Increasing returns, monopolistic competition, and international trade: Revisiting gains from trade

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Sergey Kokovin [c,*], Pavel Molchanov [c,d], Igor Bykadorov [a,b]

[a] Sobolev Institute of Mathematics SB RAS, Russian Federation
[b] Novosibirsk State University, Russian Federation
[c] HSE University, Russian Federation
[d] Aix-Marseille University, CNRS, EHESS, Centrale Marseille, AMSE, France

A B S T R A C T

We study the canonical Krugman (1979) trade model with non-CES preferences that yield au-tarky at finite trade costs. We prove a non-monotone impact of *gradual* trade liberalization. At first, near autarky, emerging trade reduces world welfare, while at free trade it becomes large enough to be beneficial (Krugman's result). This non-monotonicity persists under heteroge-nous firms. The harmful small-scale trade is explained by variable markups and underpriced imports, which become socially excessive. Unlike protectionists, we argue that "liberalization should go far". On the other hand, we show that anti-dumping measures can be viewed as a remedy for the aforementioned imports distortion.

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* Corresponding author.
E-mail addresses: skokovin@hse.ru (S. Kokovin), mrmps1992@gmail.com (P. Molchanov), bykadorov.igor@mail.ru (I. Bykadorov).

1. Introduction

It is a well-known theoretical result since Krugman (1979), that a complete trade integration, formulated as a switch from autarky to free trade, is welfare-enhancing for consumers under reasonable assumptions.[1] The gains from trade are due to economies of scale, better exploited in an integrated market. What remained unclear, however, is how gains from trade unfold during a *gradual* transition from autarky to free trade.

To fill this gap, we revisit Krugman's question and ask whether countries necessarily gain at each step of trade liberalization, i.e., a decline in transportation costs. We maintain Krugman's original setup with two symmetric countries and arbitrary additive utilities with non-constant elasticity of substitution (non-CES). The paper focuses on welfare gains from trade liberalization in a range around autarky. For autarky to exist at finite trade costs, the demand system is assumed to have a choke price, which is the only additional assumption.[2] This finite autarky point enables clear comparative statics via marginal reductions in trade costs.

We prove that trade gains are non-monotone: positive near free trade but negative at the beginning of trade liberalization. Moreover, there is a whole interval of trade costs near autarky where trade is detrimental to consumers in both countries. In other words, *small-scale trade liberalization is harmful* under classical monopolistic competition with any neo-classical additive utility that enables autarky (at finite trade costs).

How can a technological improvement, i.e., lower trade costs, become socially undesirable? The immediate observation is a fall in the mass of firms, i.e., variety. It is in contrast with the benchmark CES case, where the mass of firms is constant and welfare improves monotonically with liberalization (because equilibria coincide with social optima, see Dhingra and Morrow, 2019 for a closed-economy scenario). This comparison suggests that, in our case, the observed non-monotone welfare should be explained by non-optimality—market *distortions* created by non-CES preferences. Specifically, there are two kinds of distortion in this environment.

The first distortion involves *excessive imports*—consumers spend more on imports and transportation than is socially desired. Mathematically, this outcome stems from the increasing (absolute value of) price elasticity of demand (IED).[3] In such IED situations (see Weyl and Fabinger, 2013), firms practice incomplete pass-through: they set higher markups for cheaper products. These unequal markups indirectly subsidize exports via higher prices of domestic goods. This generates "relative dumping", meaning that a product's domestic price times the trade costs coefficient exceeds its export price.

Under relative dumping, the export price does cover the variable production and transportation costs. In that sense, there are no direct social losses from exports. However, there are opportunity costs: an inefficiently high amount of resources is spent on transportation costs, instead of being utilized for domestic production. Consumers spend too much on foreign varieties due to lower export markups relative to domestic varieties. A social planner, who maximizes world welfare, would forbid exports/imports in some neighborhood of autarky where trade gains are negative, postponing trade to further stages of trade liberalization.

The second margin of potential inefficiency, the Dixit–Stiglitz quantity versus diversity trade-off, turns out to be not so important. The mass of varieties in autarky can be insufficient or excessive.[4] If variety is insufficient, the initial trade liberalization *aggravates* this distortion by reducing variety, contributing to the welfare losses. In the opposite case of excessive variety, liberalization *mitigates* the Dixit–Stiglitz distortion. However, even in this case, the export distortion explained above outweighs the Dixit–Stiglitz effect, so that overall welfare consequences of liberalization are negative near autarky. This leads to the idea that the entry distortion plays only a secondary role in our harmful trade effect. To demonstrate it formally, we consider a thought experiment where a regulator corrects the entry distortion at autarky through a license fee; we find that the welfare losses remain intact.

The export distortion raises a natural question of its source. Section 4 suppresses one-by-one several features of the market to identify the necessary element for the harmful trade effect via the following three thought experiments.

First, we ask if decreasing variety *per se* plays a major role in the mechanism of welfare losses. To this end, we consider a regulator who prohibits entry/exit of firms during trade liberalization, i.e., *the mass of firms is fixed* at the autarky level. We show that welfare decreases with liberalization near autarky even in this setting. Moreover, the welfare losses are accompanied by a reduction in gross consumption. This reduction is a bypass channel for welfare loss when variety is fixed, whereas the primary source is a waste of resources in transportation.

Second, we ask if the general equilibrium effects, such as business-stealing and increased competition, are necessary to generate the welfare loss. We construct a thought experiment, where the regulator *fixes the individual output* of each firm at the autarky level but allows for free entry. This regulation seems similar to the previous one because the equilibrium mass of firms remains unchanged due to the fixed labor supply. However, the behavior of firms is different: the output constraint now enters directly into the firm's profit maximization problem. In this setting, a reduction in trade costs affects neither competition nor profits (unlike Brander and Krugman, 1983). Yet, the harmful trade effect remains even without general equilibrium effects and the pressure of competition. In other words, wasteful transportation and dumping originate from the individual behavior of each firm.

[1] These are further specified in Mrázová and Neary (2014), Bykadorov et al. (2015a).

[2] An elementary utility u has a choke price if it exhibits a finite derivative at zero: $u'(0) < \infty$.

[3] IED does not imply and does not follow from another important property, decreasing elasticity of utility (DEU). They generate two different distortions (Dhingra and Morrow, 2019). Near autarky, both DEU and IED locally arise for imports without the need to be assumed everywhere (see Section 4).

[4] Variety is insufficient/excessive under increasing/decreasing elasticity of utility (IEU/DEU) (Dixit and Stiglitz, 1977; Dhingra and Morrow, 2019).

Third, to demonstrate the necessity of dumping for the harmful trade result, we construct a regulation where *dumping and related market segmentation are eliminated*, i.e., markups are forced to be equal across markets. Thereby, we exclude the export/domestic distortion, defined as a discrepancy between the consumer's marginal rate of substitution and the producer's marginal rate of transformation. Numerical simulations show that, in this environment, unlike the previous regulations, harmful trade disappears; welfare grows monotonically during trade liberalization. Analytically, we are able to prove that when such an anti-dumping policy is combined with fixed entry or fixed output constraint, the world welfare grows monotonically because the equilibria are second-best socially optimal, i.e., the domestic/imported consumption ratio remains optimal. This welfare-based argument advocates an anti-dumping policy during small-scale trade liberalization. However, we are not able to prove such an analytical statement in a general Krugman's setting with entry of firms.

Summarizing, various modifications of Krugman's model point to incomplete pass-through and dumping as the main driving forces for the harmful trade effect.

Further, we discuss the *size and scope* of our harmful effect. Though it is proven locally near autarky, a numerical exercise with homogenous firms in Section 2 shows a substantial negative welfare effect, which extends non-locally. Specifically, in this example, countries do not enjoy any benefits from trade until trade costs are reduced by as much as a quarter from the choke level. Furthermore, in all numerical examples, the gains from trade have a U-shaped form. They behave approximately quadratically in volumes of trade: welfare grows slowly near its minimum and attains a sharp peak at free trade. These observations on non-linear effects suggest that “liberalization should go far” even in cases when the interval of negative gains is small.

The *generality of our findings* is investigated in Section 5. Notably, our benchmark model is extended to a non-CES heterogenous firms setting la Melitz (2003), which becomes canonical nowadays. In this framework, we prove that the welfare losses near autarky remain intact. However, simulations show that heterogeneity mitigates these losses and squeezes the related interval of trade costs. Effectively, under heterogeneity, a better selection of firms (by productivity) partially compensates for the export distortion. In other extensions, we check that our result is robust to the introduction of multiple asymmetric countries and unequal trade costs.

Finally, coming to the practical relevance of our findings, we are not sure that the losses from trade can be detected empirically. They may be overshadowed by the Melitzian selection effect or by the Ricardian comparative advantage. Rather, for an economist, the take-aways from our study are the ideas about the export distortion, dumping associated with IED, and non-monotone gains from trade, eventually increasing with liberalization. Our arguments suggest that small-scale trade is not going to bring large welfare gains. That is why an international policymaker should *liberalize trade substantially*: welfare increments should not fade with globalization but, rather, become ever larger. In addition, we find a novel, welfare-based, motive for anti-dumping policies when trade is modest in volume.

1.1. Related literature

Our paper is a part of the New Trade literature, which reveals various channels of “new” gains from trade. Those are related to monopolistic competition, product diversity, and economies of scale. The attention to welfare gains has been renewed since Arkolakis et al. (2012) examined the approaches to modeling and measuring these gains. (See also a discussion in Melitz and Redding, 2015, and a review in Costinot and Rodríguez-Clare, 2014.) Most of the related papers use the convenient CES assumption, which generally precludes losses from trade.

Our work, similar to that of Zhelobodko et al. (2012) and Arkolakis et al. (2019), steps away from the standard CES modeling by assuming more general preferences with variable elasticity of substitution (VES). This approach allows for autarky at finite trade costs; it also makes markups variable, which is strongly supported by empirical evidence (De Loecker et al., 2016; Behrens et al., 2020). Trade applications of such demand specifications include Feenstra (2003), Dhingra (2013) and, more relevant to us, Mrázová and Neary (2014).[5] The latter paper formulates a sufficient condition for positive gains from a special kind of trade liberalization: increasing the number of countries involved. The condition includes the IED property, equivalent to *decreasing* elasticity of substitution, considered to be realistic.[6] The IED property is of interest because related pro-competitive effects (markups decreasing with one or another kind of trade liberalization) can bring additional trade gains.

This idea of additional gains is studied in Arkolakis et al. (2019) under heterogenous firms and variable markups. Their baseline model is very close to our heterogenous setting, but predicts *always positive* gains from trade liberalization, contrary to our harmful-trade result. This discrepancy is explained by the fact that Arkolakis et al. (2019) assume (i) zero *fixed export* costs; (ii) unbounded productivity, which excludes autarky at finite trade costs (see Section 5). Either of these assumptions nullifies the export distortion.

We should distinguish our setting from other papers involving harm from trade liberalization. In particular, we do not touch on multi-sector or dynamic (“infant industry”) considerations. Among one-sector static models, Brander and Krugman (1983) assume a *fixed* number of firms in a partial equilibrium with oligopolistic Cournot competition and a homogenous good (no love for variety). Similarly to us, they demonstrate that trade can become “wasteful” because of reciprocal dumping. However, there are

[5] Behrens et al. (2016) also estimate welfare and market distortions under monopolistic competition with VES, while Behrens et al. (2017) study related theoretical caveats.

[6] “Though there is no clear consensus, the balance of empirical and other evidence suggests that sub-convex demands (decreasing elasticity of substitution) are more realistic than super-convex (increasing elasticity of substitution).” Mrázová and Neary (2014, p. 300).

important differences (see Section 4): their strategic dumping happens independently of product differentiation and love for variety. Moreover, their welfare loss from trade disappears under free entry.

Further, in a slightly generalized Krugman's model, Bykadorov et al. (2015a) find another case of harmful trade under VES preferences during a complete trade liberalization, i.e., a jump from autarky to free trade. In the original Krugman (1979) paper, gains from trade are based on IED&DEU combination of assumptions (increasing elasticity of demand and decreasing elasticity of utility). By contrast, Bykadorov et al. (2015a) find that IED&IEU or DED&DEU combination of properties is a necessary and sufficient condition for *harmful free trade*.[7] The explanation ivolves "misaligned revenue and utility", characteristic for these combinations (see Dhingra and Morrow, 2019). Unlike Bykadorov et al. (2015a), the present paper finds harmful *costly* trade under *any* preferences with a choke price. Finally, Chen and Zeng (2014) find similar harmful small-scale trade in a two-factor trade model with footloose capital. Their explanation rests on labor/capital substitution and international capital flows. In a setting with indirectly additive preferences, Bertoletti and Etro (2017) find a non-monotonic welfare effect like ours. All the above-mentioned studies differ from ours not only in their settings but also in the mechanisms determining welfare losses from trade.

As to similar settings, we can mention a recent discussion paper, Morgan et al. (2020), that closely follows our results and extends them to a multi-sector economy. Also, Bertoletti and Epifani (2014) find non-monotonicity of markups under liberalization without deriving the non-mononotone welfare implication.

The rest of the paper is structured as follows. Section 2 lays out the model. Sections 3 and 4 present the main result and intuitions. Section 5 extends the result to the case of firms' heterogeneity la Melitz (proven in Appendix B) and mentions other extensions. Section 6 concludes.

2. Model

We present the standard Krugman (1979) model of international trade with VES and two symmetric countries. Our single-sector economy exhibits monopolistic competition and involves an endogenous interval $[0, N]$ of identical firms producing varieties of a differentiated good, one variety per firm. The only production factor is labor, supplied inelastically by L identical consumers/workers in each country.

Each consumer maximizes her utility using two kinds of variables: x_ω is the consumption of the ω-th domestic variety and z_ζ is the consumption of the ζ-th imported variety. A representative consumer maximizes her utility subject to a budget constraint,

$$\max_{(x_\omega, z_\omega)} \int_0^N u(x_\omega)d\omega + \int_0^N u\left(z_\zeta\right)d\zeta \ s.t.$$

$$\int_0^N p_\omega^x x_\omega d\omega + \int_0^N p_\zeta^z z_\zeta d\zeta \leqslant 1,$$

where prices p_ω^x, p_ζ^z correspond to consumption volumes x_ω and z_ζ respectively. Due to the symmetry of countries, wages are equalized across countries and taken as a numeraire: $w \equiv 1$.

To ensure the existence and uniqueness of each consumer's/producer's choice in any market situation, we impose restrictions standard for VES models. As in Mrázová and Neary (2014), the elementary utility $u(\cdot)$ is thrice continuously differentiable, strictly concave, increasing at least on some interval $[0, \check{z})$, where $\check{z} \equiv \arg\max_z u(z)$ denotes the satiation point, which can be infinite (for HARA utility) or finite (for quadratic utility). Additionally, using the Arrow–Pratt concavity measure $r_g(z) \equiv -\frac{zg''(z)}{g'(z)}$ (defined for any function g), we restrict throughout the concavity of u, u', and behavior of our functions at zero, as

$$\{0 < r_u(z) < 1 \ \& \ r_{u'}(z) < 2 \ \ \forall z \in (0, \check{z})\}, \ \ u(0) = 0, \tag{1}$$

$$u'(0) < \infty, \ \ u''(0) > -\infty, \ \ u'''(0) \in (-\infty, \infty).$$

Notably, we deviate from Krugman (1979) in two ways. First, we impose a choke price to allow for autarky at finite trade costs. This implies that u must have finite derivatives at zero, which differs from the CES assumption. Second, we do not require decreasing elasticity of utility, since this feature naturally arises near the zero-consumption point.

[7] Such a detrimental combination is exotic, it holds only in specially constructed mathematical examples in Bykadorov et al. (2015a) on narrow intervals of parameters.

Using these assumptions and the consumer's first-order conditions (FOCs), we derive two inverse demand functions for each variety, for domestic and imported goods:

$$p^x_\omega = \frac{u'(x_\omega)}{\lambda}, \quad p^z_\zeta = \frac{u'\left(z_\zeta\right)}{\lambda}. \tag{2}$$

Here λ is the Lagrange multiplier for the budget constraint. Being the marginal utility of income, λ serves as the main market aggregator, similar to the price index.[8]

2.1. Producers

The output (firm size) of the $\omega - th$ firm is given by

$$Q_\omega(x_\omega, z_\omega) = L \cdot x_\omega + \tau \cdot L \cdot z_\omega,$$

where $\tau \geq 1$ is the usual iceberg trade cost. The total firm's costs (in labor units) are given by

$$C(Q_\omega) = c \cdot Q_\omega + F,$$

where F is the fixed cost of production and c is the marginal cost. Our assumptions ensure uniqueness and symmetry of producers' behavior (see Zhelobodko et al., 2012), so that from now on it is possible to omit indices ω and ζ. Introducing the "normalized revenue" function $R(\xi) \equiv u'(\xi) \cdot \xi$ and using the demand functions, the profit maximization program of a firm can be written as

$$\max_{(x,z)} \pi \equiv L \cdot \frac{R(x)}{\lambda} + L \cdot \frac{R(z)}{\lambda} - C(Q(x,z)). \tag{3}$$

Under monopolistic competition, each firm perceives the market aggregate λ as given.

Labor market clearing condition, meaning full employment of labor at the equilibrium, is written as

$$N \cdot C(Q) = L. \tag{4}$$

Zero-profit (free-entry) condition at the equilibrium is

$$\pi = 0 \tag{5}$$

and producer's FOCs are

$$\frac{\partial \pi}{\partial x} = 0, \quad \frac{\partial \pi}{\partial z} = 0. \tag{6}$$

The second-order condition (SOC) under linear costs can be rewritten in terms of normalized revenue R as

$$R''(\xi) < 0, \quad \xi \in \{x, z\},$$

which holds true under our assumptions (namely $r_{u'}(z) < 2$) and guarantees symmetry of producers' behavior (see Mrázová and Neary, 2014).

The trade balance equation, which requires the value of all exported goods to be equal to the value of imported goods, is omitted here, being satisfied trivially under the symmetry of countries.

Equilibrium (symmetric) is the bundle of consumptions, prices, the mass of firms, and the market aggregator

$$\left(x^*, z^*, p^{x^*}, p^{z^*}, N^*, \lambda^*\right)$$

that satisfies all the requirements imposed, namely, (i) utility maximization (2); (ii) profit maximization (6); (iii) free entry (5); and (iv) labor market clearing (4).[9] As usual, the budget constraints are implied by labor market clearing and so they are omitted.

By substituting N, p^x, p^z, the equilibrium condtions can be reduced to three equations in (x, z, λ), which are two FOCs and the zero-profit condition:

$$c = \frac{R'(x)}{\lambda}, \quad c(\tau^a - \varepsilon) = \frac{R'(z)}{\lambda}, \tag{7}$$

[8] In the CES case, the Lagrange multiplier is simply the inverse of the price index, up to a monotone transformation of the utility function.

[9] The possibility of *asymmetric* equilibria, i.e., uniqueness of equilibria, lies outside the focus of this paper. Uniqueness typically holds for *two* countries. As for *multiple* countries, see Allen et al. (2020).

$$C(Q) = \frac{R(x)}{\lambda} + \frac{R(z)}{\lambda}. \quad (8)$$

2.2. Welfare

The welfare (per consumer) of each country is expressed as

$$W = N \cdot (u(x) + u(z)). \quad (9)$$

Using the labor balance (4), the welfare function can be reformulated without variety variable N. In this form, it expresses "utility per cost" of two kinds of goods, domestic and foreign:

$$W(x,z) = L \cdot \left(\frac{u(x) + u(z)}{C(Q(x,z))} \right) = \frac{u(x) + u(z)}{c \cdot (x + \tau z) + F/L}. \quad (10)$$

In our economy, there is no "price index" measuring welfare, but expression (10) is instructive: welfare is represented as social benefits divided by social costs. A social planner would maximize this function in x, z without constraints. By the envelope theorem, the impact of $\partial x^{opt}(\tau)/\partial\tau$, $\partial z^{opt}(\tau)/\partial\tau$ on welfare (10) is zero, whereas (under $z^{opt} > 0$) costs τz in the denominator pull the optimal welfare $W^{opt}(\tau) \equiv W(x^{opt}(\tau), z^{opt}(\tau))$ *down* monotonically.

This proof of monotone optimum $W^{opt}(\tau)$ may support the intuitive belief that the equilibrium welfare $W^*(\tau) \equiv W(x^*(\tau), z^*(\tau))$ (though being potentially smaller than the optimal welfare) should also change monotonically in the same direction. However, the section below shows that, unlike the case of CES, this belief need not always come true under VES. The reason is that liberalization near autarky inefficiently shifts the equilibrium consumption toward imports z^*. The equilibrium numerator and the denominator of welfare in Eq. (10) both change when $(x^*(\tau), z^*(\tau)) \neq (x^{\rm opt}(\tau), z^{\rm opt}(\tau))$, so, their joint impact generates non-monotone welfare.

3. Initial losses and eventual gains from trade liberalization

This section states our main result—non-monotone welfare in trade costs—and continues with a discussion of its economic mechanism.

To formulate comparative statics of equilibria in trade costs, we treat domestic/import consumptions x, z and variety N as functions $x(\tau)$, $z(\tau)$, $N(\tau)$ of the trade costs parameter $\tau = (1 + \delta)$, modified through the trade cost increment δ. In particular, a directional derivative of import consumption z in trade cost increment is denoted as

$$z'_\delta \equiv \lim_{\delta \to 0^+} \frac{z(1+\delta) - z(1)}{\delta}.$$

The comparative statics starts with changes near free trade. Here welfare decreases in trade costs, as the following remark shows (see the proof in the earlier versions Bykadorov et al., 2015b, 2016).

Remark 1. Consider the Krugman model with two symmetric countries near free trade, when trade costs $\tau = (1 + \delta)$ increase, departing from $\tau = 1$. Then, (i) the domestic per variety consumption x increases ($x'_\delta > 0$), imports decrease ($z'_\delta < 0$); (ii) the mass of firms N and gross domestic consumption $X \equiv Nx$ both decrease, resulting in a welfare decrease:

$$N'_\delta < 0, \quad X'_\delta < 0, \quad W'_\delta < 0.$$

Having shown quite predictable behavior of welfare near free trade, now we examine the complete path of trade liberalization. A numerical example in Fig. 1 demonstrates non-monotone evolution of welfare $U = N \cdot (u(x) + u(z))$. It includes also the decomposition of this effect into non-monotonicity of two ingredients: variety N and gross consumption Q.[10] One can see a sufficiently big interval $(1.76, 2.33) \ni \tau$ of harmful trade. It means that a social planner would increase welfare by *prohibiting trade* for all trade costs $\tau \in (1.76, 2.33)$, i.e., enforcing autarky until costs fall below 1.76. Thus, under variable markups, social optima can differ substantially from the market outcome. In particular, here, more than one-quarter of the liberalization process could be in vain.

Now we show such non-monotonicity of welfare formally, focusing on the right end of the trade liberalization path $[1, \tau^a]$. Here τ^a is the autarky trade cost, defined as a maximal value for which countries do trade: $\tau^a \equiv \sup\{\tau | z(\tau) > 0\}$. To formulate a

[10] Here, the employed utility is CARA: $u(x) = 1 - e^{-bx}$, with $b = 0.25$. Other parameters are $L = 1$, $c = 3.33$, $F = 3$.

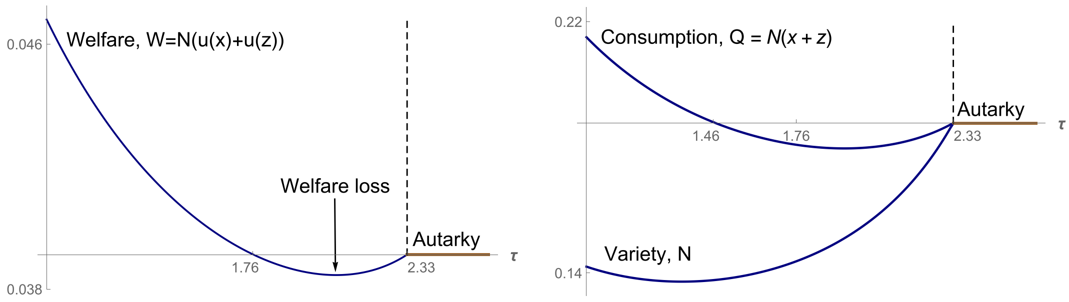

Fig. 1. Non-monotone variety (mass of firms)N, gross consumption $Q = N(x+z)$, and consumer welfare $U(x,z)$. Variety N is rescaled by a factor of 1.5 for better exposition.

related proposition in terms of trade liberalization, we express the downward directional departure from the autarky trade costs as $\tau(\varepsilon) = \tau^a - \varepsilon$ where $\varepsilon \geq 0$. For example, a directional derivative of import consumption z in trade liberalization is denoted as

$$z'_\varepsilon \equiv \lim_{\varepsilon \to 0^+} \frac{z(\tau^a - \varepsilon) - z(\tau^a)}{\varepsilon}.$$

Proposition 1. *Consider the Krugman model with two symmetric countries at the beginning of trade liberalization, when trade costs $\tau = \tau^a - \varepsilon$ depart from the autarky level τ^a,and the countries start trading. Then (i) the domestic per variety consumption x changes negligibly ($x'_\varepsilon = 0$); (ii) the consumption of import increases ($z'_\varepsilon > 0$); (iii) the mass of firms N and gross domestic consumption $X \equiv Nx$ both decrease, resulting in a welfare decrease:*

$$N'_\varepsilon < 0, \quad X'_\varepsilon < 0, \quad W'_\varepsilon < 0.$$

Corollary. *There is a non-empty cost interval $(\bar{\tau}, \tau^a)$ near autarky, where welfare decreases with trade liberalization, i.e., increases in trade costs: $W'_\tau(\tau) > 0, \ \forall \tau \in (\bar{\tau}, \tau^a)$.*

Proof. Showing $x'_\varepsilon = 0$, $\lambda'_\varepsilon = 0$, $z'_\varepsilon > 0$, and $N'_\varepsilon < 0$ is rather straightforward and is relegated to the Appendix A.1. It requires one to take a total derivative of the system of equilibrium Eqs. (7) and (8) with respect to the trade liberalization parameter ε and evaluate it at the autarky point $\tau = \tau^a$. As it turns out, the domestic per variety consumption remains unchanged, while imports increase at the expense of shrinking variety.

Showing $W'_\varepsilon < 0$ is more instructive. To find the welfare increment near autarky, use Eq. (9) and consider the representative consumer's welfare function, maximized in (x,z):

$$V(p^x, p^z, N) \equiv \max_{(x,z)} N(u(x) + u(z)) \ \ \boldsymbol{s.t.} \ N(p^x x + p^z z) = 1.$$

When we change ε, the increment of V reflects related variation in $(p^x(\varepsilon), p^z(\varepsilon), N(\varepsilon))$. The result, by the envelope theorem, can be expressed as a direct effect on the objective function and as an indirect effect (through re-optimization) of changing the budget constraint:

$$\mathrm{dV} = \underbrace{(u(x) + u(z))\mathrm{dN}}_{\text{directeffectof dN}} - \lambda \underbrace{\left(\mathrm{Nxdp}^x + \mathrm{Nzdp}^z + (p^x x + p^z z)\mathrm{dN}\right)}_{\text{budgetchangedueto } (\mathrm{dp}^x, \mathrm{dp}^z, \mathrm{dN})}, \tag{11}$$

where $\mathrm{dp}^x = (p^x)' d\varepsilon$, $\mathrm{dp}^z = (p^z)' d\varepsilon$, $\mathrm{dN} = N'_\varepsilon d\varepsilon$ are the changes in these magnitudes along the path of trade liberalization. Here, $dp^x = 0$ because firms do not change the price $p^x = \frac{u'(x)}{\lambda}$ of the domestic good, having $x'_\varepsilon = 0$ and $\lambda'_\varepsilon = 0$. Simultaneously, consumers at autarky do not buy any imports. Therefore, no money is saved after an import-price decline, $zdp^z = 0$. Thus, *no welfare changes stem from prices.*

What remains are the direct and indirect effects of the change in variety N. To find the overall effect, substitute the expressions for prices from the consumer FOC (Eq. (2)) and set $z=0$ at autarky to obtain

$$\mathrm{dV} = \left(u(x) - u'(x)x\right)\mathrm{dN} < 0. \tag{12}$$

This is true since $\frac{u'(x)x}{u(x)} < 1$ means concavity of utility u, and we already know $N'_\varepsilon < 0$. So, near autarky, *the welfare effect of trade liberalization is negative*, proving the final part of Proposition 1. **Q.E.D.**

To interpret result (12) through the main economic variables, let us imagine the disequilibrium dynamics of trade liberalization. During the initial step from autarky to trade, the trade costs fall and firms start exporting, $dz > 0$, without changing their domestic production, $dx = 0$. Consequently, the *profit-maximizing size of a firm increases*. Then, the competitive pressure causes some firms to exit the market, $dN < 0$. Put differently, the amount $c(x+F)\mathrm{dN}$ of labor resources released is relocated now to production of exports. From the consumer's point of view, this amount corresponds to spendings previously captured by the exiting firms, which is now relocated to imports. The positive welfare impact of such relocation is represented by the second summand of dV in (11). This summand, however, is less than the first one, which represents the direct change in welfare due to decreasing variety. As we explain below, a monopolist captures the consumer surplus only imperfectly, and total welfare falls.

Specifically, consider consumers who mirror the shift in trade costs by reallocating money saved on (exiting) domestic varieties, $-p^x x dN \approx p^z N dz$, to purchases of imports that are now cheaper. Multiplying such reallocation of expenditures by the marginal utility of income λ and expressing $p^x = \frac{u'(x)}{\lambda}$, consumers evaluate the direct utility of their increased spending on imports as $|u'(x)x\mathrm{dN}|$. This value, however, does not take into account the losses from decreasing variety, $|u(x)\mathrm{dN}| > |u'(x)x\mathrm{dN}|$.[11] As a consequence, the overall welfare effect (12) of trade liberalization is negative.

In contrast to this technical description, the next section explores the harmful trade effect from the causal perspective.

4. Explanations of harmful trade and market regulations

What economic mechanism can lie behind the *negative* welfare effect (12) of trade liberalization near autarky? How can it be that supplying goods to foreign consumers becomes cheaper, yet welfare declines?

Intuition suggests the influence of some market *distortion* aggravated by trade. A possible candidate is the famous Dixit-Stiglitz "excessive entry" distortion: the mass of firms is too high, and firm output is too low when the utility function belongs to DEU class, i.e., has a decreasing elasticity. Our framework allows for such a utility but also shows that trade liberalization *mitigates* this entry distortion (because N goes down) instead of aggravating it. So, there should be some other sources of market failure.

To resolve this puzzle, we rely on Dhingra and Morrow (2019). They note that, when the market chooses between technologically cheap and expensive methods of production (domestic and export production, in our case), there can be a distortion between the two. Namely, in a very general (for any preferences) Melitz-type closed economy, they find a remarkable distinction: as with homogenous firms, the Dixit–Stiglitz "excessive entry" distortion occurs only under decreasing elasticity of utility (DEU), whereas "waste of resources for inefficient production" occurs under increasing elasticity of demand (IED). Somewhat similarly, for various industrial structures, Weyl and Fabinger (2013) show that *increasing elasticity of demand* (IED) typically generates prices that bear *incomplete pass-through*, i.e., *the additional costs are not fully translated into a price increase*.

We argue that this feature of demand (IED) and incomplete pass-through is responsible for our harmful trade near autarky. Though IED property is not assumed in Proposition 1, but it naturally arises near autarky. It means that the inverse demand for imports $p_z = u'(z)/\lambda$ displays an increasing in z (absolute value of) elasticity $|E_z u'(z)| = r_u(z) \equiv -\frac{zu''(z)}{u'(z)}$ near $z \approx 0$. Indeed, at $z \approx 0$, we have zero elasticity $E_z u'(z) = 0$ because of finite non-zero derivatives u', u'' of the utility function and zero consumption level. Combining this conclusion with positive demand elasticity outside zero $|E_z u'(z)| > 0$, generally assumed for monopolistic competition (1), we obtain IED near zero consumption.[12]

As a result of this property, producers near autarky are under-pricing their exports, relative to the domestic price times trade costs:

$$p_z = \frac{\tau c}{1 - r_u(z)} < \tau p_x = \frac{\tau c}{1 - r_u(x)},$$

because of IED, where $r'_u(\cdot) > 0,\ x > z$.[13] In other words, near autarky, a consumer buys too much of imported products, paying more than a social planner would spend on producing and transporting imports. Importantly, the selling price is not lower than the marginal cost τc, and there is no absolute waste of resources or *absolute dumping*. However, there is a *relative* waste/dumping because the same resources should rather be spent on domestic production. Of course, this effect of variable markups could not be noticed under traditional CES modeling, which excludes any distortions or autarky at finite trade costs.

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[11] In other words, the utility of increasing (small-scale) per variety import consumption is too small to outweigh the decreasing utility of gross domestic consumption, which shrinks together with N.

[12] Similarly, near autarky, the elementary utility of imports $u(z)$ must display *decreasing* elasticity because, at $z \approx 0$, elasticity $E_z u(z) = 1$ by definition of u', whereas outside zero, this elasticity is less than one for any concave utility u.

[13] Such "relative dumping" in trade is shown in Brander and Krugman (1983) for oligopoly and in Kichko et al. (2014) for monopolistic competition with VES.

To further clarify our explanation and the general mechanism of trade distortions in VES monopolistic competition, let us conduct the following thought experiments with possible governmental regulations.

4.1. Entry distortion corrected by licensing

As the first step of our discussion, we investigate the role of entry inefficiency in generating the discovered effect. We put forward the following question: Are welfare losses near autarky caused by the aggravation of the *existing* Dixit–Stiglitz distortion or by the *creation* of a new distortion?[14]

Consider an autarky economy, where a social planner has corrected the excessive variety distortion by setting the appropriate license cost $f > 0$ for any firm, which makes the equilibrium match the social optimum. Respectively, each firm now faces bigger fixed costs $F + f$. Unlike cost F, the proceeds f from licensing are redistributed to consumers whose income becomes $1 + Nf/L$, instead of $w = 1$. What changes in the equilibrium Eqs. (2)–(6) is that the price is normalized by income $1 + Nf/L$, instead of $w = 1$, and the total costs $C(Q)$ are replaced by $C(Q) + f$ in the zero-profit condition.

Does the harmful trade effect disappear when the mass of firms is regulated to be at a socially optimal level?

Remark 2. When trade liberalization starts from any regulated equilibrium with (optimal or not) license cost $f \geq 0$, the main conclusion of Proposition 1 and its corollary remains valid, in the sense that there is a non-empty cost interval $(\bar{\tau}, \tau^a)$ near autarky, where welfare decreases with trade liberalization, i.e., increases in trade costs: $W'_\tau(\tau) > 0,\ \forall \tau \in (\bar{\tau}, \tau^a)$.

Proof. See Appendix A.2.

4.2. Restricted entry, output or dumping

Having established that the initial entry distortion does not play an essential role in harmful trade, now we turn to other competing (but not necessarily contradicting) explanations. We explore these via various governmental regulations. First, by forbidding entry/exit we show that *decreasing variety*, though being the main element of the key formula (12), is not the sole cause of the losses. Second, by forbidding changes in firms' output we demonstrate that business stealing (the *general equilibrium effect*) is not necessary for harmful trade. Finally, in contrast to these two regulations, we show that preventing *dumping* does eliminate the welfare losses. In this sense, incomplete path-through and dumping are more unambiguously connected with harmful trade than decreasing variety or business stealing.

4.2.1. Fixed entry/variety

We demonstrate that, even in a regulated economy where the mass of firms is forced to be constant, marginal trade liberalization near autarky remains harmful.

This setting assumes a governmental regulation that prohibits launching or closing new firms during trade liberalization. We fix mass N at the autarky level $N = \bar{N}$. (More generally, our results hold under any other fixed mass N, and any other fixed cost $f \geq 0$, but we stick to $\bar{N}$ and initial f that allow for a better comparison.) Respectively, to maintain a general equilibrium model, we should assume that the government covers the firms' losses (if any) via lump-sum transfers from consumers. Put differently, any profits (positive or negative), defined as $\pi = (p^x - c)x + (p^z - \tau c)z$, are now redistributed to consumers, and we specify such profit transfer in the budget constraint.

For such a regulated economy, a *fixed-variety equilibrium*

$$\left(x^{\#}, z^{\#}, p^{x^{\#}}, p^{z^{\#}}, \lambda^{\#}\right)$$

is defined by: (a) producers' FOCs (6); (b) inverse demands from consumers' FOCs (2):

$$c = \frac{R'(x)}{\lambda}, \quad c \cdot \left(\tau^a - \varepsilon\right) = \frac{R'(z)}{\lambda}; \tag{13}$$

and (c) the labor balance (here and below we normalize population $L = 1$ for simplicity):

$$\bar{N} \cdot (cQ + F) = 1.$$

[14] We are grateful to Andrés Rodríguez-Clare for suggesting this question.

Consequently, the fixed mass of firms entails a fixed output,

$$x+\tau z=\bar{Q}\equiv\frac{1/\bar{N}-F}{c}.$$

Before demonstrating (in Remark 3) that harm from trade remains even in such a regulated economy with fixed N, we introduce now a similar setup with fixed output.

4.2.2. Fixed output

Now, instead of fixing the mass of firms and redistributing profits, assume that the government restricts the firm's output at the autarky level $Q=\bar{Q}$, allowing the market competition to regulate entry and profits. One could expect exactly the same equilibrium as the previous one, because the market clearing condition entails the fixed mass of firms (connected with the fixed output). However, the direct restriction on Q modifies the behavior of firms. Instead of choosing x and export z, now any firm maximizes profit only in x as

$$\max_x \pi[x,(\bar{Q}-x)/\tau],$$

deriving variable z from the constraint $x+\tau z=\bar{Q}=\text{const}$. The related FOC reads as

$$\left(\frac{u'(x)}{\lambda}(1-r_u(x))-c\right)-\frac{1}{\tau}\left(\frac{u'\left(\frac{\bar{Q}-x}{\tau}\right)}{\lambda}\left(1-r_u\left(\frac{\bar{Q}-x}{\tau}\right)\right)-c\tau\right)=0. \tag{14}$$

A *fixed-output equilibrium*

$$\left(x^{\#},z^{\#}=(\bar{Q}-x)/\tau,p^{x^{\#}},p^{z^{\#}},\lambda^{\#},N^{\#}\right)$$

is defined by: (a) producers' FOC (14); (b) inverse demands from consumers' FOCs (2); (c) zero-profit condition; and (d) the labor balance.

In this situation, during the trade liberalization, the mass of firms remains stable, firms only increase export z and reduce x to keep the output size fixed. Near autarky, the competition index λ remains constant (see Appendix A.3) and the zero-profit condition holds. Therefore, the general equilibrium effects (the spillovers among firms) are essentially eliminated. In this setup welfare also decreases with (small-scale) trade.

The following remark extends Proposition 1 and summarizes the outcomes of both thought experiments above.

Remark 3. Initial departure from autarky is harmful in a regulated equilibrium with fixed (autarky) variety level $\bar{N}$, as well as in a regulated equilibrium with fixed (autarky) output level $\bar{Q}$.

Namely, in both cases there is a non-empty cost interval $(\bar{\tau},\tau^a)$ near autarky, where welfare decreases with trade liberalization, i.e., increases in trade costs: $W'_\varepsilon(\tau-\varepsilon)<0,\ \forall\tau\in(\bar{\tau},\tau^a)$.

Proof. See Appendix A.3.

The above thought experiments suggest the following interpretations of the harmful trade effect.

In the setting with fixed N, the loss in variety is eliminated but market spillovers remain. Specifically, trade liberalization has two opposite effects on a firm's profit. On the one hand, the firm experiences a positive foreign demand shock, i.e. a lower trade cost τ reduces export prices and allows the firm to marginally increase its profits by starting trade. On the other hand, since foreign firms act symmetrically, the domestic consumers relocate a part of their consumption from domestic to imported varieties. As it is shown in the proof of Remark 3(i), here the overall competitive pressure λ among firms increases with liberalization. This shift means a negative (domestic) demand shock. By the envelope theorem, consumption switching (the substitution effect) has a zero first-order effect on the firm's profit. The resulting change is only due to $d\lambda$ and $d\tau$:

$$d\pi=-\frac{u'(x)}{\lambda}\frac{d\lambda}{\lambda}x+\left(-\frac{u'(z)}{\lambda}\frac{d\lambda}{\lambda}-\text{cd}\tau\right)z.$$

One can see that the change in export profit is negligible because $z=0$ at the initial autarky point. However, the indirect effect that firms exert on each other through competition has a non-negligible negative impact on profits. In other words, each firm, seeking to marginally increment its export profit, creates an externality on foreign firms via the competition mechanism. In this medium-run equilibrium, welfare falls because negative profits are covered directly from the pocket of consumers. In the long run, some firms exit the market and the zero-profit condition is restored.

The setting with fixed Q is also instructive, because it indicates that the named externality effects are not necessary to explain the harmful trade effect. Indeed, here, all aggregate variables such as λ and N do not change near autarky, yet, the harmful trade effect is preserved. The remaining factor to explore is dumping. It leads to the export distortion, formally defined as a wedge between the marginal rate of substitution (x for z) and the marginal rate of technical substitution. The following argument emphasizes the role of variable markups and pricing to market as the most important driving force of harmful trade, working independently from the general equilibrium effects.

4.2.3. Anti-dumping

To highlight the export distortion as the main source of the harmful effect, let us consider an *anti-dumping policy*, targeted to restore efficiency. Assume that the governments of both countries either cooperatively impose the direct anti-dumping pricing requirement: $p_z \equiv \tau p_x$, or indirectly suppress dumping by facilitating competitive resale between domestic and foreign markets (such competitive arbitrage also eliminates market-segmentation pricing).

Then the ratio of consumer FOCs (2) implies the relation $\tau u'(x) = u'(z)$, equivalent to $MRS = MRT$. This relation, combined with fixed output $x + \tau z = Q = (1/\bar{N} - F)/c$, or with a fixed number of firms, determines the equilibrium variables x, z. Let us ensure that welfare in such a setting grows monotonically with trade liberalization. Indeed, one can note that the same solution x, z as in the equilibrium, would result from the direct maximization (by a social planner) of the gross utility $u(x) + u(z)$ under the same quantity constraint. The optimal value always grows monotonically when constraints are relaxed, which proves our claim.

Second-best optimality of such no-dumping equilibria leads to the hypothesis that it is dumping that bears the main responsibility for harmful trade in other settings. As to the important question of similar regulation in the original Krugman's model, our simulations confirm the hypothesis that the anti-dumping policy eliminates harmful trade, as with the setting with fixed mass of firms.[15]

Comparing our conclusions with Brander and Krugman (1983), we note that our setting with fixed N and its outcome both resemble the export/domestic misallocation of resources in their paper. There, like in our model, dumping leads to wasteful transportation and harmful trade (in their baseline model with fixed entry). However, they depart from our setting in assuming quasi-linear preferences, homogenous good and a Cournot oligopoly. This departure explains why harmful trade disappears when they introduce free entry, unlike our main setting.

To summarize, in all three cases (Proposition 1, Remark 2, Remark 3), we see that harmful trade is explained by misallocation of consumption rather than by changes in variety per se or by aggravated distortion of entry. In all these situations, the initial phase of trade liberalization becomes harmful because *the resources saved in this phase are spent on transportation of imports*. Such transportation is *relatively wasteful* near autarky. It could have increased social welfare if spent on domestic production. This tendency holds with or without fixing the mass of firms and with or without correcting excessive entry.

Having discussed the nature of the harmful effect, we now turn to its robustness by considering a series of extensions.

5. Heterogenous firms and other extensions

5.1. Heterogenous firms

As the main robustness check, we analyze a heterogenous firm model. It is worth considering because endogenous firm selection provides an additional welfare margin (see Melitz and Redding, 2015). Does better selection of productive firms under trade eliminate the harmful trade effect?

To answer this question (limiting our attention to two symmetric countries), we extend the Krugman VES model to a Melitz-type world economy with arbitrary non-CES preferences and an autarky point τ^a. This model differs from Melitz (2003) in two ways: (i) productivity is bounded from above (to allow for autarky at finite trade costs); (ii) preferences are non-CES.

The layout of our heterogenous model, relegated to Appendix B.1, relies on similar VES models for a closed economy proposed in Zhelobodko et al. (2012) and Dhingra and Morrow (2019). Among trade studies, our setting is very close to the baseline model in Arkolakis et al. (2019) but we assume non-zero fixed costs of production/export $f_x > 0$, $f_z > 0$ and a bounded productivity distribution. Both assumptions make a difference for the welfare conclusion, which we explain in more detail in this subsection.

We have proven an analog of Proposition 1 under reasonable conditions on $f_*, u(\cdot)$. In particular, unlike the homogenous case, decreasing elasticity of utility is now required for the main result.

Proposition 2. *Consider a Melitz-type heterogenous trade model with two symmetric countries, VES preferences that exhibit a choke price, and a bounded maximum productivity. (See Appendix B.1 for details.)*

Denote by τ^a the autarky trade cost, at which consumers stop purchasing imports from even the most productive foreign firms, then:

(i) *Under bounded fixed trade cost $f_z \leq f_x$ and a decreasing elasticity of utility function u, there is an interval $(\bar{\tau}, \tau^a)$ of marginal trade costs where welfare decreases in trade liberalization (a reduction in marginal trade cost τ).*

[15] Proving such a theorem for Krugman's model remains a problem.

(ii) *The same conclusion holds true when trade liberalization goes through a reduction in the fixed trade cost f_z, holding the marginal trade cost $\tau \equiv \tau^a$ fixed (welfare decreases in trade liberalization on some interval of f_z).*

Proof. See Appendix B.1.

5.1.1. Discussion

The essence of this new proposition looks similar in nature to Proposition 1. Intuitively, when trade costs initially decrease, only the most efficient firms with productivity $\varphi_{max} < \infty$ start exporting. So, at this moment, firms that react to decreasing trade costs constitute an almost homogenous population of type φ_{max}. Thus, one could think that, near autarky, the heterogeneity of exporting firms should not play a visible role. However, it is not the case. Unlike Proposition 1, the mechanism of welfare losses now operates through the change in the export cutoff, and not only through increasing export of already trading firms. It is increasing range of trading firms that becomes the main driver of the (inefficient) gross export increase, with similar consequences for welfare.

As for the importance of the assumptions used, the decreasing elasticity of utility (DEU) property is usually perceived as realistic.[16] Further, $f_z \le f_x$ is another reasonable assumption, meaning that the fixed costs of exporting are not bigger than the fixed costs of production.[17]

Turning to other assumptions, we should explain why Arkolakis et al. (2019), who also suppose a choke price in their heterogenous model, do *not* find welfare losses from trade. There are two differences between their baseline model and ours. First, they assume an unbounded productivity distribution and so there is no autarky at finite trade costs. Second, unlike our generalized Melitz approach, Arkolakis et al. (2019) employ a generalized Melitz–Ottaviano approach, without fixed costs of production or exporting. Only the choke price determines the firms' exit from domestic and foreign markets. Then, the baseline model of Arkolakis et al. (2019) generates a *fixed* mass of entrants during trade liberalization and no harm from trade. In simulations Arkolakis et al. (2019) separately explore the model versions either with fixed costs or with a bounded distribution of productivity—and again do not observe harmful trade. By contrast, the combination of bounded productivity and positive fixed costs generates our harmful effect under heterogenous firms. To verify the necessity of fixed costs, in Appendix B.2 we prove Remark 4, showing that the harmful trade effect vanishes when fixed export costs tend to zero.

Continuing on the role of fixed costs, we note that the conclusion of Remark 4 differs from Proposition 1 (homogenous model), where the welfare loss occurs *without* fixed exporting costs. To explain this discrepancy, in Appendix B.3, we present a formula for changes in the production costs of a firm, that highlights the difference. In the homogenous case, the welfare loss from trade liberalization operates through the intensive margin of trade: an increase in exports per firm. In the heterogenous case, the intensive margin of trade is negligible, because of the negligibly small mass of firms that start trading. However, the extensive margin is non-negligible: the export cutoff falls and the range of trading firms increases. This leads to a non-negligible change in total exports and a welfare loss. By contrast, if fixed exporting costs were zero, firms would enter into the export market with almost zero export volume z. This would nullify the welfare impact of the extensive margin, and thereby the overall change in welfare (see Appendix B.3). That is why fixed costs of exporting are important for welfare loss in the heterogenous model but not in the homogenous one.

Does the difference in economic mechanisms in homogenous/heterogenous models generate notably different *magnitudes* of the harmful effect? There are several important papers that differently estimate gains from trade and the impact of heterogeneity: Arkolakis et al. (2012, 2019), Melitz and Redding (2015), and Fernandes et al. (2019). We refer to these papers for detailed discussion and confine ourselves to the following numerical example.

Fig. 2 illustrates the size of harmful effect under heterogeneity.[18] Our welfare simulation uses the utility function and parameter specifications as in the baseline calibration by Arkolakis et al. (2019), but supplement these with non-zero fixed production and export costs, f_x and f_z and bound $\varphi_{max} = 2.85$ from Melitz and Redding (2015). Like in the homogenous economy, we observe an interval of trade costs where trade is harmful. However, the welfare loss (measured in equivalent variation) amounts to only 0.01% of GDP, as shown in the magnified graph (right panel).

Fig. 2 suggests that the trade loss is essentially *mitigated* by heterogeneity and selection of productive firms into exporting. A very small fraction of firms starts exporting near autarky, unlike *all* firms in the homogenous case. Moreover, firms that start trading are among the most productive ones, so, the iceberg transportation costs incurred by these firms are small. As a result, the overall welfare loss appears rather small.

Let us look at the range of trade costs where the harmful effect appears. In the simulation above, welfare losses happen at trade cost $\tau \in (2.4, 3)$. This interval lies above $\tau \approx 1.83$, the contemporary trade cost level estimated in the literature (Melitz and Redding, 2015). Taking into account this remote range and the small magnitude of losses, we expect that it is improbable to observe the trade loss empirically. What matters for economists is not the U-shaped welfare evolution per se, but rather the

[16] "I find it more plausible that revenues should be a declining fraction of the aggregate benefits generated by a product as price is lowered and quantity increased." Spence (1976, p. 233).

[17] In principle, when the difference $f_z < f_x$ is too big, it could generate an unconventional ordering of cut-offs, i.e., firms exporting without domestic production. However, it does not create a problem for Proposition 2 because near autarky $\bar{\varphi}_z \approx \varphi_{\max} > \bar{\varphi}_x$, by construction.

[18] The second graph is a magnified version of the first one and shows welfare losses from trade near autarky point $\tau = 3$. The utility function is $u = \frac{(x+\alpha)^{1+\gamma} - \alpha^{1+\gamma}}{1+\gamma}$ with $\gamma = -0.253$ and $\alpha = 1.5$. Other parameters are $L = 1, f_e = 1$ (sunk cost of entry), $f_x = 1, f_z = 0.535$. The productivity distribution is Pareto $G(\varphi) = \frac{1-\left(\frac{\varphi_{\min}}{\varphi}\right)^{\theta}}{1-\left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^{\theta}}$ with upper bound $\varphi_{max} = 2.85$, lower bound $\varphi_{min} = 1$ and the shape parameter $\theta = 5$.

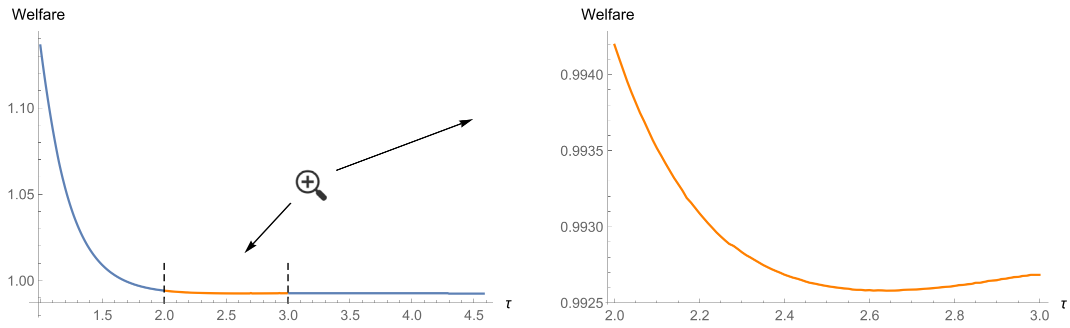

Fig. 2. Welfare gains from trade (measured in equivalent variation) relative to the reference point $\tau = 1.68$.

long flat tail (L-shape) of welfare evolution, caused by the mechanism described in our paper. Similarly, Melitz and Redding (2015, p. 1133) note that welfare gains from a 1% decrease in trade costs are higher "when the economy is relatively open than when it is relatively closed."

Common sense would expect the first breath of trade freedom to be the most desired one, unlike the last step, where consumers are "satiated with freedom." Modeling tells us the opposite: the last step contains the greatest rewards.

5.2. Harmful small-scale trade without autarky

Previous sections have shown that all VES utility functions that feature a choke-price should generate harmful trade. This subsection intends to extend this class, showing that our assumption of choke-price is not necessary. Namely, we can transform *any* utility function u that generates harmful trade into a function u_ε that preserves this effect, but does not have a choke-price. To do so, we slightly disturb u by adding a CES function as follows:

$$u_\varepsilon(x) \equiv u(x) + \varepsilon \cdot x^\rho, \qquad \rho \in (0, 1). \tag{15}$$

Under sufficiently small ε such convex combination generates non-monotone welfare without creating a finite autarky point.

To prove this claim, we follow Dhingra and Morrow (2019) method of finding a "potential function" that imitates equilibria. Namely, one can show that the equilibrium equations of Krugman's model are the same as equations of the optimization program that maximizes the gross world revenue:

$$\max_{(x,z,N)\geq 0} U(x,z,N,\varepsilon) \equiv N \cdot (xu'_\varepsilon(x) + zu'_\varepsilon(z)) \qquad s.t. \qquad N \cdot (x + \tau z + f) \leq L. \tag{16}$$

The constraint at any maximum becomes active, so, we can plug N into the objective function as $L \cdot (xu'_\varepsilon(x) + zu'_\varepsilon(z)/(x + \tau z + f)$. This is a strictly quasiconcave function, because it is a strictly concave function divided by a linear function of x, z. Therefore, any equilibrium is an argmaximum of a strictly quasiconcave function of x, z on a convex domain R^2_+. Besides, the objective function changes *continuously* in parameter ε. Consequently, by the Maximum theorem, the argmaximum (x, z) also changes *continuously* in parameter ε. This means that a usual equilibrium obtained under initial utility $u(\cdot)$ and any costs τ—can be *arbitrarily closely approximated*, using a sufficiently small parameter ε. So, the whole non-monotone path of initial equilibria (obtained with initial utility $u(\cdot)$, with all $\tau \in [1, \infty)$) can be arbitrarily closely approximated by a path of new equilibria, obtained with u_ε. So, the new path can be made non-monotone, as we needed to prove.

5.3. Asymmetric countries or non-uniform trade costs

To check the robustness of our harmful effect, we explored some additional extensions for the homogenous firm model.

5.3.1. Non-uniform distances/costs among the trading countries

This setting involves $K + 1$ countries as in Proposition 1, but they are symmetrically located on a Salop circular space. Thus, each distinguishes its neighbors from remote partners. Starting from free trade, an increase in the common trade costs coefficient τ makes some partners (couples) stop their exchange, starting with the most remote ones, then closer ones, and so on. In this setting, it remains beneficial to stop too-small trade between any couple of countries, keeping intact the trade that is more essential in volume. (See Online Appendix and an earlier version of this paper, Bykadorov et al., 2016). Therefore, harmful trade is not driven by equal distances between several traders or complete termination of trade. Partial termination may also be beneficial. Somewhat similarly, we studied non-equal sizes: several big and several small countries (see Bykadorov et al., 2016). The harmful effect remains the same.

5.3.2. Non-uniformly changing trade coefficients

This setting involves countries that are symmetric in size but different in trade costs. There are two groups of countries so that intra-group trade costs (such as railway costs within each continent) differ from the inter-group trade costs (such as sea transportation between two continents). When we change only the inter-group trade costs, the harmful small-scale trade effect remains intact (see Online Appendix Section B.2). This shows the robustness of harm to non-uniform changes in costs.

6. Conclusion

We have studied welfare changes during gradual trade liberalization within the canonical Krugman and Melitz models of trade, generalized to any VES (variable elasticity of substitution) preferences that generate autarky. Krugmanian trade gains appear non-monotone: they are positive near free trade but negative near autarky. Thus, trade needs a critical mass: when small in volume, it does not bring welfare gains.

The harm from small-scale trade is explained by a sort of "wasteful export" distortion. When a small reduction in trade costs opens up trade, consumer's welfare falls, because, under our assumptions, firms use "relative dumping." It means pricing their export below the domestic price times the trade coefficient (though above production and transport costs). It is not an absolute waste of resources, but a *relative* one, compared to its alternative use for gross domestic production.

Under firm heterogeneity, the harmful effect is considerably mitigated by the selection effect. Taking this into account, it is not likely that the effect can be detected empirically. Rather, we emphasize a non-monotone welfare and the intricate mechanisms of distortions in the canonical Krugman and Melitz models with general VES preferences (variable markups). Our study of distortions provides a welfare-based argument against relative dumping when trade is small in size. It also confronts the populist idea that "too vigorous trade liberalization is harmful," supporting the opposite view.

Appendix A

A.1. Proof of Proposition 1

For simplicity, let $L = 1$. The equilibrium equations are two first-order conditions (FOCs) and the zero-profit condition (A.1) and (A.2):

$$c = \frac{R'(x)}{\lambda}, \quad c(\tau^a - \varepsilon) = \frac{R'(z)}{\lambda}, \tag{A.1}$$

$$C(Q) = \frac{R(x)}{\lambda} + \frac{R(z)}{\lambda}. \tag{A.2}$$

To show that $x'_\varepsilon = 0, \lambda'_\varepsilon = 0$ and $z'_\varepsilon > 0$ at the autarky point, we take the directional total derivatives ($d\varepsilon > 0$) of these conditions in ε at $\varepsilon = 0$:

$$0 = \frac{R''(x)}{\lambda} x'_\varepsilon - \frac{R'(x)}{\lambda^2} \lambda'_\varepsilon, \tag{A.3}$$

$$-c = \frac{R''(z)}{\lambda} z'_\varepsilon - \frac{R'(z)}{\lambda^2} \lambda'_\varepsilon, \tag{A.4}$$

$$c \cdot (x'_\varepsilon - z + \tau^a z'_\varepsilon) = \frac{R'(x(\tau^a))}{\lambda} x'_\varepsilon + \frac{R'(z(\tau^a))}{\lambda} z'_\varepsilon - \left(\frac{R(x)}{\lambda^2} + \frac{R(z)}{\lambda^2}\right) \lambda'_\varepsilon. \tag{A.5}$$

Recalling that $z = 0 = R(0)$ at autarky, and substituting Eqs. (A.1) into (A.5), we obtain the negligible derivative of λ:

$$c \cdot (x'_\varepsilon + \tau^a z'_\varepsilon) = c \cdot (x'_\varepsilon + \tau^a z'_\varepsilon) - \left(\frac{R(x)}{\lambda^2}\right) \lambda'_\varepsilon \;\Rightarrow\; \lambda'_\varepsilon = 0.$$

Plugging this result into the remaining Eqs. (A.3) and (A.4), we achieve zero x increment, $x'_\varepsilon = 0$, and then (since $R''(z) < 0$ by SOC) we find that z grows:

$$-c = \frac{R''(z)}{\lambda} z'_\varepsilon \;\Rightarrow\; z'_\varepsilon > 0.$$

Since variety is $N = \frac{1}{cx+c\tau z+F}$, it is straightforward to conclude that it decreases, $N'_\varepsilon<0$.

The rest of the proof of Proposition 1 is included in Section 3.

A.2. Proof of Remark 2

Here we repeat the proof of Proposition 1 but with additional fixed parameter f. Consequently, when deriving formula (11) for welfare change, the indirect effect via the budget constraint is augmented by an additional transfer, licensing cost f. As a result, in formula (12), expression $u'(x)x\mathrm{dN}$ will be replaced by $(u'(x)x+\lambda f)\mathrm{dN}$. One can see that a new positive term f in the final expression results in

$$W'_\varepsilon = (u(x)-u'(x)x-\lambda f)\mathrm{dN}<0,$$

i.e., *decreasing* welfare near autarky, and the effect extends to some interval (by continuity of solutions). **Q.E.D.**

A.3. Proof of Remark 3

The directional total derivatives ($d\varepsilon>0$) of the equilibrium conditions in ε near $\varepsilon=0$ are

$$0=\frac{R''(x)}{\lambda}x'_\varepsilon-\frac{R'(x)}{\lambda^2}\lambda'_\varepsilon,\quad -c=\frac{R''(z)}{\lambda}z'_\varepsilon-\frac{R'(z)}{\lambda^2}\lambda'_\varepsilon,\quad x'_\varepsilon=-\tau z'_\varepsilon-z.$$

Recalling that $z=0=R(0)$ at autarky and rearranging the system of equations, we find *growing* competition and imports:

$$\lambda'_\varepsilon=c\left(\frac{R'(x)}{\lambda^2}\frac{R''(z)}{R''(x)}\cdot\frac{1}{\tau^a}+\frac{R'(z)}{\lambda^2}\right)^{-1}>0,\quad z'_\varepsilon=-\frac{1}{\tau^a}\frac{R'(x)}{R''(x)}\frac{\lambda'_\varepsilon}{\lambda}>0,$$

that entails *falling* domestic consumption,

$$x'_\varepsilon=-\tau^a z'_\varepsilon<0.$$

To find the welfare consequences of increasing imports and decreasing domestic consumption, we again take the consumer's indirect utility function,

$$V(p^x,p^z,\bar{N})\equiv\max_{(x,z)}\ \bar{N}(u(x)+u(z))\ \ \boldsymbol{s.t.}\ \ \bar{N}(p^x x+p^z z)=1+\bar{N}\pi(x,z), \tag{A.6}$$

where the mass of firms, $\bar{N}$, is now fixed, and profits are redistributed to consumers. Then, by the envelope theorem, the consequences of the trade liberalization $d\varepsilon>0$ can be expressed simply as

$$\mathrm{dV}=\lambda\cdot\underbrace{\left(-\bar{N}\mathrm{xdp}^x-\bar{N}\mathrm{zdp}^z+\bar{N}d\pi\right)}_{\text{budgetchangedueto }(\mathrm{dp}^x,\ \mathrm{dp}^z,\ d\pi)}=\lambda\bar{N}\cdot\left(-\mathrm{xdp}^x+\mathrm{xdp}^x+(p^x-c)\mathrm{dx}+(p^z-\tau c)\mathrm{dz}\right)<0. \tag{A.7}$$

Here we have excluded some terms due to equality $dN=0$ and equality $z=0$ at autarky. We see that the impact dp_x of domestic prices cancels out because it is present in expenditures and profits at the same time and $(p^z-\tau c)=0$ at autarky. This leaves us with only the negative direct effect of domestic consumption $dx<0$ on profits. Thus, *welfare decreases* during trade liberalization near autarky on some interval (by continuity of solutions). **Q.E.D.**

Appendix B

B.1. The model of harmful trade under heterogenous firms and related proofs

This section generalizes our main finding—harmful small trade—to heterogenous firms. The setting combines the Melitz trade model with general non-CES preferences, formalized in Zhelobodko et al. (2012) and Dhingra and Morrow (2019). We consider two countries, symmetric in their production technologies and population sizes. Such symmetry leads to a trivially satisfied trade balance and equal wages. For the simpler exposition, we normalize the country's population size $L\equiv1$ and the country's wage $w\equiv1$. Each variety ω can belong either to the set Ω_x of domestically produced varieties or to imported varieties, Ω_z.

B.1.1. Consumer's program

A consumer chooses consumption $\{x, z\} = \{x_\omega, z_\zeta\}_{\omega\in\Omega_x,\zeta\in\Omega_z}$ to maximize her utility from domestic (denoted as x) and imported (denoted as z) horizontally differentiated varieties subject to her budget constraint:

$$\max_{\{x,\ z\}} \int_{\omega\in\Omega_x} x(\omega)d\omega + \int_{\zeta\in\Omega_z} z(\omega)d\zeta, \tag{B.1}$$

$$\int_{\omega\in\Omega_x} p(\omega)x(\omega)d\omega + \int_{\zeta\in\Omega_z} p(\omega)z(\omega)d\zeta \le 1.$$

Using the dual variable λ and differentiating the Lagrangian, we obtain the consumer's inverse demands,

$$p(x(\omega)) = \frac{u'(x(\omega))}{\lambda}, \qquad p(z(\zeta)) = \frac{u'(z(\zeta))}{\lambda}.$$

B.1.2. Producer's program

A firm that decides to enter the market must bear sunk entry cost $f_e > 0$. Upon entry, it discovers its marginal productivity φ drawn from some cumulative distribution $G(\varphi)$ with density $g(\varphi)$, the marginal cost being $1/\varphi$. The firm then decides either to start producing or to exit the market. To produce, a firm needs fixed production cost $f_x > 0$. If producing something, a firm can also gain access to the foreign market after paying additional fixed cost $f_z > 0$ of exporting. Since each firm can be characterized by its productivity φ, and all φ-firms behave symmetrically, we can substitute indices $\omega \in \Omega_x$ and $\zeta \in \Omega_z$ with their corresponding productivities $\varphi\in[\bar{\varphi}_x, \varphi_{\max}]$ and $\varphi\in[\bar{\varphi}_z, \varphi_{\max}]$, multiplying the integral by the mass of entrants M_e. Here, the minimal productivities $\bar{\varphi}_x<\bar{\varphi}_z$ are the endogenous cut-offs for production/export, while φ_{max} is the maximal productivity feasible.

Any φ-firm's per purchase profit $\pi_z(\varphi)$ from exporting involves iceberg transportation cost τ, while domestic profit $\pi_x(\varphi)$ does not:

$$\pi_z(\varphi) \equiv \frac{u'\left(z_\varphi\right)}{\lambda} z_\varphi - \frac{1}{\varphi}\tau z_\varphi - f_z,$$

$$\pi_x(\varphi) \equiv \frac{u'\left(x_\varphi\right)}{\lambda} x_\varphi - \frac{1}{\varphi} x_\varphi - f_x.$$

A firm with productivity φ maximizes its composite profit π:

$$\begin{cases} \pi(\varphi) \equiv \pi_x(\varphi) + \pi_z(\varphi), & \text{if } \varphi \text{ decides } \textit{to export}, \\ \pi(\varphi) \equiv \pi_x(\varphi), & \text{otherwise.} \end{cases}$$

These formulas mean that exporters bear $f_x + f_z$ fixed costs: they must build a domestic plant using f_x units of labor and hire a foreign distributor for f_z units of labor. Differentiating the profit functions in outputs x, z, one obtains the firm's FOCs:

$$\frac{u'\left(z_\varphi\right)}{\lambda} + \frac{u''\left(z_\varphi\right)}{\lambda} z_\varphi = \frac{1}{\varphi}\tau \quad \forall\varphi, \tag{B.2}$$

$$\frac{u'\left(x_\varphi\right)}{\lambda} + \frac{u''\left(x_\varphi\right)}{\lambda} x_\varphi = \frac{1}{\varphi} \quad \forall\varphi. \tag{B.3}$$

Further, the fixed costs of production and export determine the productivity cut-offs $\bar{\varphi}_x$ and $\bar{\varphi}_z$, below which it is not profitable to engage in production or export. A firm with the cut-off productivity $\bar{\varphi}_x$ ($\bar{\varphi}_z$) appears indifferent between producing (exporting) or not. This dependence determines both the domestic-exit condition and the export-exit condition:

$$\pi_x(\bar{\varphi}_x) = \frac{u'\left(x_{\bar{\varphi}_x}\right)}{\lambda} x_{\bar{\varphi}_x} - \frac{1}{\bar{\varphi}_x} x_{\bar{\varphi}_x} - f_x = 0, \tag{B.4}$$

$$\pi_z(\bar{\varphi}_z) = \frac{u'\left(z_{\bar{\varphi}_z}\right)}{\lambda} z_{\bar{\varphi}_z} - \frac{1}{\bar{\varphi}_z} \tau z_{\bar{\varphi}_z} - f_z = 0. \tag{B.5}$$

The ex-ante decision to enter the market is determined by the zero-expected-profit condition (free-entry condition):

$$\int_{\bar{\varphi}_x}^{\varphi_{\max}}\left(\frac{u'\left(x_\varphi\right)}{\lambda}x_\varphi-\frac{1}{\varphi}x_\varphi-f_x\right)g(\varphi)d\varphi+\int_{\bar{\varphi}_z}^{\varphi_{\max}}\left(\frac{u'\left(z_\varphi\right)}{\lambda}z_\varphi-\frac{1}{\varphi}\tau z_\varphi-f_z\right)g(\varphi)d\varphi-f_e=0. \tag{B.6}$$

Finally, normalizing the mass of workers $L=1$, the equilibrium mass M_e of entering firms is derived from the labor balance in the economy:

$$M_e\cdot\left(\int_{\bar{\varphi}_x}^{\varphi_{\max}}\left(\frac{1}{\varphi}x_\varphi+f_x\right)g(\varphi)d\varphi+\int_{\bar{\varphi}_z}^{\varphi_{\max}}\left(\frac{1}{\varphi}\tau z_\varphi+f_z\right)g(\varphi)d\varphi+f_e\right)=1. \tag{B.7}$$

B.1.3. Definition

Trade equilibrium is a bundle $\{\lambda, M_e, \bar{\varphi}_x, \bar{\varphi}_z, \{x_\varphi, z_\zeta\}_{\varphi>0,\zeta>0}\}$ (consisting of intensity of competition, mass of entrants, two cutoffs, and two consumption schedules) that satisfies the equilibrium Eqs. (B.2)–(B.7). *Welfare* is defined as each consumer's utility level Eq. (B.1).

B.1.4. Comparative statics

Our next goal is to find how the equilibrium and the consumer's welfare react to a decrease in transportation cost τ.

Proposition. *Assume two symmetric countries with heterogenous firms in the generalized Melitz model, where the equilibrium is defined by* (B.2)–(B.7).

Denote by τ^a the autarky trade cost, at which consumers stop purchasing imports from even the most productive foreign firms, then:

(i) *Under bounded fixed trade cost $f_z \le f_x$ and a decreasing elasticity of utility function u, there is an interval $(\bar{\tau}, \tau^a)$ of marginal trade cost where welfare decreases in trade liberalization (a reduction in marginal trade cost τ).*

(ii) *The same conclusion holds true when trade liberalization goes through a reduction in the fixed trade cost f_z, holding the marginal trade cost $\tau \equiv \tau^a$ fixed (welfare decreases in trade liberalization on some interval of f_z).*

Proof. To prove claim (i), we perform the comparative statics of five equations that determine the equilibrium. We take the left-hand total derivatives of the Eqs. (B.4)–(B.6) with respect to falling trade cost τ from autarky, expressed as $\tau(\varepsilon)=\tau^a-\varepsilon$ where $\varepsilon \ge 0$ is the trade liberalization parameter. Correspondingly, the directional derivatives of the variables are denoted as $\lambda'(\varepsilon)$, $x'_\varphi(\varepsilon)$, $z'_\varphi(\varepsilon)$, $\bar{\varphi}'_x(\varepsilon)$, $\bar{\varphi}'_z(\varepsilon)$ (where e.g. $z'_\varepsilon \equiv \lim_{\varepsilon\to 0^+}\frac{z(\tau^a-\varepsilon)-z(\tau^a)}{\varepsilon}$). Here we use the envelope theorem to ignore variables $x'_\varphi(\varepsilon)$, $z'_\varphi(\varepsilon)$ among the derivatives of the profit function. Differentiating Eq. (B.6), we may also ignore changes in the productivity cutoff $\bar{\varphi}$ where it serves as the lower limit of integration because the integrand here is zero, due to the zero-profit conditions. Thus we obtain

$$\int_{\bar{\varphi}_x}^{\varphi_{\max}}\left(-\frac{u'\left(x_\varphi\right)}{\lambda}x_\varphi\frac{\lambda'(\varepsilon)}{\lambda}\right)g(\varphi)d\varphi+\int_{\bar{\varphi}_z}^{\varphi_{\max}}\left(-\frac{u'\left(z_\varphi\right)}{\lambda}z_\varphi\frac{\lambda'(\varepsilon)}{\lambda}+\frac{1}{\varphi}z_\varphi\right)g(\varphi)d\varphi=0,$$

$$-\frac{u'\left(x_{\bar{\varphi}_x}\right)}{\lambda}x_{\bar{\varphi}_x}\frac{\lambda'(\varepsilon)}{\lambda}+\frac{1}{\bar{\varphi}_x}x_{\bar{\varphi}_x}\frac{\bar{\varphi}'_x(\varepsilon)}{\bar{\varphi}_x}=0,$$

$$-\frac{u'\left(z_{\bar{\varphi}_z}\right)}{\lambda}z_{\bar{\varphi}_z}\frac{\lambda'(\varepsilon)}{\lambda}+\frac{1}{\bar{\varphi}_z}\tau z_{\bar{\varphi}_z}\frac{\bar{\varphi}'_z(\varepsilon)}{\bar{\varphi}_z}+\frac{1}{\bar{\varphi}_z}z_{\bar{\varphi}_z}=0,$$

$$\left(2\frac{u''\left(x_\varphi\right)}{\lambda}+\frac{u'''\left(x_\varphi\right)}{\lambda}x_\varphi\right)x'_\varphi(\varepsilon)-\left(\frac{u'\left(x_\varphi\right)}{\lambda}+\frac{u''\left(x_\varphi\right)}{\lambda}x_\varphi\right)\frac{\lambda'(\varepsilon)}{\lambda}=0\quad\forall\varphi,$$

$$\left(2\frac{u''\left(z_\varphi\right)}{\lambda}+\frac{u'''\left(z_\varphi\right)}{\lambda}z_\varphi\right)z'_\varphi(\varepsilon)-\left(\frac{u'\left(z_\varphi\right)}{\lambda}+\frac{u''\left(z_\varphi\right)}{\lambda}z_\varphi\right)\frac{\lambda'(\varepsilon)}{\lambda}=-\frac{1}{\varphi}\quad\forall\varphi.$$

At the autarky point, the selection effect pushes the exporting cut-off to its maximum: $\bar{\varphi}_z\to\varphi_{\max}$. Here, the second summand of the first formula above becomes negligible because of coinciding lower and upper limits of integration. However, $\bar{\varphi}_x<\varphi_{\max}$, and we obtain a local estimate of one-sided total derivative $\lambda'(\varepsilon)$:

$$\int_{\bar{\varphi}_x}^{\varphi_{\max}}\left(-\frac{u'\left(x_\varphi\right)}{\lambda}x_\varphi\frac{\lambda'(\varepsilon)}{\lambda}\right)g(\varphi)d\varphi+0=0\quad\Rightarrow\lambda'(\varepsilon)=0.$$

Using this fact, we can further simplify the remaining equations as follows:

$$\bar{\varphi}'_x(\varepsilon)=0,\qquad x'_\varphi(\varepsilon)=0\ \ \forall\varphi,$$

$$\frac{\bar{\varphi}'_z(\varepsilon)}{\bar{\varphi}_z}=-\frac{1}{\tau}<0\quad\text{if}\quad z_{\bar{\varphi}_z}\neq 0,$$

$$\left(2\frac{u''\left(z_\varphi\right)}{\lambda}+\frac{u'''\left(z_\varphi\right)}{\lambda}z_\varphi\right)\cdot z'_\varphi(\varepsilon)=-\frac{1}{\varphi}<0\quad\forall\varphi.$$

Thus the domestic cut-off does not react, while the export cut-off productivity decreases as τ falls. The left-hand side of the last equation includes the SOC of profit maximization; therefore the term in parentheses term is negative, implying that imports increase as τ falls:

$$z'_\varphi(\varepsilon)>0\quad\forall\varphi.$$

Now we can analyze the consumer's welfare (where the mass M_e of entrants is plugged):

$$W=\frac{\int_{\bar{\varphi}_x}^{\varphi_{\max}}u\left(x_\varphi\right)g(\varphi)d\varphi+\int_{\bar{\varphi}_z}^{\varphi_{\max}}u\left(z_\varphi\right)g(\varphi)d\varphi}{\int_{\bar{\varphi}_x}^{\varphi_{\max}}\left(\frac{1}{\varphi}x_\varphi+f_x\right)g(\varphi)d\varphi+\int_{\bar{\varphi}_z}^{\varphi_{\max}}\left(\frac{1}{\varphi}\tau z_\varphi+f_z\right)g(\varphi)d\varphi+f_e}.$$

(i) The one-sided total derivative of the numerator as τ falls from τ^a (using our findings) is

$$\int_{\bar{\varphi}_x}^{\varphi_{\max}}u'\left(x_\varphi\right)x'_\varphi(\varepsilon)g(\varphi)d\varphi+\int_{\bar{\varphi}_z}^{\varphi_{\max}}u'\left(z_\varphi\right)z'_\varphi(\varepsilon)g(\varphi)d\varphi$$

$$-u\left(x_{\bar{\varphi}_x}\right)g(\bar{\varphi}_x)\bar{\varphi}'_x(\varepsilon)-u\left(z_{\bar{\varphi}_z}\right)g(\bar{\varphi}_z)\bar{\varphi}'_z(\varepsilon)$$

$$=-u\left(z_{\bar{\varphi}_z}\right)g(\bar{\varphi}_z)\bar{\varphi}'_z(\varepsilon).$$

(ii) The directional total derivative of the welfare denominator, denoted by C, is rearranged using $x'_\varphi(\varepsilon)=0$ and $\varphi_{\max}=\bar{\varphi}_z$ as

$$\frac{dC}{d\varepsilon}=\int_{\bar{\varphi}_x}^{\varphi_{\max}}\left(\frac{1}{\varphi}x'_\varphi(\varepsilon)\right)g(\varphi)d\varphi+\int_{\bar{\varphi}_z}^{\varphi_{\max}}\left(\frac{1}{\varphi}\tau z'_\varphi(\varepsilon)-\frac{1}{\varphi}z_\varphi\right)g(\varphi)d\varphi$$

$$-\left(\frac{1}{\bar{\varphi}_x}x_{\bar{\varphi}_x}+f_x\right)g(\bar{\varphi}_x)\bar{\varphi}'_x(\varepsilon)-\left(\frac{1}{\bar{\varphi}_z}\tau z_{\bar{\varphi}_z}+f_z\right)g(\bar{\varphi}_z)\bar{\varphi}'_z(\varepsilon)$$

$$= -\left(\frac{1}{\bar{\varphi}_z}\tau z_{\bar{\varphi}_z} + f_z\right) g(\bar{\varphi}_z)\bar{\varphi}_z'(\varepsilon).$$

Therefore, the total derivative of the welfare function (multiplied by C^2) is

$$W'(\varepsilon)C^2 = \left(-u\left(z_{\bar{\varphi}_z}\right) g(\bar{\varphi}_z)\bar{\varphi}_z'(\varepsilon)\right) \cdot \left(\int_{\bar{\varphi}_x}^{\varphi_{\max}} \left(\frac{1}{\varphi} x_\varphi + f_x\right) g(\varphi) d\varphi + f_e\right)$$

$$-\left(-\left(\frac{1}{\bar{\varphi}_z}\tau z_{\bar{\varphi}_z} + f_z\right) g(\bar{\varphi}_z)\bar{\varphi}_z'(\varepsilon)\right) \cdot \int_{\bar{\varphi}_x}^{\varphi_{\max}} u\left(x_\varphi\right) g(\varphi) d\varphi\bigg).$$

Using the free-entry condition and the zero cut-off export-profit condition, this expression simplifies to

$$W'(\varepsilon)C^2 = \left(-u\left(z_{\bar{\varphi}_z}\right) g(\bar{\varphi}_z)\bar{\varphi}_z'(\varepsilon)\right) \cdot \left(\int_{\bar{\varphi}_x}^{\varphi_{\max}} \frac{u'\left(x_\varphi\right)}{\lambda} x_\varphi g(\varphi) d\varphi\right)$$

$$+\left(\frac{u'\left(z_{\bar{\varphi}_z}\right)}{\lambda} z_{\bar{\varphi}_z} g(\bar{\varphi}_z)\bar{\varphi}_z'(\varepsilon)\right) \cdot \int_{\bar{\varphi}_x}^{\varphi_{\max}} u\left(x_\varphi\right) g(\varphi) d\varphi\bigg) = \tag{B.8}$$

$$= \int_{\bar{\varphi}_x}^{\varphi_{\max}} \frac{u'\left(x_\varphi\right)}{\lambda} x_\varphi \cdot \left(u\left(z_{\bar{\varphi}_z}\right) g(\bar{\varphi}_z)\bar{\varphi}_z'(\varepsilon)\right)$$

$$\times \left\{\frac{u\left(x_\varphi\right)}{u'\left(x_\varphi\right) x_\varphi} \frac{u'\left(z_{\bar{\varphi}_z}\right) z_{\bar{\varphi}_z}}{u\left(z_{\bar{\varphi}_z}\right)} - 1\right\} g(\varphi) d\varphi,$$

which, using notation E for elasticities, simplifies to

$$W'(\varepsilon)C^2 = \frac{u\left(z_{\bar{\varphi}_z}\right) g(\bar{\varphi}_z)\bar{\varphi}_z'(\varepsilon)}{\lambda} \cdot \int_{\bar{\varphi}_x}^{\varphi_{\max}} u'\left(x_\varphi\right) x_\varphi \cdot \left\{\frac{E\left[u\left(z_{\bar{\varphi}_z}\right)\right]}{E\left[u\left(x_\varphi\right)\right]} - 1\right\} g(\varphi) d\varphi. \tag{B.9}$$

To evaluate the sign of this expression, we need $z_{\bar{\varphi}_z} < x_{\bar{\varphi}_x}$. In order to show it, note that, when combining zero-profit conditions ((B.4) and (B.5)) and FOCs ((B.2) and (B.3)), we get

$$-\frac{u''\left(x_{\bar{\varphi}_x}\right)}{\lambda} x_{\bar{\varphi}_x}^2 = f_x,$$

$$-\frac{u''\left(z_{\bar{\varphi}_z}\right)}{\lambda} z_{\bar{\varphi}_z}^2 = f_z.$$

The left-hand side of each equation is a monotonically increasing function of consumption because

$$-\left(\frac{u''\left(x_{\bar{\varphi}_x}\right)}{\lambda} x_{\bar{\varphi}_x}^2\right)'_{x_{\bar{\varphi}_x}} = -2\frac{u''\left(x_{\bar{\varphi}_x}\right)}{\lambda} x_{\bar{\varphi}_x} - \frac{u'''\left(x_{\bar{\varphi}_x}\right)}{\lambda} x_{\bar{\varphi}_x}^2 = -\frac{u''\left(x_{\bar{\varphi}_x}\right)}{\lambda} x_{\bar{\varphi}_x} \cdot \left(2 - r_{u'}\left(x_{\bar{\varphi}_x}\right)\right) > 0$$

due to the SOC. Therefore, in the case when $f_z \le f_x$, the consumption quantities can be ordered as $x_\varphi \ge x_{\bar{\varphi}_x} \ge z_{\bar{\varphi}_z}$.

We use this fact and decreasing elasticity $\mathcal{E}[u(z)] \equiv zu'(z)/u(z)$ of utility to conclude that, for all φ, the fraction $\mathcal{E}[u(z_{\bar{\varphi}_z})]/\mathcal{E}[u(x_\varphi)] > 1$ and the integrand in $W'(\varepsilon)C^2$ is positive, as well as other multipliers except $\bar{\varphi}_z'(\varepsilon)$. So the welfare directional (one-sided) total derivative $W'(\varepsilon)<0$ is negative at autarky point, as we needed to prove. Extension of this sign to some interval follows by continuity.

To prove claim (ii), the falling trade cost f_z near autarky is expressed as $f_z(\varepsilon)=f_z^a-\varepsilon$, where $\varepsilon \geq 0$ is the trade liberalization parameter. Correspondingly, the directional derivatives in $\varepsilon \geq 0$ of our variables are again denoted as $\lambda'(\varepsilon)$, $x'_\varphi(\varepsilon)$, $z'_\varphi(\varepsilon)$, $\bar{\varphi}'_x(\varepsilon)$, $\bar{\varphi}'_z(\varepsilon)$ (where, e.g. $z'_\varepsilon \equiv \lim\limits_{\varepsilon\to 0^+} \frac{z(f_z^a-\varepsilon)-z(f_z^a)}{\varepsilon}$)

Using these, one can just repeat the proof (i) in all details, changing only two total derivatives: formula (B.5) for zero cut-off profit and (B.2) for export FOC. Now the derivatives take the form:

$$-\frac{u'\left(z_{\bar{\varphi}_z}\right)}{\lambda} z_{\bar{\varphi}_z} \frac{\lambda'(\varepsilon)}{\lambda}+\frac{1}{\bar{\varphi}_z} \tau z_{\bar{\varphi}_z} \frac{\bar{\varphi}'_z(\varepsilon)}{\bar{\varphi}_z}+1=0,$$

$$\left(2\frac{u''\left(z_\varphi\right)}{\lambda}+\frac{u'''\left(z_\varphi\right)}{\lambda} z_\varphi\right) z'_\varphi(\varepsilon)-\left(\frac{u'\left(z_\varphi\right)}{\lambda}+\frac{u''\left(z_\varphi\right)}{\lambda} z_\varphi\right) \frac{\lambda'(\varepsilon)}{\lambda}=0 \quad \forall \varphi.$$

All derivatives here demonstrate the same sign as in the proof of claim (i), only now the export sales do not change: $z'_\varphi(\varepsilon)=0\ \forall\varphi$. This novelty does not influence the main estimate (B.9) of the welfare consequences of liberalization, because in both cases the upper and lower bounds of integration coincide. Hence, the conclusion for decreasing f_z is the same as for decreasing τ. **Q.E.D.**

B.2. Heterogenous firms and vanishing fixed export costs

To show the role of fixed export costs for the harmful trade effect, we extend the proof displayed in Appendix B.1 to a situation without f_z.

Remark 4. When fixed export costs vanish: $f_z \to 0$, negative welfare gains near autarky also tend to zero.

To prove this remark, we first note that the model remains valid and the export productivity cut-off in Eq. (B.5) is determined by a choke price. Let us reexamine formula (B.9), which expresses the welfare change. When fixed export costs vanish ($f_z \to 0$), related cut-off production $z_{\bar{\varphi}_z}$ tends to zero too, by formulae (B.2)–(B.5), i.e., $z_{\bar{\varphi}_z} \underset{f_z\to 0}{\longrightarrow} 0$. The ordering $x_\varphi \geq x_{\bar{\varphi}_x} \geq z_{\bar{\varphi}_z}$ is preserved. Consequently, the welfare change in formula (B.9) remains negative, due to $\left\{\frac{E\left[u\left(z_{\bar{\varphi}_z}\right)\right]}{E\left[u\left(x_\varphi\right)\right]}-1\right\} \geq 0$, but approaches zero, because $u\left(z_{\bar{\varphi}_z}\right) \to 0$, i.e., $W' \to 0$. **Q.E.D.**

B.3. A comparison between homogenous and heterogenous models

This subsection describes the difference in the mechanism of welfare losses in the heterogenous firm model, compared to the homogenous firm model. To do so, it is instructive to analyze the reallocation of labor resources in production costs during trade liberalization from an autarky level.

Consider the cost function of a firm in the homogenous firm model:

$$C_{\text{homog}}(\varepsilon)=\mathrm{c}x+c\tau(\varepsilon)z+f.$$

Trade liberalization from finite autarky trade costs is represented by parameter $\varepsilon \geq 0$, where $\tau(\varepsilon)=\tau^a-\varepsilon$. As was shown in the proof of Proposition 1, the directional derivatives of domestic and export sales per capita with respect to the trade liberalization parameter ε are $x'(\varepsilon)=0$ and $z'(\varepsilon)>0$ respectively. Then (normalizing $L=1$), the directional derivative of the total costs of a firm is

$$C'_{\text{homog}}(\varepsilon)=\underbrace{\mathrm{c}x'(\varepsilon)}_{=0}-\underbrace{\mathrm{c}z}_{=0}+\underbrace{c\tau z'(\varepsilon)}_{>0}, \tag{B.10}$$

where $cz=0$ because firms do not incur fixed export costs and gradually increase their exports from zero autarky level, $z=0$. Therefore, initially, production costs and exports increase solely due to the intensive margin of trade, where *all* firms export.

An analog of a firm's costs in the heterogenous firm model is the expected firm's production costs,

$$C_{\text{heterog}}(\varepsilon) = \int_{\bar{\varphi}_x}^{\varphi_{\max}} \left(\frac{1}{\varphi} x_\varphi + f_x\right) g(\varphi) d\varphi + \int_{\bar{\varphi}_z}^{\varphi_{\max}} \left(\frac{1}{\varphi} \tau(\varepsilon) z_\varphi + f_z\right) g(\varphi) d\varphi + f_e.$$

We showed in the proof of Proposition 2 that the directional derivatives of sales per capita and productivity cutoffs with respect to trade liberalization parameter ε are

$$x'_\varphi(\varepsilon) = 0,\ z'_\varphi(\varepsilon) > 0,\ \bar{\varphi}'_x(\varepsilon) = 0,\ \bar{\varphi}'_z(\varepsilon) < 0.$$

Then, the directional derivative of the total expected costs of a firm is

$$C'_{\text{heterog}}(\varepsilon) = \underbrace{\int_{\bar{\varphi}_x}^{\varphi_{\max}} \left(\frac{1}{\varphi} x'_\varphi(\varepsilon)\right) g(\varphi) d\varphi}_{=0} - \underbrace{\int_{\bar{\varphi}_z}^{\varphi_{\max}} \left(\frac{1}{\varphi} z_\varphi\right) g(\varphi) d\varphi}_{=0} + \underbrace{\int_{\bar{\varphi}_z}^{\varphi_{\max}} \left(\frac{1}{\varphi} \tau(\varepsilon) z'_\varphi(\varepsilon)\right) g(\varphi) d\varphi}_{=0}$$

$$\underbrace{-\frac{1}{\bar{\varphi}_x^2} x_{\bar{\varphi}_x} \bar{\varphi}'_x(\varepsilon)}_{=0} + \underbrace{-\frac{1}{\bar{\varphi}_z^2} z_{\bar{\varphi}_z} \tau \bar{\varphi}'_z(\varepsilon)}_{<0}\Bigg). \tag{B.11}$$

There are two principal differences with the homogenous firm model. First, during trade liberalization, firms start trading at a non-negligible positive exports $z_\varphi > 0$ (for $\varphi \in [\bar{\varphi}_z, \varphi_{\max}]$) due to positive fixed export costs, $f_z > 0$. This renders the last term in the formula above to be positive. Second, in the neighborhood of autarky, only a negligibly small population of firms enter the export market, $\bar{\varphi}_z = \varphi_{\max}$, which collapses the integrals in the second and third terms due to equality between lower and upper limits of integration. In this case, the gross intensive margin of trade (total increase in export sales per firm) is zero because only a negligibly small mass of firms was trading in the beginning. Contrarily, the extensive margin of trade is positive: there is a reallocation of resources toward exports due to a non-negligible entry of less productive firms into the export market. We note that for the "harmful" result to hold, it is necessary that firms start trading with a positive export level $z_\varphi > 0$. When there are no fixed export costs, $f_z = 0$, this requirement does not hold and the extensive margin of trade also nullifies due to $z_{\bar{\varphi}_z} = 0$.[19]

To recapitulate, the expression (B.10) and (B.11) explain the key difference in the mechanism of welfare change near autarky between the two models. In the homogenous firm model, all firms do trade near autarky, and there is only a change in the intensive margin of trade (export quantities). In the heterogenous firm model, the intensive margin of trade is absent, but the extensive margin of trade (the change in the export productivity cutoff) plays a major role.

This insight also explains a significant difference in the size of the welfare loss between the two models. Namely, it is almost negligible in the heterogenous firm model because of a relatively slow entry of new firms into the export market.

Appendix C. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jinteco.2022.103595.

References

Allen, T., Arkolakis, C., Li, X., 2020. On the Equilibrium Properties of Network Models with Heterogeneous Agents. NBER, Cambridge, USA Working Paper 27837 (27837).

Arkolakis, C., Costinot, A., Donaldson, D., Rodríguez-Clare, A., 2019. The elusive pro-competitive effects of trade. Rev. Econ. Stud. 86 (1), 46–80.

Arkolakis, C., Costinot, A., Rodríguez-Clare, A., 2012. New trade models same old gains? Am. Econ. Rev. 102 (1), 94–130.

Behrens, K., Kanemoto, Y., Murata, Y., 2017. On Measuring Welfare Changes When Varieties are Endogenous. CEPR, London, UK Discussion Paper DP11774.

Behrens, K., Mion, G., Murata, Y., Suedekum, J., 2016. Distorted Monopolistic Competition. CEPR Discussion Paper DP11642.

Behrens, K., Mion, G., Murata, Y., Suedekum, J., 2020. Quantifying the gap between equilibrium and optimum under monopolistic competition. Quart. J. Econ. 135 (4), 2299–2360.

Bertoletti, P., Epifani, P., 2014. Monopolistic competition: ces redux? J. Int. Econ. 93 (2), 227–238.

Bertoletti, P., Etro, F., 2017. Monopolistic competition when income matters. Econ. J. 127 (603), 1217–1243.

Brander, J., Krugman, P., 1983. A reciprocal dumping' model of international trade. J. Int. Econ. 15 (3-4), 313–321.

Bykadorov, I., Ellero, A., Funari, S., Kokovin, S., Molchanov, P., 2016. Painful Birth of Trade under Classical Monopolistic Competition. NRU Higher School of Economics, Moscow, Russia Working Paper BRP 132/EC.

Bykadorov, I., Gorn, A., Kokovin, S., Zhelobodko, E., 2015a. Why are losses from trade unlikely? Econ. Lett. 129, 35–38.

Bykadorov, I., Molchanov, P., Kokovin, S., 2015b. Elusive Pro-Competitive Effects and Harm from Gradual Trade Liberalization. Sobolev Institute of Mathematics SB RAS, Novosibirsk Preprint No 295.

Chen, C.-M., Zeng, D.-Z., 2014. Home Market Effects: Beyond the Constant Elasticity of Substitution. SSRN WP, Rochester, NY, USA 2538066.

[19] Theoretically, instead of $f_z > 0$ we could have assumed zero fixed exporting costs, $f_z = 0$, but exogenously require firms to start trading at some fixed positive level $z_\varphi > 0$. Then, harmful trade effect would still hold without any fixed production or export costs. This possibility highlights the idea that economies of scale and the Dixit-Stiglitz distortion do not play the principal role for our main result, as was suggested by Remark 3.

Costinot, A., Rodríguez-Clare, A., 2014. Trade theory with numbers: quantifying the consequences of globalization. In: Gopinath, G., Helpman, E., Rogoff, K. (Eds.), Handbook of International EconomicsVol. 4. Elsevier, pp. 197–261.
De Loecker, J., Goldberg, P.K., Khandelwal, A.K., Pavcnik, N., 2016. Prices, markups, and trade reform. Econometrica 84 (2), 445–510.
Dhingra, S., 2013. Trading away wide brands for cheap brands. Am. Econ. Rev. 103 (6), 2554–2584.
Dhingra, S., Morrow, J., 2019. Monopolistic competition and optimum product diversity under firm heterogeneity. J. Pol. Econ. 127 (1), 196–232.
Dixit, A.K., Stiglitz, J.E., 1977. Monopolistic competition and optimum product diversity. Am. Econ. Rev. 67 (3), 297–308.
Feenstra, R.C., 2003. A homothetic utility function for monopolistic competition models without constant price elasticity. Econ. Lett. 78 (1), 79–86.
Fernandes, A.M., Klenow, P.J., Meleshchuk, S., Pierola, D., Rodriguez-Clare, A., 2019. The Intensive Margin in Trade. CESifo, Munich, Germany Working Paper 7540 (7540).
Kichko, S., Kokovin, S., Zhelobodko, E., 2014. Trade patterns and export pricing under non-ces preferences. J. Int. Econ. 94 (1), 129–142.
Krugman, P., 1979. Increasing returns, monopolistic competition, and international trade. J. Int. Econ. 9 (4), 469–479.
Melitz, M.J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. Econometrica 71 (6), 1695–1725.
Melitz, M.J., Redding, S.J., 2015. New trade models, new welfare implications. Am. Econ. Rev. 105 (3), 1105–1146.
Morgan, J., Tumlinson, J., Várdy, F., 2020. Bad Trade: The Loss of Variety. SSRN WP 3529246, Rochester, NY, USA.
Mrázová, M., Neary, J.P., 2014. Together at last: trade costs, demand structure, and welfare. Am. Econ. Rev. 104 (5), 298–303.
Spence, M., 1976. Product selection, fixed costs, and monopolistic competition. Rev. Econ. Stud. 43 (2), 217–235.
Weyl, E.G., Fabinger, M., 2013. Pass-through as an economic tool: principles of incidence under imperfect competition. J. Pol. Econ. 121 (3), 528–583.
Zhelobodko, E., Kokovin, S., Parenti, M., Thisse, J.-F., 2012. Monopolistic competition: beyond the constant elasticity of substitution. Econometrica 80 (6), 2765–2784.