Pollution versus Inequality: Tradeoffs for Fiscal Policy

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Abstract

In this paper, we investigate the impact of redistribution and polluting commodity taxation on inequality and pollution in a dynamic setting. We build a two-sector Ramsey model with a green and a polluting good. Households are heterogeneous, which allows for income inequality, and have a level of subsistence consumption for the polluting commodity, modeled by non-homothetic preferences. Increasing the tax rate has a mixed effect depend on the level of subsistence consumption. A low level allows to tackle both the pollution and inequality issues. Under a high level of it, pollution increases: if inequality can be reduced through redistribution, taxation does not allow to solve for environmental degradation. Looking at the stability properties of the economy, we find that the level of subsistence consumption and the externality matter. A high subsistence level of polluting consumption leads to instability or indeterminacy of the steady-state, while the environmental externality plays a stabilizing role in the economy. This leaves room for taxation and redistribution: increasing the tax rate and redistributing more towards workers play a key role in the occurrence of indeterminacy and instability.

JEL classification: E62; H23; Q52.

Keywords: Externalities; Heterogeneous agents; Inequality; Pollution; Redistribution; Taxation.

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1. Introduction

The 2018 IPCC report emphasises the role of social justice and equity in limiting global warming to 1.5°C.[1] This importance has been again highlighted in the Glasgow Climate Pact of November 2021. Indeed, high income households are the ones who pollute the most (Chancel, 2021), so that the climate change and inequalities issues are linked: both are tried to be tackled in order to (i) preventing temperatures to rise above 1.5°C and (ii) preventing inequality to rise too much, which in turn impacts pollution. When it comes to reduce polluting emissions, taxation can be seen as an efficient tool regarding the internalization of the externality. Environmental taxation has two benefits: it corrects the inefficiency induced by the externality, and it provides revenue for the government. This leads to a broad use of environmental taxes in Europe and America (World Bank, 2020). Yet, environmental tax reforms face some opposition in the society, for example the Yellow Vests movement in France. Overestimating their losses, people reject the implementation of a carbon tax even if the tax revenue is directly redistributed towards them (Douenne & Fabre, 2019). Indeed, environmental taxes are usually considered regressive, as low income households spend a higher share of their income on polluting goods and hence bear most of the tax burden (Grainger & Kolstad, 2010). In this case, reducing emissions works against inequality reduction: this is the so-called equity-efficiency tradeoff. Understanding the distributional effect of environmental tax reforms is thus key when trying to reduce both emissions and inequality at the same time, while making environmental taxes socially acceptable. Recycling the tax revenue can be used to reconcile both goals.

This paper aims at analyzing the impact of environmental taxation and revenue recycling on pollution and inequality. More precisely, we study whether pollution mitigation and inequality reduction are compatible in a dynamic setting with two goods, a green and a polluting one. Inequality comes from the fact that some households are financially constrained and other are not (Kaplan et al., 2014, Aguiar et al., 2020). Indeed, heterogeneity in discount factors allows to differences in propensity-to-save and hence to recover hand-to-mouth agents who only receive a labor income (and potentially transfers). We build a two-sector Ramsey model with heterogeneous households and a subsistence level of consumption for the polluting commodity. Polluting production impacts households through their utility function. A tax is set on polluting consumption, and its revenue is given back to households through lump-sum transfers. At the equilibrium, households spend a constant share of their expenditure on each good, augmented by the subsistence level of consumption for the polluting commodity. Heterogeneity in discount factors leads to inequality: the most patient household holds all the capital.

We study the impact of both redistribution and environmental taxation on environmental quality and inequality. Because of homogeneous preferences, decreasing income inequality through lump-sum transfers does not have any impact on pollution. Hence redistribution cannot be used to tackle both inequality and pollution at the same time. Increasing the tax rate has a mixed effect on pollution. Under a low level of subsistence consumption, polluting emissions are reduced: when redistributing enough to workers, both the inequality and pollution issues are handled. Under a high level of subsistence consumption, increasing the tax rate leads to an increase in polluting emissions: environmental taxation fails to tackle the pollution problem. This comes from two effects: a negative price effect and a positive redistribution effect. The redistribution

[1]"Social justice and equity are core aspects of climate-resilient development pathways [...] as they address challenges and inevitable trade-offs, [...], without making the poor and disadvantaged worse off."

effect depends on subsistence consumption, the redistirbution rate and the expenditure share on polluting consumption. The higher these parameters the stronger this effect, such that it dominates the negative price effect after some threshold. Hence, the redistribution rate and the subsistence level of consumption matter when it comes to reduce both inequalities and pollution using taxation. They also matter when it comes to the welfare effect of environmental taxation. A low subsistence of polluting consumption and an intermediate rdistribution rate allow to increase welfare for both workers and capitalists, while at the same time reducing pollution. A high subsistence level increases welfare of each type if and only if the environmental externality does not widely impact households. If the externality has a huge impact on utility, increase the tax rate has a negative impact both on pollution and welfare. Finally, we study the dynamics of the model under a local analysis. Focusing on the neighborhood of the steady state, we show that the level of subsistence consumption and the externality matter in the stability properties of our model: a high level of subsistence consumption favors indeterminacy and instability, while an important externality brings back some stability. The environmental fiscal policy also has a role to play: a higher tax rate and greater redistribution towards workers promote indeterminacy and instability. Therefore, polluting commodity taxation and redistribution must be handled carefully when taking into account a subsistence level of polluting consumption, both for the effectiveness of the environmental tax reform and the stability of the steady state.

Our paper is linked to three strands of the literature. First, a huge part of the environmental literature analyzes the link between redistribution and pollution. Empirically, mixed or no effects are found, as for example in Lin & Li (2011). Berthe and Elie (2015) gather this literature and explain the differences in models (both theoretically and empirically) leading to this ambiguous effect. From a more macroeconomic point of view, Oueslati (2015) shows that in a two-sector endogeneous growth model, lump-sum transfers have no impact on aggregate variables, and hence on pollution. Rausch & Schwartz (2016) find that heterogeneity and non-homotheticity of preferences matter when it comes to the impact of redistribution on aggregate variables. Along the literature, we find that there is no effect of redistribution on pollution.

A second strand of interest studies the link between environmental taxation and inequality. Scalera (1996) and Hofkes (2001) look at the long-term effect of environmental taxation of endogeneous growth models. Part of this literature focuses on taxation and revenue recycling, showing the importance of the recycling scheme when it comes to make environmental taxes progressive. Klenert & Mattauch (2016) analyze the importance of subsistence consumption in the distributional effect of environmental tax reforms. Klenert et al. (2018) show the importance of lump-sum transfers in order to reduce inequality. Our paper analyzes both the impact of taxation and redistribution on pollution and inequality. Like part of the literature on redistribution and pollution we find that homogeneous preferences leads to no impact of redistribution on pollution. Alike the literature on taxation finding a positive relationship between higher taxation and lower pollution (Bovenberg and de Mooij, 1997, Bosquet, 2000), we find that introducing a subsistence level of polluting consumption mitigates this result: under a high level of subsistence consumption, the environmental tax backfires and polluting emissions rise.

Finally, our paper is linked to the literature concerned by the stability properties of models with an environmental component. Antoci et al. (2005) show that, in a growth model with two goods, consumption choices can lead to indeterminacy. E. Koskela and M. Puhakka (2006) show the possibility of two-period bifurcations in an OLG model with Stone-Geary preferences. Itaya (2008) analyzes the impact of environmental taxation on long-run growth in a model with a

representative agent and an environmental externality. He finds that the impact of taxation on growth depends on the indeterminacy of the balanced growth path. To the best of our knowledge, our paper is the first analyzing the stability properties of a Ramsey economy with both externalities and non-homothetic preferences. We find that non-homothetic preferences can lead to instability and indeterminacy, while the externality brings back stability. In each case, the fiscal policy plays a key role in the occurrence of instability and indeterminacy. Increasing the tax rate promotes instability while redistirbuting more towards workers promotes the occurrence of endogeneous cycles.
The rest of the paper is organized as follows. In Section 2, we present our framework. Sections 3 and 4 state and analyze the equilibrium and the steady state of the economy. Section 5 studies the impact of a change in taxation and in redistribution of the tax revenue. In Section 6, we analyze the local stability properties of our economy. Section 7 provides concluding remarks. All proofs are relegated to the Appendix.

2. The Model

We consider a infinite-horizon two-sector model with an environmental externality. There are three types of agents: households, firms and a government. Households are infinitely-lived and heterogeneous in their discount factors. Firms produce either a clean good, or a polluting good that exerts an externality on households' utilities. Government's intervention, through taxation and redistribution, aims at reducing environmental damages as well as income and consumption inequalities.

2.1. Firms

There are 2 sectors ($j = p, g$) in the economy, each composed of a representative firm: the clean sector produces Y_{gt} and the polluting sector produces Y_{pt} at every period t. Each sector uses capital K_{jt} and labor N_{jt} to produce output according to a Cobb-Douglas production function:

$$Y_{jt} = A_j K_{jt}^{\eta} N_{jt}^{1-\eta} \tag{1}$$

with A_j a productivity parameter. We assume $A_g \neq A_p$.

In the polluting sector, output is a consumption good c_{pt} only, while the clean sector produces a good that is consumed, denoted c_{gt}, and invested trough capital K_t. The capital good includes immaterial and non-polluting inputs used in the production process, such as R&D investment and human capital.

Each firm j ($j = p, g$) seeks to maximize its profit:

$$\pi_{jt} = p_{jt} Y_{jt} - r_t K_{jt} - w_t N_{jt}$$

with $p_{gt} = p_t$, $p_{pt} = 1$, r_t the rental rate of capital and w_t the wage rate.
Assuming that capital and labor are perfectly mobile across sectors, first-order conditions for

profit maximization give:

$$r_t = \eta p_t A_g K_{gt}^{\eta-1} N_{gt}^{1-\eta} = \eta A_p K_{pt}^{\eta-1} N_{pt}^{1-\eta} \tag{2}$$

$$w_t = (1-\eta) p_t A_g K_{gt}^{\eta} N_{gt}^{-\eta} = (1-\eta) A_p K_{pt}^{\eta} N_{pt}^{-\eta} \tag{3}$$

2.2. Pollution

We consider pollution as a flow due to production in the polluting sector. One can think for example of gases emitted during the production process such as sulfur dioxide and carbon monoxide, which have a short lifetime and hence can be considered as flow pollutants (Liu & Liptak, 2000).
Environmental quality is given by:

$$E_t = \frac{1}{\gamma Y_{pt}} \tag{4}$$

with γ the emission factor.

2.3. Households

The economy is composed of two types of infinitely-lived households ($i = \{1, 2\}$). For simplification purposes, we assume there is one household of each type. This allows to acknowledge for income inequality as well as financial constraint of part of the population which only receives and spend her labor income. At every period t, each household consumes a clean good c_{git} at price p_t and a polluting good c_{pit} which is the numeraire and taxed at a rate τ. There exists a subsistence consumption level c_0 of the polluting good, so that households cannot consume zero of it. This can be seen for instance as the minimum level of energy a household needs to consume in order to live in a decent manner. Consumption can be summarized by a basket of good C_{it} purchased at price P_{it}. Bundle price is household specific as it depends on individual bundle consumption. Households also derive some utility from environmental quality E_t. Finally, they can invest a_{it+1} in the capital good (so that $a_{1t} + a_{2t} = K_t$), supply $n_i = \frac{1}{2}$ of labor at the wage rate w_t, and receive a lump-sum transfer T_{it} from the government.
Intertemporal utility writes

$$\sum_{t=0}^{\infty} \beta_i^t E_t^{\mu} \frac{((c_{pit} - c_0)^{\alpha} c_{git}^{1-\alpha})^{1-\sigma}}{1-\sigma}, \tag{5}$$

with $\sigma \in (0, 1)$ the inverse of the elasticity of intertemporal substitution in consumption, $(c_{pit} - c_0)^{\alpha} c_{git}^{1-\alpha} = C_{it}$ and $\alpha \in (0, 1)$ the share of consumption devoted to the polluting good. Pollution is taken as given by households, such that it only plays the role of an externality in the utility function. β_i is the discount factor of household i.
We consider that:

Assumption 1 $\beta_1 > \beta_2$,

i.e. household 1 is more patient then household 2. This assumption accounts for the relationship between inequality and time discounting (Epper et al., 2020).

The budget constraint at time t writes

$$c_{pit}(1+\tau) + c_{git}p_t + p_t(a_{it+1} - (1-\delta)a_{it}) = \frac{w_t}{2} + r_t a_{it} + T_{it}.$$

Households maximize their discounted lifetime utility with respect to their budget and borrowing constraints $a_{it+1} \geq 0$.
The first-order conditions for a solution to the households' problem are:

$$c_{git}p_t = (1-\alpha)P_{it}C_{it} \tag{6}$$

$$(c_{pit} - c_0)(1+\tau) = \alpha P_{it}C_{it} \tag{7}$$

$$\frac{C_{it}^{-\sigma}}{P_{it}}E_t^{\mu}p_t \geq \beta_i \frac{C_{it+1}^{-\sigma}}{P_{it+1}}E_{t+1}^{\mu}(p_{t+1}(1-\delta) + r_{t+1}) \tag{8}$$

$$P_{it} = \frac{(1+\tau)^{\alpha}p_t^{1-\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}} + \frac{(1+\tau)c_0}{C_{it}} \tag{9}$$

with (8) holding at equality when $a_{it} > 0$. Because households consume a positive minimal amount of the polluting good, the aggregate price depends on individual consumption and hence on households' types.
Equations (6) and (7) allow to rewrite the budget constraint of household i as:

$$P_{it}C_{it} + p_t(a_{it+1} - (1-\delta)a_{it}) = \frac{w_t}{2} + r_t a_{it} + T_{it} \tag{10}$$

Equations (6) and (7) are standard optimality conditions for Cobb-Douglas utility functions: agents spend a constant share of their consumption expenditure in each good, augmented by the subsistence consumption level for the polluting commodity. Equation (8) states that households either smooth consumption using their savings or are prevented from borrowing and hence are financially constrained.

2.4. The Government

The government levies a tax on the polluting good. The tax revenue is used to reduce income inequality: a share ε is redistributed to household 1 and the rest to household 2. The government faces a balanced budget rule:

$$T_t = T_{1t} + T_{2t} = \tau(c_{p1t} + c_{p2t}) = \tau c_{pt}, \tag{11}$$

with $T_{1t} = \varepsilon T_t$ and $T_{2t} = (1-\varepsilon)T_t$ the transfers given to households 1 and 2 respectively.

3. Equilibrium

Because household 1 is more patient than household 2, we prove in the next section that at the steady state, the most patient household holds all the capital and smoothes consumption according to a binding Euler equation. On the contrary, the impatient household does not smooth consumption and holds no asset, so that $a_{1t} = K_t > 0 = a_{2t}$. We focus on equilibria around the steady-state, i.e in which this result holds.

Using equations (2)-(3) and market clearing conditions, we obtain:

$$K_t = \frac{K_{pt}}{N_{pt}} = \frac{K_{gt}}{N_{gt}} \tag{12}$$

$$K_t = K_{pt} + K_{gt} \tag{13}$$

$$N_{pt} + N_{gt} = 1 \tag{14}$$

$$p_t \equiv p = \frac{A_p}{A_g} \tag{15}$$

$$\tag{16}$$

Plugging (12) and (15) in (2)-(3) yields:

$$r_t = \eta A_p K_t^{\eta-1} \tag{17}$$

$$w_t = (1-\eta) A_p K_t^{\eta} \tag{18}$$

Substituting (17)-(18) into (8) and (10), and using the market clearing condition on the capital market for each household gives:

$$\frac{C_{1t}^{-\sigma}}{P_{1t}} E_t^{\mu} = \beta_1 \frac{C_{1t+1}^{-\sigma}}{P_{1t+1}} E_{t+1}^{\mu} \left(1 - \delta + \frac{r_{t+1}}{p}\right) \tag{19}$$

$$P_{1t}C_{1t} = \left(\tfrac{(1+\eta)}{2} + \tfrac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right) A_p K_t^{\eta} - (1 + \tfrac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}) p(K_{t+1} - (1-\delta)K_t) + \tfrac{2\varepsilon\tau(1+\tau)}{1+\tau(1-\alpha)} c_0 \tag{20}$$

$$\frac{C_{2t}^{-\sigma}}{P_{2t}} E_t^{\mu} > \beta_2 \frac{C_{2t+1}^{-\sigma}}{P_{2t+1}} E_{t+1}^{\mu} \left(1 - \delta + \frac{r_{t+1}}{p}\right) \tag{21}$$

$$P_{2t}C_{2t} = \frac{(1-\eta)(1+\tau)}{2(1+\tau(1-\alpha(1-\varepsilon)))} A_p K_t^{\eta} + (1-\varepsilon)\tau \left(\frac{\alpha P_{1t}C_{1t} + 2c_0(1+\tau)}{(1+\tau(1-\alpha(1-\varepsilon))}\right) \tag{22}$$

As we focus on an equilibrium around the steady-state, we use equations (20) and (22) evaluated at the steady-state and look for the conditions under which we have $C_{1t} > 0$ and $C_{2t} > 0$:

Assumption 2 *$c_0 \leq c_{01}$ whenever $\frac{1}{2\tau} + \frac{1-\alpha}{2} \geq \varepsilon$ and by $c_0 \leq c_{02}$ whenever $\varepsilon \geq \frac{1+\alpha}{2} - \frac{1}{2\tau}$. Values can be found in Appendix A.2.*

Households' income partly depends on the redistribution rate ε. When very few of the tax revenue is given back towards one household, the budget constraint gets tighter. In this case, a too high subsistence level of consumption for the polluting good would make the constraint impossible to hold: the minimal level of consumption would be too high compared to the disposable income.

At equilibrium, $E_t = \frac{1}{\gamma c_{pt}}$. Using this and substituting (17) in equations (19)-(20) gives the dynamic equations of the model:

$$K_{t+1} = \frac{\left(\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{(1+\tau(1-\alpha))}\right) A_p K_t^{\eta} + \frac{2\varepsilon\tau(1+\tau)}{1+\tau(1-\alpha)} c_0 - P_{1t}C_{1t}}{p\left(1 + \frac{\alpha\varepsilon\tau}{(1+\tau(1-\alpha))}\right)} + (1-\delta)K_t \tag{23}$$

$$C_{1t+1}^{\sigma} = \beta_1 \frac{C_{1t}^{\sigma}\left(P + \frac{(1+\tau)c_0}{C_{1t}}\right)}{P + \frac{(1+\tau)c_0}{C_{1t+1}}} \left(\frac{\alpha P C_{1t} + (1+\tau)c_0 + \frac{\alpha(1-\eta)}{2} A_p K_t^{\eta} + 2(1+\tau)c_0}{\alpha P C_{1t+1} + (1+\tau)c_0 + \frac{\alpha(1-\eta)}{2} A_p K_{t+1}^{\eta} + 2(1+\tau)c_0}\right)^{\mu} \left(1 - \delta + \eta A_g K_{t+1}^{\eta-1}\right) \tag{24}$$

Definition 1 *Under Assumptions 1-2, an equilibrium of the economy is a sequence* (K_t, C_{1t}) *such that equations (23)-(24) are satisfied, given* $K_0 \geq 0$.[2]

Equations (6), (7), (20), (22) and market clearing conditions allow to recover aggregate consumption for both goods:

$$c_{pt} = \sum_i c_{pit} = \frac{\alpha}{1+\tau(1-\alpha)}\left(A_p K_t^\eta - p(K_{t+1} - (1-\delta)K_t)\right) + \frac{2(1+\tau)}{1+\tau(1-\alpha)}c_0 \quad (25)$$

$$c_{gt} = \sum_i c_{git} = \frac{(1-\alpha)(1+\tau)}{p(1+\tau(1-\alpha))}\left(A_p K_t^\eta - p(K_{t+1} - (1-\delta)K_t) + 2\tau c_0\right) \quad (26)$$

4. Steady State

In this section, we show the existence of a unique steady-state in which the most patient household holds a positive amount of capital, while the impatient one does not save at all.

The Euler equation for agent i writes $1 \geq \beta_i(1 - \delta + \frac{r}{p})$, holding with equality if $a_i > 0$. Recall that $\beta_1 > \beta_2$, so that $\beta_1(1-\delta+\frac{r}{p}) > \beta_2(1-\delta+\frac{r}{p})$. Thus, we have $a_1^* = K^* > 0 = a_2^*$. Having the most patient agent holding all the capital and the least patient consuming all her disposable income is a standard result in the literature (Becker, 1980, Becker & Foias, 1987, Sorger, 1994), supported by empirical evidence (Krussell & Smith, 1998, Kaplan et al., 2014, Carroll et al., 2017). Rewriting the left hand-side of the inequality gives $\frac{r^*}{p} = \frac{1-\beta_1}{\beta_1} + \delta$. As we found that p is a constant, we have $p = \frac{A_p}{A_g}$ and $P_i^* = \frac{(1+\tau)^\alpha p^{*(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{1-\alpha}} + (1+\tau)\frac{c_0}{C_i^*}$. Using (17)-(18), capital-labor ratios remain constant, with $K^* = \frac{K_p^*}{N_p^*} = \frac{K_g^*}{N_g^*} = \left(\frac{\eta A_g}{\frac{1}{\beta_1}-(1-\delta)}\right)^{\frac{1}{1-\eta}}$, and the wage rate writes $w^* = (1-\eta)A_p\left(\frac{\eta A_g}{\frac{1}{\beta_1}-(1-\delta)}\right)^{\frac{\eta}{1-\eta}}$.

Writing (6)-(7) at the steady state gives individual consumptions. Equations (20) and (22)-(26) allow to recover overall consumption for each household, as well as aggregate consumption in both sectors:

$$P_1^*C_1^* = K^*p\left(\frac{r^*/p}{\eta}\left(\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right) - \delta\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\right) + \frac{2\varepsilon\tau(1+\tau)}{1+\tau(1-\alpha)}c_0 \quad (27)$$

$$P_2^*C_2^* = K^*p\left[\left(\frac{r^*/p}{\eta} - \delta\right)\frac{\alpha(1-\varepsilon)\tau}{1+\tau(1-\alpha)} + \frac{r^*/p}{\eta}\frac{1-\eta}{2}\right] + \frac{2(1-\varepsilon)\tau(1+\tau)}{1+\tau(1-\alpha)}c_0 \quad (28)$$

$$c_p^* = \frac{\alpha}{1+\tau(1-\alpha)}K^*p\left(\frac{r^*/p}{\eta} - \delta\right) + \frac{2(1+\tau)}{1+\tau(1-\alpha)}c_0 \quad (29)$$

$$c_g^* = \frac{(1-\alpha)(1+\tau)}{(1+\tau(1-\alpha))}K^*\left(\frac{r^*/p}{\eta} - \delta\right) + \frac{2\tau(1+\tau)(1-\alpha)}{p(1+\tau(1-\alpha))}c_0 \quad (30)$$

Finally, market clearing conditions give $Y_p^* = c_p^*$ and $Y_g^* = c_g^* + \delta K^*$, and pollution writes $E = \frac{1}{\gamma c_p^*}$.

[2] At the equilibrium, optimality conditions, market-clearing conditions and the government's budget constraint hold. All variables are given by the sequence (K_t, C_{1t}).

Proposition 1 *Under Assumptions 1 -2, there exists a unique steady state in the economy* (K^*, C_1^*) *solution to (23)-(24), characterised by equations (27)-(30) ans defined by parameters of the model. At the steady state, household 2 is constrained and household 1 holds all the capital.*

From now on, we refer to household 1 as the capitalist and household 2 to as the worker.

5. Public Policy

The government has two tools: lump-sum transfers and commodity taxation. This part aims at analyzing the impact of both on pollution and inequalities. More precisely, we study whether one fiscal tool can reduce both pollution and inequality, and how does this depend on the other tool.

5.1. Taxation

There are two effects at play when increasing the tax rate in our model: a price effect and a redistribution effect. By increasing the consumer price for the polluting commodity, purchasing power of both households is lowered so that for the same income, they consume less. Increasing the consumer price also leads to a substitution between the two goods: agents increase their consumption in the clean good as its relative price decreases. Increasing the tax rate increases the government revenue, and hence the amount received through lump-sum transfers. Everything else equal, this rise in disposable income allows for a higher level of consumption: this is the redistribution effect.

Effect on consumption and environmental quality

We first investigate the effect of an increase in τ on aggregate consumptions, c_p^* and c_g^*.

Proposition 2 *Under Assumptions 1-2, an increase in commodity taxation always leads to an increase in clean consumption. Polluting consumption decreases when* $c_0 < \overline{c_0}$ *and increases for* $c_0 > \overline{c_0}$, *with* $\overline{c_0} = \frac{(1-\alpha)K^* p\left(\frac{r^*/p}{\eta} - \delta\right)}{2}$.

The increase in clean consumption comes from the substitution effect, as an increase in taxation makes the clean good relatively cheaper than the polluting one. The mixed effect on polluting consumption comes from the price and redistribution effects. The former is negative, while the latter is positive. A low subsistence level of consumption pushes the redistribution effect down so that the price effect dominates. The reverse occurs when subsistence consumption is high, hence the rise in emissions. This effect of the polluting tax is similar to what has been found by Bosi & Desmarchelier (2018) when looking at capital taxation under an environmental Kuznets curve. When it comes to mitigate pollution, subsistence consumption is hence of high importance as it shapes the efficiency of environmental taxation.

Regarding individual behaviors, equation (6) at the steady-state gives:

$$\frac{dc_{gi}^*}{d\tau} = \frac{(1-\alpha)}{p} \frac{d(P_i^* C_i^*)}{d\tau} > 0$$

for any household i. Clean consumption does not increase only at the aggregate level, but also at the individual one, which corresponds again to the susbstitution effect of the tax.

Results for individual polluting consumption are summed up in the following proposition:

Proposition 3 *Let* $\varepsilon^* = \frac{1}{2}\left(1 + \eta \frac{\frac{1-\beta_1}{\beta_1}}{\frac{r^*}{p} - \eta\delta}\right)$*. Under Assumptions 1-2, following an increase in* τ*, we obtain:*

- *For* $0 \leq c_0 < \overline{c_0}$*:*
  - *When* $\varepsilon < \varepsilon^*$*,* c^*_{p1} *decreases and* c^*_{p2} *decreases for* $\alpha < \underline{\alpha_2}$ *and increases for* $\alpha > \overline{\alpha_2}$*;*
  - *When* $\varepsilon > \varepsilon^*$*,* c^*_{p2} *decreases and* c^*_{p1} *decreases for* $\alpha < \underline{\alpha_1}$ *and increases for* $\alpha > \overline{\alpha_1}$
- *For* $c_0 > \overline{c_0}$*:*
  - *When* $\varepsilon < \varepsilon^*$*,* c^*_{p2} *increases and* c^*_{p1} *decreases for* $\alpha < \underline{\alpha_1}$ *and increases for* $\alpha > \overline{\alpha_1}$*;*
  - *When* $\varepsilon > \varepsilon^*$*,* c^*_{p1} *increases and* c^*_{p2} *decreases for* $\alpha < \underline{\alpha_2}$ *and increases for* $\alpha > \overline{\alpha_2}$*.*

From Proposition 3, three variables are key when analyzing which effect dominates the other: the redistribution rate, the size of subsistence consumption and the spending share on the polluting commodity. These three variables play a role on the redistribution effect, as the tax revenue depends on both c_0 and α, and by definition on ε. When at least two variables out of three are high enough for household i, the redistribution effect dominates the price effect. The reverse occurs when at least two are low.
When the government redistributes few of the tax revenue towards household i and that c_0 is low, no matter α the size of the redistribution effect is very low: the price effect dominates and polluting consumption decreases. On the opposite, when a high share of the tax revenue is redistributed and c_0 is high, the redistribution effect overrules the price effect, leading to an increase in polluting consumption for that household.
When c_0 is low (resp. high) and a high (resp. low) share of the tax revenue is redistributed towards household i, then which effect dominates the other depends on the size of α. When α is low, the redistribution effect is pushed down by c_0 and α, so that the price effect dominates. In contrast, a high α pushes the redistribution effect up as a lot of the tax revenue is given to the household, hence it dominates the price effect.

Looking at inequality, we get the following proposition:

Proposition 4 *Under Assumptions 1-2,* $\frac{\partial(P^*_2 C^*_2)}{\partial\tau} > \frac{\partial(P^*_1 C^*_1)}{\partial\tau}$ *whenever* $\varepsilon < \frac{1}{2}$*.*

Recall that $P^*_i C^*_i$ is equal to disposable income net of capital holdings. Increasing the tax rate decreases inequality if and only if the government redistributes a higher share towards the worker. Hence, having a low level of subsistence consumption and redistributing more towards the worker allows to reconcile the tradeoff between pollution mitigation and inequality reduction. A tradeoff occurs when c_0 is high and ε is low, or when c_0 is low and ε is high: in these cases, the government is able either to reduce inequality, or to reduce pollution but cannot tackle both at the same time. When subsistence consumption is high and ε is high, increasing polluting commodity taxation leads to a deadend for both inequality and pollution reduction.

Effect on welfare

Changing the commodity tax rate affects individual welfare through consumption and environmental quality:

$$\frac{dU_i^*}{d\tau} = \mu \frac{dE^*}{d\tau} E^{*\mu-1} \frac{C_i^{*1-\sigma}}{1-\sigma} + \frac{dC_i}{d\tau} C_i^{*-\sigma} E^{*\mu} \tag{31}$$

The environmental effect on utility depends on the value taken by c_0: a low subsistence level of consumption contributes positively to environmental quality when increasing the tax rate. The consumption effect depends on both the positive impact on green consumption and the mixed effect on polluting consumption. If polluting consumption decreases, the consumption effect depends on whether the increase in clean consumption offsets the decrease in polluting consumption. As for the impact on polluting consumption, this is determined by the sizes of α, c_0 and ε. Whenever the environmental and consumption effects go in the same direction, the impact on individual utilities is straightforward. Otherwise, the overall impact on utilities depends on which effects outweighs the other. This is settled by the size of μ.

Proposition 5 *Under Assumptions 1-2, there exist ε_1, $\mu_1^*(\tau,\varepsilon)$ and $\mu_1^*(\tau,\varepsilon)$ such that increasing the tax rate on the polluting commodity:*

- *For $c_0 < \overline{c_0}$:*
  - *Increases welfare for the capitalist when $\varepsilon > \varepsilon_1$ and $\alpha > \hat{\alpha}$ or $\varepsilon < \varepsilon_1$ and $\mu > \mu_1^*(\tau,\varepsilon)$;*
  - *Increases welfare for the worker when $\varepsilon < \frac{1+\alpha}{2}$ or $\varepsilon > \frac{1+\alpha}{2}$ and $\mu > \mu_2^*(\tau,\varepsilon)$.*
- *For $c_0 > \overline{c_0}$:*
  - *Increases welfare for the capitalist when $\varepsilon > \varepsilon_1$ and $\mu < \mu_1^*(\tau,\varepsilon)$;*
  - *Increases welfare for the worker when $\varepsilon < \frac{1+\alpha}{2}$ and $\mu < \mu_2^*(\tau,\varepsilon)$.*

When $c_0 < \overline{c_0}$ and no matter the size of μ, there always exists a way to play on ε so that increasing commodity taxation is welfare increasing for at least the worker. When $c_0 > \overline{c_0}$, the size of μ matters: if it is high enough, then increasing the tax rate is welfare decreasing for both households, no matter how much does the government redistribute of the tax revenue to each of them, because the (negative) environmental effect dominates the consumption effect. When the envionmental externality does not matter much for both households, increasing the tax rate can be welfare beneficial for at least one household, depending on the redistribution rate.

5.2. Redistribution

Effect on consumption and emissions

The effect of redistribution through a change in the redistribution rate ε is straightforward. From equations (29)-(30), redistribution has no impact on aggregate consumptions c_p^* and c_g^*. Capital is given by the modified golden rule, so that redistribution has no impact on it either.

The only impact of ε is hence on individual variables, i.e on individual consumption. As consumption of both goods are functions of overall consumption, the only thing we have to look at is the impact of a change in redistribution on C_1^* and C_2^* respectively. From (27)-(28), giving more of the tax revenue to the worker has a positive effect on her consumption, while it is negative for the capitalist. This is a pure revenue effect: increasing redistribution towards the worker raises her disposable income, and hence her consumption in both goods proportionally, while it reduces the capitalist's consumption. Yet, both trends perfectly compensate each other so that aggregate consumption, and more broadly all aggregate variables, do not change, environmental damages included. Hence, it is possible for the government to reduce income and consumption inequalities by increasing redistribution towards the worker, while not harming environmental quality. Rausch & Schwartz (2016) show that when preferences are homothetic and identical, aggregate behaviors are similar to a single-agent behavior. Thus, changing the redistribution pattern has no impact on aggregate consumption as it is similar to giving back everything to a single agent. Heterogeneous preferences should allow for an aggregate impact of redistribution. Having househoulds consuming different shares of the polluting commodity, the impact on aggregate polluting consumption would depend on which household spends a higher share on this good, as redistribution effects on both households would not offset each other. Changing the redistribution rate hence would have an impact on pollution.

Effect on welfare

Knowing that changing the redistribution rate does not affect pollution, the impact on welfare is only done through consumption. More precisely:

$$\frac{dU_i^*}{d\varepsilon} = \frac{dC_i}{d\varepsilon} C_i^{*-\sigma} E^{*\mu} \tag{32}$$

Redistributing more to the worker decrease bundle consumption for the capitalist and increase for the worker due to the revenue effect. Hence, decreasing income inequality through a lower ε decreases welfare for the capitalist and increases welfare for the worker.

Putting taxation and redistribution altogether, there always exists a way to reduce both pollution and inequality by playing on redistribution when the government taxes the polluting commodity and redistributes the tax revenue through lump-sum transfers. This assumes that the minimal amount of polluting good consumed is relatively low. Yet, this policy mix cannot be welfare improving for both households. Having a welfare improving environmental tax reform would mean that either pollution, inequalities or both increase. Subisstence consumption and the size of the externality are hence of high importance when it comes to the welfare impact of environmental tax reforms. Subsistence consumption also shapes the impact on pollution, and so the efficiency of the environmental policy.

6. Local Stability

The level of subsistence consumption and the externality play an important role in the long-run impact of environmental fiscal policies. Characterizing the stability properties near the steady-state of (23)-(24) allows us to analyze the effect of these two parameters in the shorter run, as well as the role played by the fiscal policy. For that, we look at the log-linearized system evaluated at the steady-state:

$$\begin{bmatrix} \tilde{K_{t+1}} \\ \tilde{C_{t+1}} \end{bmatrix} = \begin{bmatrix} \Omega_1 & \Omega_2 \\ \Omega_3 & \Omega_4 \end{bmatrix} \begin{bmatrix} \tilde{K_t} \\ \tilde{C_t} \end{bmatrix}$$

with

$$\Omega_1 = \frac{\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}}{\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)} \left(\frac{1}{\beta_1} + (1-\delta)\right) + 1 - \delta \tag{33}$$

$$\Omega_2 = -\frac{PC_1^*}{K^* p\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)} \tag{34}$$

$$\Omega_3 = \frac{\mu\alpha\frac{(1-\eta)}{2} r^* K^* (\Omega_1 - 1) - \beta_1 K^* f''(K^*)\Omega_1}{(1+\tau)c_0 C_1^{*-1} - \sigma - \mu\alpha C_1^* P} \tag{35}$$

$$\Omega_4 = 1 + \frac{\beta_1 C_1^* f''(K^*)P - \mu\alpha\frac{(1-\eta)}{2} PC_1^* r^*}{p\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\left((1+\tau)c_0 C_1^{*-1} - \sigma - \mu\alpha C_1^* P\right)}. \tag{36}$$

We explore the role of the externality μ and subsistence consumption c_0 on the stability properties of the economy. We first analyze the role of subsistence consumption by setting $\mu = 0$, and then relax this assumption. Following Grandmont et al. (1998) and their geometrical method, we use the trace and the determinant to study the stability properties of the system, given by:

$$Tr = \Omega_1 + \Omega_4 \tag{37}$$

$$D = \Omega_1 - \frac{\mu\alpha\frac{(1-\eta)}{2} PC_1^* r^*}{p\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\left((1+\tau)c_0 C_1^{*-1} - \sigma - \mu\alpha PC_1^*\right)} \tag{38}$$

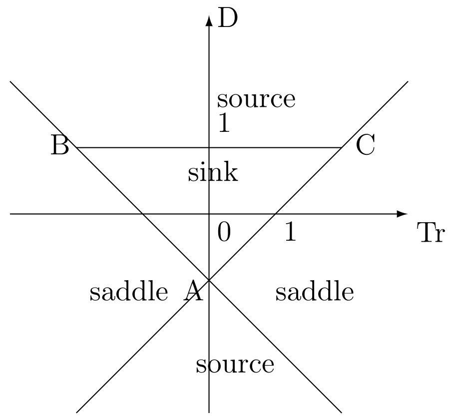

Figure 1: Geometrical method

To do so, we use the characteristic polynomial $Pol(\lambda) = \lambda^2 - \lambda Tr + D$ evaluated at $\lambda = -1$, 0 and 1 (see Figure 1). Along (AB), $Pol(-1) = 0$ (one eigenvalue is equal to -1) and along (AC), $Pol(1) = 0$ (one eigenvalue is equal to 1). On [BC], the two eigenvalues are complex conjugates with modulus one. The equilibrium is a sink (locally indeterminate) inside the triangle ABC, meaning that stochastif fluctuations occur around the steady-state. It is a saddle on the right side of both (AB) and (AC) or on the left side of them, and it is a source otherwise, i.e the equilibrium is not stable. A Flip bifurcation occurs when (TR,D) crosses (AB), leading to two-periods fluctuations. A Hopf bifurcation happens when crossing [BC], giving rise to endogeneous fluctuations.

Assumption 3 $\varepsilon \leq \frac{1}{2\tau} + \frac{1-\alpha}{2}$.

This ensures the existence of the upper-bound c_{01} on c_0, as stated in Assumption 2.

6.1. Local dynamics when $c_0 > 0$ and $\mu = 0$

When the externality is not taken into account, the trace and the determinant simplify to:

$$Tr = 1 + \Omega_1 - \frac{\beta_1 f''(K^*) C_1^* P}{\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\left(\sigma - (1+\tau)c_0 C_1^{*-1}\right)} \tag{39}$$

$$D = \Omega_1 \tag{40}$$

where C_1^* is a function of c_0. Making values of c_0 vary between 0 and c_{01}, we analyze the stability properties by looking at how do the trace and the determinant, that are functions of c_0, move on the plane described in Figure 1. Starting at $c_0 = 0$, the point $(Tr(0), D(0))$ lies on the right side of (AC). If $\eta > \overline{\eta}$, $(Tr(0), D(0))$ lies above C. It lies under it otherwise. As the determinant does not depend on subsistence consumption, increasing c_0 leads to draw a horizontal line going right on the (Tr,D) plan. At c_0^F, the horizontal line crosses (AB), leading to a source or a sink depending on the level of capital intensity. The point $(Tr(c_{01}), D(C_{01}))$ lies on (AC).
When accounting for subsistence consumption but not for the externality, some instability can emerge depending on the level of c_0 and η. As long as $c_0 < c_0^F$, saddle-path stability is ensured. When $c_0 > c_0^F(\tau, \varepsilon)$, local unstability arises when $\eta > \overline{\eta}$, and indeterminacy occurs whenever $\eta < \overline{\eta}$. We deduce that:

Proposition 6 *Under Assumptions 1 -3, a steady-state with no environmental externality and a positive level of subsistence consumption has the following stability properties:*

- *when $c_0 < c_0^F(\tau, \varepsilon)$, the steady-state is a saddle;*
- *when $c_0 > c_0^F(\tau, \varepsilon)$, the steady-state is a source for $\eta > \overline{\eta}$ and a sink otherwise.*

A flip bifurcation occurs at $c_0 = c_0^F$, and a Hopf birfurcation arises when $c_0 > c_0^F$ and η crosses the value $\overline{\eta}$.
Values of c_0^F and $\overline{\eta}$ are in Appendix A.7.

As both c_0^F and $\overline{\eta}$ depend on τ and ε, there is a role to play for both taxation and redistribution in maintaining the stability and determinacy of the equilibrium. To analyze this role, we need the following lemma:

Lemma 1 *Under Assumptions 1-3 and $\mu = 0$, we have $\frac{\partial c_0^F}{\partial \varepsilon} > 0$. The sign of $\frac{\partial c_0^F}{\partial \tau}$ is negative whenever redistributing almost all the tax revenue towards any household. $\overline{\eta}$ decreases with ε and τ.*

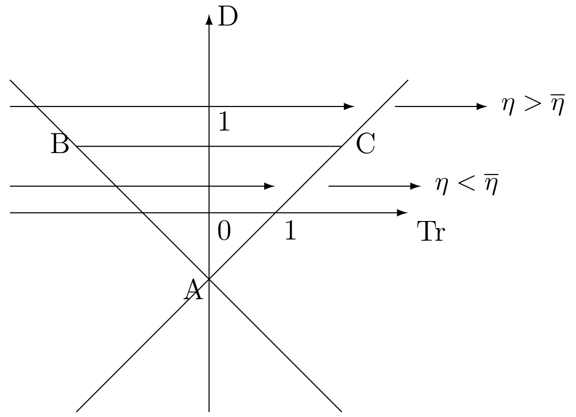

Figure 2: Local dynamics with $\mu = 0$

Increasing the tax rate promotes instability whenever the government redistributes almost all the tax revenue towards the worker, as noth $\lrcorner 0^F$ and $\overline{\eta}$ decrease with the tax rate. In the same vein, increasing the share of the tax revenue redistributed towards the worker promotes endogeneous stochastic flutucations, as c_0^F decreases and $\overline{\eta}$ increases.

To give economic intuition for cycles, recall that the dynamics are driven by equations (19) and (20). Equation (20) can be rewritten as $\frac{U'(C_{1t})}{P_{1t}} = \beta_1 \frac{U'(C_{1t+1})}{P_{1t+1}}(1 - \delta + \frac{r_{t+1}}{p})$. Anticipating a higher C_{t+1} leads to a higher $\frac{U'(C_{1t+1})}{P_{1t+1}}$ when c_0 is high. To maintain equality, r_{t+1} must decrease, which is equal to an increase in capital holdings K_{t+1}. As capital and labor incomes are positive functions of capital, they increase at $t+1$, so that next period consumption increases. Expectations are then self-fulfilling and there are oscillations because a higher investment in capital K_{t+1} also implies a lower current consumption C_{1t}. Indeterminacy and cycles are here possible only because subsistence consumption establishes interactions between prices and quantities[3]. Without a sufficiently high subsistence level of consumption, the aggregate price does not depend on consumption, and $\frac{U'(C_{1t+1})}{P_{1t+1}}$ always decreases when anticipating a higher consumption level.

Whenever subsistence consumption is low enough, i.e below $\overline{c_0}$, increasing the tax rate and redistributing more towards the worker has a positive long-run impact by reducing both inequality and pollution. Yet, opposite forces are playing in the shorter run: redistribution promotes indeterminacy and endogeneous cycles, while increasing polluting commodity taxation has a mixed effect depending on how much is redistributed towards the worker. A high subsistence consumption leads the fiscal policy to have a mixed effect both on the short- and long-run: increasing the tax rate increases pollution and does not necessarily favors stability or determinacy. Redistributing more towards the worker decreases inequality but promotes the emergence of cycles.

6.2. Local dynamics when $c_0 > 0$ and $\mu > 0$

In this case, the trace and the determinant are given by (37)-(38). We depart from the case in which there is subsistence consumption but no externality, and analyze what happens when we increase the value of μ. Doing so, the point (Tr,D) draws a half line of slope $S(c_0) =$

[3]See Chen et al. (2015) for a similar result with consumption externalities.

$\frac{\frac{1-\eta}{2}\frac{r^*}{p}(\sigma-(1+\tau)c_0C_1^{*-1})}{\frac{1-\eta}{2}\frac{r^*}{p}(\sigma-(1+\tau)c_0C_1^{*-1})+PC_1^*f''(K^*)K^*}$. The origin lies on $(Tr(c_0), D(c_0))$ for $\mu = 0$, which belongs to the horizontal line of the previous case. As c_0 moves from 0 to $+\infty$, the origin $(Tr(c_0), D(c_0))$ moves right to the limit point $(Tr(+\infty), D(+\infty))$ that lies on (AC). At the same time, $S(c_0)$ decreases (resp. increases) when it is downward (resp. upward) sloping. Whenever $c_0 < c_0^F$, the whole half line lies in the saddle region, (below (AB) and above (AC) when $c_0 > c_0^*$, below (AC) and above (AB) otherwise): local indeterminacy and instability are impossible to get, no matter the size of μ. When $c_0 > c_0^F$, one lies above (AB) and above (AC), such that for $\mu = 0$, the steady-state is a sink for $\eta < \overline{\eta}$ and a source when $\eta > \overline{\eta}$. An increase in μ makes the half line drew by (Tr, D) moving downward on the left.

When $\eta > \overline{\eta}$, the half line crosses (AB) and/or [BC] when μ increases, depending on the value taken by c_0. More precisely, the steady-state will be a source for any $\mu < \mu_H(\tau, \varepsilon)$, and a sink for any $\mu \in (\mu_H(\tau, \varepsilon), \mu_F(\tau, \varepsilon))$. A Hopf bifurcation arises when $\mu = \mu_H(\tau, \varepsilon)$. For any $\mu > \mu_F(\tau, \varepsilon)$, the steady-state lies back in the saddle region, with a Flip bifurcation occuring at $\mu = \mu_F(\tau, \varepsilon)$. In the case $\mu_H(\tau, \varepsilon) = \mu_F(\tau, \varepsilon)$, the steady state is a source for μ below this level, and a saddle otherwise. When $\eta < \overline{\eta}$, the half line crosses (AB) when $\mu = \mu_F(\tau, \varepsilon)$: a Flip bifurcation occurs. The steady state is hence a sink for any $\mu < \mu_F(\tau, \varepsilon)$ and a saddle otherwise. Results are summed up in the following proposition:

Proposition 7 *Under Assumptions 1-3 and $\mu > 0$, there exists μ_H for $\eta > \overline{\eta}$ and μ_F, such that the steady state is:*

- *a source for $c_0 > c_0^F$ and $\mu < \mu_H$;*
- *a sink for $c_0 > c_0^F$ and $\mu \in (\mu_H, \mu_F)$;*
- *a saddle otherwise.*

A Hopf bifurcation occurs when μ crosses μ_H, and a Flip bifurcation arises for $\mu = \mu_F$.

Saddle-path stability is ensured by a sufficiently important externality ($\mu > \mu_F$) and a relatively low subsistence level of consumption. When the externality is relatively weak and the subsistence level of consumption relatively high, the steady state can lose its stability through the occurrence of cycles of period two. The fiscal policy has a role to play on stability propeties of the economy, given by the following lemma:

Lemma 2 *Under Assumptions 1 -3 and $\mu > 0$:*

- *$\mu_H(\tau, \varepsilon)$ and $\mu_F(\tau, \varepsilon)$ increase when redistributing more towards the worker;*
- *$\frac{\partial \mu_H(\tau,\varepsilon)}{\partial \tau} > 0$ and $\frac{\partial \mu_F(\tau,\varepsilon)}{\partial \tau} > 0$ when $\varepsilon < \varepsilon_1$.*

The fiscal policy plays again a key role in the occurrence of instability and indeterminacy. Not only does it increase the interval for c_0 under which the steady state is unstable or indeterminate, it also increases the interval for μ under which the steady state is unstable or a sink as long as redistribution is high enough. Even if introducing the externality brings back some stability, using the fiscal tools to fight polluting emissions and inequality harms stability properties of the economy.

A level of subsistence consumption below $\overline{c_0}$ leads to a positive impact of the fiscal environmental policy in the long-run, as increasing the tax rate reduces pollution and redistributing moretowards the worker decreases inequality. Yet, the fiscal policy promotes instability and indeterminacy in the shorter-run. A higher subsistence level of polluting consumption leads to a deterioration of environmental quality when increasing the tax rate, and both fiscal tools promote again instability and indeterminacy in the short-run. Even if introducing the environmental externality brings back some stability, the fiscal policy lowers that role, getting the economy closer to the case in which it is not taken into account.

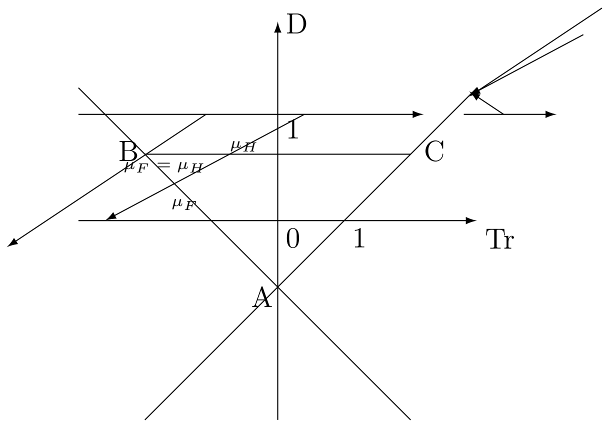

Figure 3: Local dynamics with $\eta > \overline{\eta}$

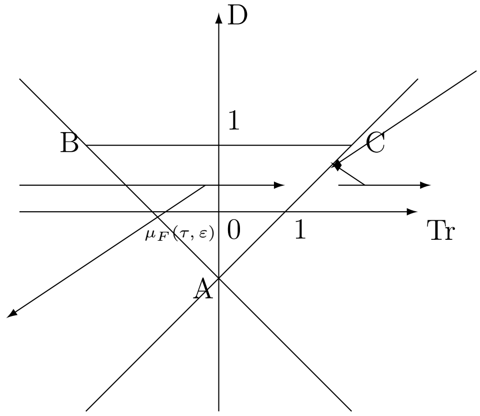

Figure 4: Local dynamics with $\eta < \overline{\eta}$

7. Concluding Remarks

In this paper, we investigate the impact of environmental commodity taxation and redistribution on pollution and income inequality. For that, we build a two-sector Ramsey model with heterogeneous households, an environmental externality and a subsistence level of consumption for the polluting good.

After characterizing the intertemporal equilibrium, we show that there exists a unique steady-state in which the most patient household holds all the capital. We then discuss the impact of taxation and redistribution on pollution and inequality. We find that redistribution only impacts inequality. Focusing on taxation, increasing the tax rate does not reduce pollution when the level of susbistence consumption is too high. Coupling both instruments, there is room to tackle both issues when the susbistence level of polluting consumption is relatively low. On the contrary, there is at best a tradeoff between inequality reduction and pollution mitigation when subsistence consumption is high. At worst, an environmental fiscal reform leads to a dead-end for inequality reduction and pollution mitigation.

Analyzing the local dynamics, we show that the level of subsistence consumption can lead to indeterminacy. Increasing polluting commodity taxation and redistributing more towards the worker decreases the threshold above which the steady state becomes unstable or indeterminate. The environmental externality has a stabilizing role in the economy, as taking it into accounting brings back stability. In this case, the environmental fiscal policy again plays a key role in the occurrence of indeterminacy. Hence, policy makers must carefully handle environmental taxation and redistribution to avoid instability and indeterminacy, and must take into account subsistence consumption when implementing an efficient and inequality reductive environmental tax reform.

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A. Appendix

A.1. Household's Problem

The household's problem writes:

$$\begin{aligned} \max_{a_{it+1}, c_{pit}, c_{git}} \quad & \sum_{t=0}^{\infty} \beta_i^t U_i(c_{pit}, c_{git}, E_t) \\ \text{subject to} \quad & c_{pit}(1+\tau) + c_{git} p_t + p_t(a_{it+1} - (1-\delta) a_{it}) = w_t n_i + r_t a_{it} + T_{it} \\ & a_{it+1} \geq 0 \end{aligned}$$

Using the following Lagrangian to solve the optimization problem:

$$\mathbb{L} = \sum_t \beta_i^t \left[E_t^{\mu} \frac{(c_{pit}^{\alpha} c_{git}^{1-\alpha})^{1-\sigma}}{1-\sigma} + \lambda_{it}^{BC}(w_t n_i + r_t a_{it} + T_{it} - c_{pit}(1+\tau) - c_{git} p_t - p_t(a_{it+1} - (1-\delta) a_{it})) + \lambda_{it}^{FC} a_{it} \right]$$

Optimality conditions:

$$\begin{aligned} \frac{1-\alpha}{c_{git}} C_{it}^{1-\sigma} E_t^{\mu} &= \lambda_{it}^{BC} p_t \\ \frac{\alpha}{c_{pit} - c_0} C_{it}^{1-\sigma} E_t^{\mu} &= \lambda_{it}^{BC} (1+\tau) \\ \lambda_{it}^{BC} p_t &= \beta_i \lambda_{it+1}^{BC} (p_{t+1}(1-\delta) + r_{t+1}) + \beta_i \lambda_{it+1}^{FC} \end{aligned}$$

Dividing the two first FOCs and rearranging gives $c_{git} p_t = \frac{1-\alpha}{\alpha}(c_{pit} - c_0)(1+\tau)$.
Denote $I_{it} = w_t n_i + r_t a_{it} - (a_{it+1} - (1-\delta) a_{it})$. Plugging the previous equation for $c_{git} p_t$ in the budget constraint of agent i yields:

$$\begin{aligned} c_{pit} &= \frac{\alpha}{1+\tau} I_{it} + c_0(1-\alpha) \\ c_{git} &= \frac{1-\alpha}{p_t}(I_{it} - c_0(1+\tau)) \end{aligned}$$

The ex post budget constraint can be written $P_{it} C_{it} = I_{it}$.
Recall $C_{it} = (c_{pit} - c_0)^{\alpha} c_{git}^{1-\alpha}$, so that $C_{it} = \left(\frac{\alpha}{1+\tau}(I_{it} - c_0(1+\tau))\right)^{\alpha} \left(\frac{1-\alpha}{p_t}(I_{it} - c_0(1+\tau))\right)^{1-\alpha}$.
Plugging this into the ex post budget constraint and rearranging yields $P_{it} = \frac{(1+\tau)^{\alpha} p_t^{1-\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}} + \frac{(1+\tau) c_0}{C_{it}}$.

A.2. Proof of Assumption 2

We have

$$C_{1t} = \frac{1}{P} \left(\left(\frac{(1+\eta)}{2} + \frac{\alpha \varepsilon \tau}{1+\tau(1-\alpha)} \right) A_p K_t^{\eta} - (1 + \frac{\alpha \varepsilon \tau}{1+\tau(1-\alpha)}) p (K_{t+1} - (1-\delta) K_t) - \frac{(1+\tau(1-\alpha-2\varepsilon))(1+\tau)}{1+\tau(1-\alpha)} c_0 \right) \quad (41)$$

$$C_{2t} = \frac{1}{P} \left(\left(\frac{(1-\eta)}{2} + \frac{\alpha(1-\varepsilon)\tau}{1+\tau(1-\alpha)} \right) A_p K_t^{\eta} - (\frac{\alpha(1-\varepsilon)\tau}{1+\tau(1-\alpha)}) p (K_{t+1} - (1-\delta) K_t) - \frac{(1-\tau(1+\alpha-2\varepsilon))(1+\tau)}{1+\tau(1-\alpha)} c_0 \right) \quad (42)$$

$$P = \frac{(1+\tau)^{\alpha} p^{1-\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}} \quad (43)$$

C_{1t} and C_{1t} are positive whenever, respectively:

$$c_0 \leq \left(\frac{(1+\eta)}{2(1+\tau(1-\alpha-2\varepsilon))(1+\tau)} + \frac{\alpha\varepsilon\tau}{(1+\tau(1-\alpha-2\varepsilon))(1+\tau)}\right) A_p K_t^\eta - \left(\frac{1+\tau(1-\alpha)}{(1+\tau(1-\alpha-2\varepsilon))(1+\tau)} + \frac{\alpha\varepsilon\tau}{(1+\tau(1-\alpha-2\varepsilon))(1+\tau)}\right) p(K_{t+1} - (1-\delta)K_t) \equiv c_{01} \tag{44}$$

$$c_0 \leq \left(\frac{(1-\eta)}{2(1-\tau(1+\alpha-2\varepsilon))(1+\tau)} + \frac{\alpha(1-\varepsilon)\tau}{(1-\tau(1+\alpha-2\varepsilon))(1+\tau)}\right) A_p K_t^\eta - \frac{\alpha(1-\varepsilon)\tau}{(1-\tau(1+\alpha-2\varepsilon))(1+\tau)} p(K_{t+1} - (1-\delta)K_t) \equiv c_{02} \tag{45}$$

A.3. Proof of Proposition 2

From (29) and (30):

$$\frac{dc_p^*}{d\tau} = \frac{-\alpha(1-\alpha)}{(1+\tau(1-\alpha))^2} K^* p \left(\frac{r^*/p}{\eta} - \delta\right) + \frac{2\alpha}{(1+\tau(1-\alpha))^2} c_0$$

$$\frac{dc_g^*}{d\tau} = \frac{1-\alpha}{p(1+\tau(1-\alpha))^2} \left(K^* p \left(\frac{r^*/p}{\eta} - \delta\right) + 2(1+2\tau(1+\tau^c(1-\alpha)))c_0\right)$$

It is straightforward that $\frac{dc_g^*}{d\tau} > 0$.
$\frac{dc_p^*}{d\tau} > 0 \Longleftrightarrow c_0 > \frac{(1-\alpha)K^* p\left(\frac{r^*/p}{\eta} - \delta\right)}{2}$

A.4. Proof of Proposition 3

For the capitalist

The derivative of c_{p1}^* with respect to τ is a second degree polynomial. Polluting consumption decreases for:

$$\tau^2 \left(K^* p(1-\alpha)\left(\frac{r^*/p}{\eta} - \eta\delta\right) + \left(\frac{r^*/p}{\eta} + \eta\left(\frac{1}{\beta_1} - 1 - \delta\right)\right)\frac{(1-\alpha)}{2\alpha\varepsilon} - \frac{2\eta c_0}{\alpha}\right) + \tau\left(\left(\frac{r^*/p}{\eta} + \eta\left(\frac{1}{\beta_1} - 1 - \delta\right)\right)\frac{(1-\alpha)}{\alpha\varepsilon} - \frac{4\eta c_0}{\alpha}\right) - \left(K^* p\left(\frac{r^*/p}{\eta} - \eta\delta - \frac{\left(\frac{r^*/p}{\eta} + \eta\left(\frac{1}{\beta_1} - 1 - \delta\right)\right)}{2\alpha\varepsilon}\right) + \frac{2\eta c_0}{\alpha}\right) > 0,$$

This is positive when $\left(2\left(\frac{r^*/p}{\eta} - \eta\delta\right) - \frac{\left(\frac{r^*/p}{\eta} + \eta\left(\frac{1}{\beta_1} - 1 - \delta\right)\right)}{\varepsilon}\right)\left(K^* p(\frac{r^*/p}{\eta} - \delta\eta)(1-\alpha) - 2\eta c_0\right) > 0$.

When this is negative, the sign of the polynomial is equal to the sign of $K^* p(1-\alpha)\left(\frac{r^*/p}{\eta} - \eta\delta\right) + \left(\frac{r^*/p}{\eta} + \eta\left(\frac{1}{\beta_1} - 1 - \delta\right)\frac{(1-\alpha)}{2\alpha\varepsilon}\right) - \frac{2\eta c_0}{\alpha}$, which is positive for $0 < \alpha < \hat{\alpha_1}$ and negative for $\hat{\alpha_1} < \alpha < 1$.

- $\varepsilon < \varepsilon^*$:
  - When $c_0 < \overline{c_0}$, the discriminant is negative and $\alpha < \hat{\alpha}$, so the polynomial is positive, meaning c_{p1}^* decreases.
  - When $c_0 > \overline{c_0}$, the discriminant is positive and there are 2 roots:

$$\tau_1^1 = \frac{\frac{4\eta c_0}{\alpha} - \left(\frac{r^*/p}{\eta} + \eta\left(\frac{1}{\beta_1} - 1 - \delta\right)\right)\frac{(1-\alpha)}{\alpha\varepsilon} - \sqrt{2K^* p\left(2\left(\frac{r^*/p}{\eta} - \eta\delta\right) - \frac{\left(\frac{r^*/p}{\eta} + \eta\left(\frac{1}{\beta_1} - 1 - \delta\right)\right)}{\varepsilon}\right)\left(K^* p(\frac{r^*/p}{\eta} - \delta\eta)(1-\alpha) - 2\eta c_0\right)}}{2\left(K^* p(1-\alpha)\left(\frac{r^*/p}{\eta} - \eta\delta + \left(\frac{r^*/p}{\eta} + \eta\left(\frac{1}{\beta_1} - 1 - \delta\right)\right)\frac{(1-\alpha)}{2\alpha\varepsilon}\right) - \frac{2\eta c_0}{\alpha}\right)}$$

$$\tau_2^1 = \frac{\frac{4\eta c_0}{\alpha} - \left(\frac{r^*/p}{\eta} + \eta\left(\frac{1}{\beta_1} - 1 - \delta\right)\right)\frac{(1-\alpha)}{\alpha\varepsilon} + \sqrt{2K^* p\left(2\left(\frac{r^*/p}{\eta} - \eta\delta\right) - \frac{\left(\frac{r^*/p}{\eta} + \eta\left(\frac{1}{\beta_1} - 1 - \delta\right)\right)}{\varepsilon}\right)\left(K^* p(\frac{r^*/p}{\eta} - \delta\eta)(1-\alpha) - 2\eta c_0\right)}}{2\left(K^* p(1-\alpha)\left(\frac{r^*/p}{\eta} - \eta\delta + \left(\frac{r^*/p}{\eta} + \eta\left(\frac{1}{\beta_1} - 1 - \delta\right)\right)\frac{(1-\alpha)}{2\alpha\varepsilon}\right) - \frac{2\eta c_0}{\alpha}\right)}$$

  The denominator is positive for $0 < \alpha < \hat{\alpha_1}$ and negative for $\hat{\alpha_1} < \alpha < 1$, and $\hat{\alpha_1} < 0$

for $\varepsilon > \frac{K^* p\left(\frac{1-\beta_1}{\beta_1}(1+\eta)+(1-\eta)\delta\right)}{4\eta c_0}$.

When $\varepsilon > \frac{K^* p\left(\frac{1-\beta_1}{\beta_1}(1+\eta)+(1-\eta)\delta\right)}{4\eta c_0}$, the denominator is negative. The numerators of τ_1^1 and τ_2^1 are always positive so $\tau_1^1 < 0$ and $\tau_2^1 < 0$ for any value of α and c_{p1}^* increases.

When $\varepsilon < \frac{K^* p\left(\frac{1-\beta_1}{\beta_1}(1+\eta)+(1-\eta)\delta\right)}{4\eta c_0}$, the numerator of τ_1^1 is an increasing function of α, and evaluating the numerator of τ_1^1 at $\hat{\alpha}_1$, we find it is equal to zero. Hence $\tau_1^1 < 0$ for any value of α. The numerator of τ_2^1 has an inverted-U shape in α, and we know that it is negative at the minimum value of α, and positive when $\alpha = 1$. Also, when $\alpha \to \hat{\alpha}$, $\tau_2^1 \to +\infty$, so there exists an $\underline{\alpha}_1 < \hat{\alpha}$ such that $\tau_2^1 > 0$ and a $\overline{\alpha}_1 < \hat{\alpha}$ such that $\tau_2^1 > 1$. We can summarize:

α	c_{p1}^*
$0 < \alpha < \underline{\alpha_1}$	decreases
$\underline{\alpha_1} < \alpha < \overline{\alpha_1}$	has an inverted U-shape
$\overline{\alpha_1} < \alpha$	increases

- $\varepsilon > \varepsilon^*$:

- When $c_0 < \overline{c_0}$, the discriminant is positive and there are 2 roots, τ_1^1 and τ_2^1.
  When $\varepsilon > \frac{K^* p\left(\frac{1-\beta_1}{\beta_1}(1+\eta)+(1-\eta)\delta\right)}{4\eta c_0}$, the denominator is negative and the numerator of τ_1^1 is a decreasing function of α. At the maximum value α can take, the numerator is positive, so the numerator is always positive on the interval and $\tau_1^1 < 0$ always. The numerator of τ_2^1 is always positive, so $\tau_2^1 < 0$ and c_{p1}^* increases.
  When $\varepsilon < \frac{K^* p\left(\frac{1-\beta_1}{\beta_1}(1+\eta)+(1-\eta)\delta\right)}{4\eta c_0}$, the numerator of τ_1^1 has an inverted U shape in α, and is positive at $max(\alpha)$. We know then it crosses 0 once at $\hat{\alpha}_1$, so $\tau_1^1 < 0$ on that interval. The numerator of τ_2^1 is an increasing function of α, and we have the numerator negative when $\alpha \to 0$, and positive when $\alpha \to max(\alpha)$. Hence we know there exists an $\underline{\alpha_1}$ such that the numerator becomes positive. We also know that the numerator is positive at $\hat{\alpha}_1$, so $\underline{\alpha_1} < \hat{\alpha}_1$. When $\alpha \to \hat{\alpha}_1$, $\tau_2^1 \to +\infty$. Therefore, there exists an $\overline{\alpha_1}$ such that on $(\overline{\alpha_1}, \hat{\alpha}_1)$, $\tau_2^1 > 1$. Results can be summarized in the following table:

α	c_{p1}^*
$0 < \alpha < \underline{\alpha_1}$	decreases
$\underline{\alpha_1} < \alpha < \overline{\alpha_1}$	has an inverted U-shape
$\overline{\alpha_1} < \alpha$	increases

- When $c_0 > \overline{c_0}$, the discriminant is negative and $\alpha > \hat{\alpha_1}$, so the polynomial is negative, i.e c_{p1}^* increases.

For the worker

The derivative of c^*_{p2} with respect to τ is a second degree polynomial. Polluting consumption decreases for:

$$\tau^2\left(K^*p(1-\alpha)\left(\frac{r^*/p}{\eta}-\eta\delta\right)+\left(\frac{r^*/p(1-\eta)}{\eta}\right)\frac{(1-\alpha)}{2\alpha(1-\varepsilon)}-\frac{2\eta c_0}{\alpha}\right)+\tau\left(\left(\frac{r^*/p(1-\eta)}{\eta}\right)\frac{(1-\alpha)}{\alpha(1-\varepsilon)}-\frac{4\eta c_0}{\alpha}\right)-\left(K^*p\left(\frac{r^*/p}{\eta}-\eta\delta-\frac{\left(\frac{r^*/p(1-\eta)}{\eta}\right)}{2\alpha(1-\varepsilon)}\right)+\frac{2\eta c_0}{\alpha}\right)>0,$$

which holds when $\left(2\left(\frac{r^*/p}{\eta}-\eta\delta\right)-\frac{\left(\frac{r^*/p(1-\eta)}{\eta}\right)}{1-\varepsilon}\right)\left(K^*p(\frac{r^*/p}{\eta}-\delta\eta)(1-\alpha)-2\eta c_0\right)>0.$
When this is negative, the sign of the polynomial is equal to the sign of

$$K^*p(1-\alpha)\left(\frac{r^*/p}{\eta}-\eta\delta+\left(\frac{r^*/p(1-\eta)}{\eta}\right)\frac{(1-\alpha)}{2\alpha(1-\varepsilon)}\right)-\frac{2\eta c_0}{\alpha} \tag{46}$$

,
which is positive for $0<\alpha<\hat{\alpha}_2$ and negative for $\hat{\alpha}_2<\alpha<1$.

- $\varepsilon<\varepsilon^*$:

- When $c_0<\overline{c_0}$, the discriminant is positive, so there are two roots:

$$\tau_1^2=\frac{\frac{4\eta c_0}{\alpha}-\left(\frac{r^*/p(1-\eta)}{\eta}\right)\frac{(1-\alpha)}{\alpha(1-\varepsilon)}-\sqrt{2K^*p\left(2\left(\frac{r^*/p}{\eta}-\eta\delta\right)-\frac{\left(\frac{r^*/p(1-\eta)}{\eta}\right)}{1-\varepsilon}\right)\left(K^*p(\frac{r^*/p}{\eta}-\delta\eta)(1-\alpha)-2\eta c_0\right)}}{2\left(K^*p(1-\alpha)\left(\frac{r^*/p}{\eta}-\eta\delta+\left(\frac{r^*/p(1-\eta)}{\eta}\right)\frac{(1-\alpha)}{2\alpha(1-\varepsilon)}\right)-\frac{2\eta c_0}{\alpha}\right)}$$

$$\tau_2^2=\frac{\frac{4\eta c_0}{\alpha}-\left(\frac{r^*/p(1-\eta)}{\eta}\right)\frac{(1-\alpha)}{\alpha(1-\varepsilon)}+\sqrt{2K^*p\left(2\left(\frac{r^*/p}{\eta}-\eta\delta\right)-\frac{\left(\frac{r^*/p(1-\eta)}{\eta}\right)}{1-\varepsilon}\right)\left(K^*p(\frac{r^*/p}{\eta}-\delta\eta)(1-\alpha)-2\eta c_0\right)}}{2\left(K^*p(1-\alpha)\left(\frac{r^*/p}{\eta}-\eta\delta+\left(\frac{r^*/p(1-\eta)}{\eta}\right)\frac{(1-\alpha)}{2\alpha(1-\varepsilon)}\right)-\frac{2\eta c_0}{\alpha}\right)}$$

The denominator is positive for $\alpha<\hat{\alpha}_2$ and negative for $\alpha>\hat{\alpha}_2$, with $\hat{\alpha}_2<0$ for $\varepsilon<1-\frac{K^*p\left(\frac{r^*/p}{\eta}\right)(1-\eta)}{4\eta c_0}$.
When $\varepsilon<1-\frac{K^*p\left(\frac{r^*/p}{\eta}\right)(1-\eta)}{4\eta c_0}$, the numerators of τ_1^2 and τ_2^2 are always positive, so $\tau_1^2<0$ and $\tau_2^2<0$ for any value of α: c^*_{p2} increases.
When $\varepsilon>1-\frac{K^*p\left(\frac{r^*/p}{\eta}\right)(1-\eta)}{4\eta c_0}$, the numerator of τ_1^2 is an increasing function of α and is equal to zero at $\hat{\alpha}_2$, so $\tau_1^2<0$ for any value of α. The numerator of τ_2^2 has an inverted U shape in α and is negative when α is close to zero and positive around its maximal value, meaning there exists an $\underline{\alpha}_2<\hat{\alpha}_2$ such that the numerator becomes positive. When $\alpha\to\hat{\alpha}_2$, $\tau_2^2\to+\infty$, meaning there exists $\overline{\alpha}_2<\hat{\alpha}_2$ such that $\tau_2^2>1$. Results can be summarized in the following table:

α	c^*_{p2}
$0<\alpha<\underline{\alpha_2}$	decreases
$\underline{\alpha_2}<\alpha<\overline{\alpha_2}$	has an inverted U-shape
$\overline{\alpha_2}<\alpha$	increases

- When $c_0>\overline{c_0}$, the discriminant is negative and $\alpha>\hat{\alpha}_2$ so c^*_{p2} increases.

- $\varepsilon > \varepsilon^*$:

  - When $c_0 < \overline{c_0}$, the discriminant is negative and $\alpha < \hat{\alpha}_2$ so the polynomial is negative, and c^*_{p2} decreases.
  - When $c_0 > \overline{c_0}$, the discriminant is positive and there are 2 roots τ^2_1 and τ^2_2. In this case, we always have $\varepsilon > 1 - \frac{K^*p\left(\frac{r^*/p}{\eta}\right)(1-\eta)}{4\eta c_0}$, i.e $a > 0$ for $\alpha < \hat{\alpha}_2$ and $a < 0$ otherwise. The numerator of τ^2_1 has an inverted-U shape in α, is negative at the minimum value of α and positive at $\alpha = 1$, so it crosses 0 once at $\hat{\alpha}$. Hence $\tau^2_1 < 0$ always. The numerator of τ^2_2 is an increasing function of α, negative at $min(\alpha)$ and positive at $\alpha = 1$. As before, there exists an $\underline{\alpha_2} < \hat{\alpha}_2$ such that the numerator becomes positive, and an $\overline{\alpha_2} < \hat{\alpha}_2$ such that $\tau^2_2 > 1$. Results are summarized in the following table:

α	c^*_{p2}
$0 < \alpha < \underline{\alpha_2}$	decreases
$\underline{\alpha_2} < \alpha < \overline{\alpha_2}$	has an inverted U-shape
$\overline{\alpha_2} < \alpha$	increases

A.5. Proof of Proposition 4

We have:

$$\frac{\partial P^*_2 C^*_2}{\partial \tau} = K^*p\left(\frac{r^*/p}{\eta} - \delta\right)\frac{\alpha(1-\varepsilon)}{(1+\tau(1-\alpha))^2} + \frac{2(1-\varepsilon)(1+2\tau+2\tau^2(1-\alpha))}{(1+\tau(1-\alpha))^2}c_0$$
$$\frac{\partial P^*_1 C^*_1}{\partial \tau} = K^*p\left(\frac{r^*/p}{\eta} - \delta\right)\frac{\alpha\varepsilon}{(1+\tau(1-\alpha))^2} + \frac{2\varepsilon(1+2\tau+2\tau^2(1-\alpha))}{(1+\tau(1-\alpha))^2}c_0$$

It is straightforward that $\frac{\partial P^*_2 C^*_2}{\partial \tau} > \frac{\partial P^*_1 C^*_1}{\partial \tau}$ for $\varepsilon < \frac{1}{2}$.

A.6. Proof of Proposition 5

Impact on overall consumption

For the capitalist: Taking the derivative of C^*_1 with respect to τ gives the following:

$$\tau^2 c_0(1-\alpha)(2\varepsilon - 1 + \alpha) + 2\tau c_0(2\varepsilon - 1 + \alpha) + K^*p(\frac{r^*/p}{\eta} - \delta)\alpha\varepsilon + c_0(2\varepsilon - 1)$$

Solving for the polynomial gives that $\frac{\partial C^*_1}{\partial \tau} > 0$ for $c_0 < \overline{c_0}$ and $\varepsilon < \frac{1-\alpha}{2}$ or $c_0 > \overline{c_0}$ and $\varepsilon > \frac{1-\alpha}{2}$.

- $c_0 < \overline{c_0}$:
  If $\varepsilon > \frac{1-\alpha}{2}$, C^*_1 increases.
  If $\varepsilon < \frac{1-\alpha}{2}$, the polynomial has two roots:

$$\tau_{11} = \frac{2c_0(1-\alpha-2\varepsilon)-\sqrt{\Delta_1}}{2c_0(1-\alpha)(2\varepsilon-1+\alpha)}$$
$$\tau_{12} = \frac{2c_0(1-\alpha-2\varepsilon)+\sqrt{\Delta_1}}{2c_0(1-\alpha)(2\varepsilon-1+\alpha)}$$

and Δ_1 the discriminant associated to $\frac{\partial C_1^*}{\partial \tau}$. $\tau_{12} < 0$ as the nominator is positive and the denominator negative.
$\tau_{11} > 0$ iff $\varepsilon > \frac{\eta c_0}{2\eta c_0 + K^* p\alpha\left(\frac{1-\beta_1}{\beta_1}+\delta(1-\eta)\right)} \equiv \varepsilon_1$. If not, $\tau_{11} < 0$ and C_1 decreases. Assuming $\varepsilon_1 < \varepsilon < \frac{1-\alpha}{2}$, we must now ensure that $\tau_{11} < 1$.

$$\tau_{11} < 1 \Longleftrightarrow \alpha^2 \eta c_0 + \alpha\left(2\varepsilon\eta c_0 - K^* p\varepsilon\left(\frac{1-\beta_1}{\beta_1}+\delta(1-\eta)\right) - 4\eta c_0\right) + 4\eta c_0(1-2\varepsilon) > 0$$

The polynomial in α has two roots, $\hat{\alpha}$ and $\hat{\hat{\alpha}}$. $\hat{\hat{\alpha}} < 0$ and $0 < \hat{\alpha} < 1$ iff $\varepsilon > \frac{\eta c_0}{6\eta c_0 + K^* p\left(\frac{1-\beta_1}{\beta_1}+\delta(1-\eta)\right)}$, which is always verified under the assumption $\varepsilon > \varepsilon_1$. Hence, we have $\tau_{11} < 1$ for $\alpha < \hat{\alpha}$ and C_1^* has an inverted U-shape, and $\tau_{11} > 1$ for $\alpha > \hat{\alpha}$, leading to an increase in C_1^*.

- $c_0 > \overline{c_0}$:
  If $\varepsilon < \frac{1-\alpha}{2}$, C_1^* decreases.
  $\varepsilon > \frac{1-\alpha}{2}$, the polynomial has two roots $\tau_{11} < 0$ and τ_{12}. $\tau_{12} > 0$ iff $\varepsilon < \varepsilon_1$. If $\varepsilon > \varepsilon_1$, C_1^* increases.
  Assuming $\varepsilon < \varepsilon_1$ and after some computations, we find that $\tau_{12} > 1$, meaning that C_1^* decreases.

For the worker: Taking the derivative of C_2^* with respect to τ gives:

$$\tau^2 c_0(1-\alpha)(2(1-\varepsilon)-1+\alpha) + 2\tau c_0(2(1-\varepsilon)-1+\alpha) + K^* p(\frac{r^*/p}{\eta} - \delta)\alpha(1-\varepsilon) + c_0(1-2\varepsilon)$$

Solving for the polynomial gives that $\frac{\partial C_2^*}{\partial \tau} > 0$ for $c_0 < \overline{c_0}$ and $\varepsilon > \frac{1+\alpha}{2}$ or $c_0 > \overline{c_0}$ and $\varepsilon < \frac{1+\alpha}{2}$.

- $c_0 < \overline{c_0}$:
  If $\varepsilon < \frac{1+\alpha}{2}$, C_2^* increases.
  If $\varepsilon > \frac{1-\alpha}{2}$, the polynomial as two roots:

$$\tau_{21} = \frac{2c_0(1-\alpha-2(1-\varepsilon))-\sqrt{\Delta_2}}{2c_0(1-\alpha)(2(1-\varepsilon)-1+\alpha)}$$
$$\tau_{22} = \frac{2c_0(1-\alpha-2(1-\varepsilon))+\sqrt{\Delta_2}}{2c_0(1-\alpha)(2(1-\varepsilon)-1+\alpha)}$$

  and Δ_2 the discriminant associated to $\frac{\partial C_2^*}{\partial \tau}$. $\tau_{22} < 0$ as the nominator is positive and the denominator negative, and after some computations we find $\tau_{21} > 1$. Hence, C_2^* decreases.

- $c_0 > \overline{c_0}$:
  If $\varepsilon > \frac{1+\alpha}{2}$, C_2^* decreases.

If $\varepsilon < \frac{1+\alpha}{2}$, the polynomial has two roots τ_{21} and τ_{12} tht are both negative. Therefore C_2^* increases.

Impact on welfare

An increase in the commodity tax rate leads to changes in utilities:

$$\frac{dU_i}{d\tau} = \mu \frac{dE^*}{d\tau} E^{*\mu-1} \frac{C_i^{*1-\sigma}}{1-\sigma} + \frac{dC_i}{d\tau} C_i^{*-\sigma} E^{*\mu}$$

The result is straightforward:

- when $c_0 < \overline{c_0}$, if C_i increases, then U_i increases. If C_i decreases, U_i increases if $\mu > \mu_i^*(\tau, \varepsilon)$ and decreases otherwise;
- when $c_0 > \overline{c_0}$, if C_i decreases, then U_i decreases. If C_i increases, U_i decreases if $\mu > \mu_i^*(\tau, \varepsilon)$ and increases otherwise.

A.7. Proof of Proposition 6

Looking at the characteristic polynomial:

$$Pol(-1) = 2(1+D) - \frac{\beta_1 f''(K^*) C_1^* P}{\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha}\right)\left(\sigma - (1+\tau)c_0 C_1^{*-1}\right)}$$
$$Pol(1) = \frac{\beta_1 f''(K^*) C_1^* P}{\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha}\right)\left(\sigma - (1+\tau)c_0 C_1^{*-1}\right)}$$

If $c_0 < c_0^*$, then $\sigma C_1^* - (1+\tau)c_0 > 0$ and we have a saddle.
If $c_0 > c_0^*$, $Pol(1) > 0$ so we must check the sign of $Pol(-1)$ in order to know whether there is indeterminacy of the equilibrium or not.
Looking at $Pol(-1)$, we have $Pol(-1) > 0$ if $c_0 > c_0^F$, meaning the equilibrium is either a sink or a source, depending on whether $D > 1$ or not, i.e. whether $\eta > \overline{\eta}$ or not.
Setting $D = 1$, we have $\overline{\eta} = \frac{\delta}{\frac{1}{\beta_1} - 1 + \delta} - \left(\frac{1}{\beta_1} - 1\right)\left(1 + \frac{2\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)$.

$Pol(-1) = 0$ is a second degree polynomial in c_0. After some computation, we find that the first root is greater than c_{01}. Hence, c_0^F is the second root of this polynomial, and $Pol(-1) < 0$ whenever $c_0 \in (c_0^*, c_0^F)$, and greater than zero otherwise.

c_0^F is given by:

$$\frac{(1+\tau)(1+D)\left(1+\frac{\sigma(1+\tau(1-\alpha-2\varepsilon))}{P(1+\tau(1-\alpha))}\right) - \frac{\beta_1 f''(K^*)K^*}{P\left(1+\frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)}\left(\frac{r^*/p}{\eta}\left(\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right) - \delta\left(1+\frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\right)\frac{(1+\tau)(1+\tau(1-\alpha-2\varepsilon)}{1+\tau(1-\alpha)} - \sqrt{\Delta}}{-\frac{\frac{\beta_1}{p} f''(K^*)}{P\left(1+\frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)}\left(\frac{(1+\tau)(1+\tau(1-\alpha-2\varepsilon)}{1+\tau(1-\alpha)}\right)^2}$$

with Δ the discriminant associated with $Pol(-1)$.

A.8. Expressions for critical values of μ

The trace and determinant of the characteristic polynomial when $c_0 > 0$ and $\mu > 0$ are given by

$$Tr(c_0) = \frac{r^*}{p}\left(\frac{\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}}{1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}}\right) + 2 - \delta + \frac{\frac{\beta_1}{p} f''(K^*) P C_1^* - \alpha\mu \frac{(1-\eta)r^*}{2p} P C_1^*}{\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\left((1+\tau)c_0 C_1^{*-1} - \sigma - \alpha\mu P C_1^*\right)} \tag{47}$$

$$D(c_0) = \frac{r^*}{p}\left(\frac{\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}}{1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}}\right) + 1 - \delta - \frac{\alpha\mu \frac{(1-\eta)r^*}{2p} P C_1^*}{\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\left((1+\tau)c_0 C_1^{*-1} - \sigma - \alpha\mu P C_1^*\right)} \tag{48}$$

$\mu_H(\tau, \varepsilon)$ is such that $D = 1$, which yields

$$\mu_H(\tau,\varepsilon) = \frac{\left((1+\tau)c_0 C_1^{*-1} - \sigma\right)\left(\frac{r^*}{p}\left(\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right) - \delta\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\right)}{\alpha P C_1^* \left(\frac{r^*}{p} - \delta\right)\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)} \tag{49}$$

$\mu_F(\tau, \varepsilon)$ is such that 1+Tr+D=0. We obtain

$$\mu_F(\tau,\varepsilon) = \frac{(1+\tau)c_0 C_1^{*-1} - \sigma}{\alpha P C_1^*} + \frac{\frac{\beta_1}{p} f''(K^*)}{2\alpha\left(\frac{r^*}{p}\left(\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right) + (2-\delta)\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\right)} \tag{50}$$

A.9. Proof of Lemma 2

The derivatives of μ_H and μ_F with respect to ε are given by:

$$\frac{\partial \mu_H}{\partial \varepsilon} = \frac{\left(\sigma - \frac{2(1+\tau)c_0}{C_1^*}\right)\frac{\partial C_1^*}{\partial \varepsilon}\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\left(\frac{r^*}{p}\left(\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right) - \delta\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\right) + \left(\frac{(1+\tau)c_0}{C_1^*} - \sigma\right) C_1^* \frac{r^*(1-\eta)\alpha\tau}{2p(1+\tau(1-\alpha))}}{\alpha P\left(\frac{r^*}{p} - \delta\right)\left(C_1^*\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\right)^2}$$

$$\frac{\partial \mu_F}{\partial \varepsilon} = \frac{\left(\sigma - \frac{2(1+\tau)c_0}{C_1^*}\right)\frac{\partial C_1^*}{\partial \varepsilon}}{\alpha P C_1^{*2}} - \frac{\frac{\beta_1}{p} f''(K^*)\left(\frac{r^*}{p} + (2-\delta)\right)\frac{\tau}{(1+\tau(1-\alpha))}}{2\left(\frac{r^*}{p}\left(\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right) + (2-\delta)\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\right)^2}$$

Both derivatives are positive when $\frac{\partial C_1^*}{\partial \varepsilon} < 0$, i.e when decreasing ε and redistributing more towards workers.

Derivatives with respect to τ yield:

$$\frac{\partial \mu_H}{\partial \tau} = \frac{\left(\left(\sigma - \frac{2(1+\tau)c_0}{C_1^*}\right)\frac{\partial C_1^*}{\partial \tau} + c_0\right)\left(\frac{r^*}{p}\left(\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right) - \delta\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\right)\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right) + \left(\frac{(1+\tau)c_0}{C_1^*} - \sigma\right) C_1^* \frac{r^*(1-\eta)\alpha\varepsilon}{2p(1+\tau(1-\alpha))^2}}{\alpha P\left(\frac{r^*}{p} - \delta\right)\left(C_1^*\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\right)^2}$$

$$\frac{\partial \mu_F}{\partial \tau} = \frac{\left(\sigma - \frac{2(1+\tau)c_0}{C_1^*}\right)\frac{\partial C_1^*}{\partial \tau} + c_0}{\alpha P C_1^{*2}} - \frac{\frac{\beta_1}{p} f''(K^*)\left(\frac{r^*}{p} + (2-\delta)\right)\frac{\varepsilon}{(1+\tau(1-\alpha))^2}}{2\left(\frac{r^*}{p}\left(\frac{1+\eta}{2} + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right) + (2-\delta)\left(1 + \frac{\alpha\varepsilon\tau}{1+\tau(1-\alpha)}\right)\right)^2}$$

also positive when $\frac{\partial C_1^*}{\partial \tau} < 0$, which happens under the condition $\varepsilon < \varepsilon_1$.