arranging equations (K.6) and (K.7) gives the Keynes-Ramsey rule

$$\dot{c}=-\frac{u_c\left(c\right)}{u_{cc}\left(c\right)}\left[R_S\left(S\right)-\rho-\frac{\bar{\psi}h_S\left(S\right)}{u_c\left(c\right)}\right]$$

At the steady state, we have

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(R(S) + \Gamma(M) - Q_2(M))} = 0$$

Because we use linear specifications for analytical tractability, we have the optimum level of mitigation investment as $M^* = \frac{1}{P_M}$. Then, we can reformulate the function $G(S)$ as

$$G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 + \left(\frac{\bar{\psi}\bar{h}}{P_M} - 2gS\right) + (g - \rho) = 0 \tag{K.8}$$

where $a_1 = -2g\bar{\psi}\bar{h}$, $b_1 = 2g\bar{\psi}\bar{h}$, $c_1 = \frac{\bar{\psi}\bar{h}}{P_M} - 2gS$ and $d_1 = g - \rho$. Equation (K.8) is similar to $G(S)$ of the benchmark economy without an environmental policy. The only term that is different in the economy with mitigation policy compared to the benchmark economy is the term c_1 of equation (I.1). The term c_1 is higher in the case with a mitigation policy than in the benchmark economy case.

We look at the effect of a higher value c_1 on the upper and lower extremes of the inequality (I.5). By assuming $c_1 < 0$ and $d_1 > 0$, the derivative of the upper and lower extremes are as follows

$$\frac{\partial L_E}{\partial c_1} = \frac{1}{2}\frac{\underbrace{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)}_{>0}\underbrace{(2c_1 - 18d_1)}_{<0}}{\left(\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)^2\right)^{\frac{1}{2}}} - \frac{1}{2}\left(\frac{\underbrace{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)}_{>0}\underbrace{(2c_1 - 18d_1)}_{<0} + 3.64d_1c_1^2}{\left(\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)^2 + 64d_1c_1^3\right)^{\frac{1}{2}}}\right) > 0$$

$$\frac{\partial U_E}{\partial c_1} = \frac{1}{2}\frac{\underbrace{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)}_{>0}\underbrace{(2c_1 - 18d_1)}_{<0}}{\left(\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)^2\right)^{\frac{1}{2}}} + \frac{1}{2}\left(\frac{\underbrace{\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)}_{>0}\underbrace{(2c_1 - 18d_1)}_{<0} + 3.64d_1c_1^2}{\left(\left(c_1^2 - 18c_1d_1 - 27d_1^2\right)^2 + 64d_1c_1^3\right)^{\frac{1}{2}}}\right) < 0$$

We observe that the occurrence of multiple equilibria with a mitigation policy is relatively unlikely because there is a smaller range of values for damage $\bar{\psi}$ that cause multiple equilibria.

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